S 12

PROBABILITY DISTRIBUTIONS FOR DISCRETE VARIABLES

⇒ A probability distribution is a mathematical model that describes probabilities for all possible outcomes of an experiment or a sample space.

⇒ The sum of all probabilities in any distribution is 1.

⇒ A random variable is a quantity that can have a range of values. A random variable is denoted by a capital X (Y, Z), with individual values designated by a lower-case x (y, z) with a numerical subscript.

- A discrete random variable is a variable that can only have certain values within a given range. (Number of H when a fair coin is tossed 4 times, sum of two numbers when two dice are rolled once, number of students on the Honour Roll, number of years one lived in the Yukon, …).
- A continuous random variable is a variable that can have infinite number of possible values in a given range. (Time needed to complete a test, time spent on commuting to school, time a certain flight is delayed throughout a year, …).

⇒ A probability distribution is often shown as a table/graph of probability versus the value of the random variable. The graph is called a probability histogram.

Example 1:

a)

x	Value of x_i (number rolled on a die)	$P(x)$
x_1	1	$\frac{1}{6}$
x_2	2	$\frac{1}{6}$
x_3	3	$\frac{1}{6}$
x_4	4	$\frac{1}{6}$
x_5	5	$\frac{1}{6}$
x_6	6	$\frac{1}{6}$

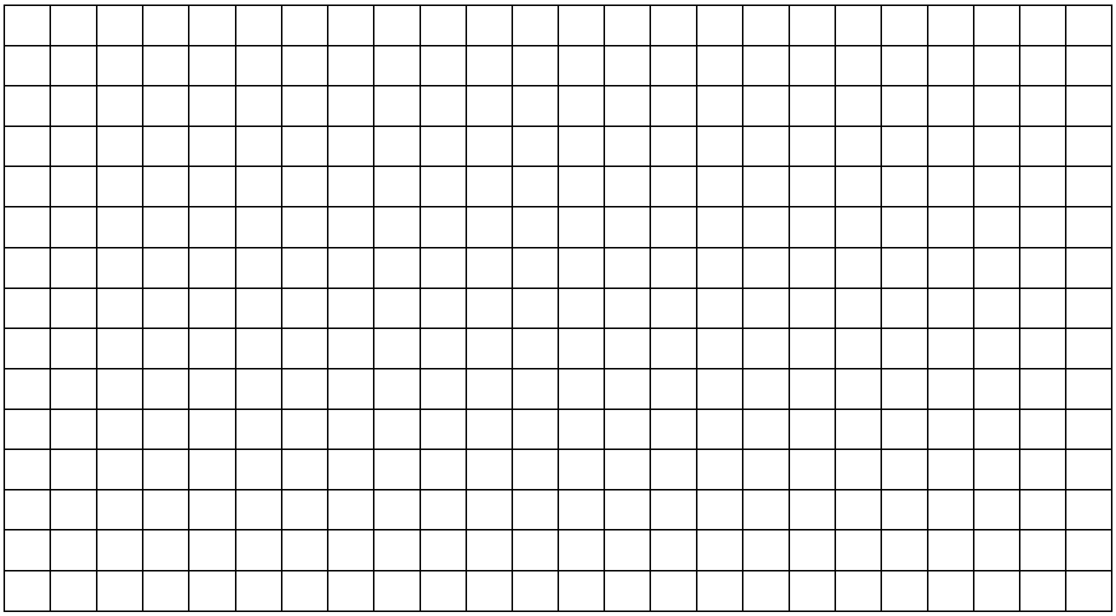
b)

x	Value of x_i (number of siblings)	$P(x)$
x_1	0	
x_2	1	
x_3	2	
x_4	3	
x_5	4	
x_6	5	

⇒ A probability histogram is a graph of a probability distribution in which equal interval are marked on the horizontal axis and the probabilities associated with these intervals are indicated by the areas of the bars.

Example 2:

A probability histogram for the number of siblings



Weighted Mean

⇒ The mean (= average) of a set of numbers that are given weightings based on their frequency.

- Create a frequency table
- Multiply each number (value of the discrete variable) by its weight (= frequency) and divide by the sum of the weights (=number of respondents, experiments,…)

x	Value of x_i (number of siblings)	Frequency
x_1	0	
x_2	1	
x_3	2	
x_4	3	
x_5	4	
x_6	5	

Weighted mean =

<u>Expected Value = expectation = E(X)</u>

- ⇒ **<u>The Expectation</u>** of a probability distribution is the predicted average of all possible outcomes. In other words, it is the weighted average value of the random variable.
- ⇒ It is important to keep in mind that the value of the expectation can be a decimal or a fraction even if the value of the random variable is always integral. Furthermore, the value of the expectation can be an integer that is never possible as a value of individual outcomes.

- ⇒ **<u>Formula:</u>**

Example 3. Find the E(x) of the number of siblings.

UNIFORM DISTRIBUTION

- A uniform distribution occurs when, in a single trial, all outcomes are equally likely.
- For a uniform distribution P(x) = 1/n, where n is the number of possible outcomes in the experiment.
- Formula for E(X) =

  - When calculating E(X), you can find the sum of the numbers from 1 to n using a formula:

Note: The expectation of a fair game is equal to zero.

- Examples of uniform distributions:
  - Rolling a six-sided die once.
  - Tossing a fair coin once.
  - Selecting a single card from a standard deck of cards.
  - Choosing a number from 10 digits available.