S 12

Permutations with Non-Ordered Elements

(Arrangements of items chosen from a selection that includes items of the same type.)

The number of permutations of n objects, when p of one type are identical, q of another type are identical, r of another type are identical, and so on, is given by:

$$n(A) = \frac{n!}{p! \ q! \ r!\dots}$$

Example 1: Compare the number of arrangements of the sets of letters:

a) $AB_1B_2B_3$ and ABBB

b) A₁A₂B₁B₂ and AABB

Example 2:

A hockey team ended its season with 12 wins, 8 losses, and 4 ties. In how many orders could these outcomes happen?

Example 3: Compare the number of orders of the above hockey team's wins, losses, and ties with those of a team that had eight wins, eight losses, and eight ties. Would you expect the number of orders to be higher or lower in this scenario? Why?

If a number of distinct objects need to remain in a specific order in a permutation, divide by the factorial of that number.

Example 1: How many ways are there to arrange the letters in the word NUMBER if the consonants must remain in the original order?

Example 2: How many permutations are the of the letters in the word EXPLAIN if the vowels must be in alphabetical order?