S12

Introduction to Sets and Venn Diagrams

1. A **set** is a collection of items (=elements) that share certain properties.
   - Sets are denoted by capital letters.
   - Items that belong into a particular set are enclosed in braces (=curly brackets) and separated by a comma when listed.

Examples:

2. A **subset** is a collection of none, some or all items in a particular set.
   - Subsets are denoted by capital letters.
   - A set A is a subset of a set B if and only if every element in A is also in B.
     - Notation: ____________
   - A set A is **a proper subset** of a set B if and only if every element in A is also in B, and there exists at least one element in B that is not in A.
     - Notation:_____________
     - Note, by the above definitions, a set can be its own subset.

3. A **universal set** is a collection of everything that has certain properties.
   - Sometimes "…" is used to show that there is a pattern in the list of elements in the set.
   - Notation: _____________

4. An **empty set = Null set** is a set that contains no elements.
   - An empty set is a subset of every set including an empty set.
   - Notation: ___________ or ___________

5. A **<u>Cardinality = order of a set</u>** is the number of elements in the set (= the size of the set).
    - Order in which elements are listed in a set is not important. For example, given A={1,2, 3} and B = {3, 1, 2}, sets A and B are **<u>equal.</u>**

Example:

6. **<u>Union = OR= "in either set or both"</u>**
    - Symbol: __________
    - Example:

7. **<u>Intersection = AND = "only in both sets"</u>**
    - Symbol:____________
    - Example:

8. **<u>Difference = "in one set but not in the other"</u>**
    - Symbol:____________
    - Example:

Venn Diagrams

1. Intersection of events E and F

2. Union of events C and D

3. Mutually Exclusive Events A and B

4. Complement of event A = A^c

Mutually Exclusive and Non-Mutually Exclusive Events

- Mutually events have different properties = events that can never occur at the same time (=simultaneously).

Examples:

Additive Principle = Rule of Sum for Mutually Exclusive Events

The probability of either of two mutually exclusive events, A or B, is given by:

$$\mathbf{P(A\ or\ B) = P(A) + P(B) = P(A \cup B)}$$

Principle of Inclusion and Exclusion

If A and B are non-mutually exclusive events, then the total number of favourable outcomes is given by:

$$\mathbf{n(A\ or\ B) = n(A) + n(B) - n\ (A\ and\ B)}$$

Probability of Non-Mutually Exclusive Events

The probability of either of two non-mutually exclusive events, A or B, is number of given by:

$$\mathbf{P(A\ or\ B) = P(A) + P(B) - P\ (A\ and\ B)}$$

$$\mathbf{P(A \cup B) = P(A) + P(B) - P\ (A \cap B)}$$

Example 1: What is the probability of selecting either hearts or a face card from a regular deck of cards?

Example 2: What is the probability of selecting an ace or a face card from a regular deck of cards?

Independent and Dependent Events

- When one event has no effect on the probability of another event, the events are called **independent.**

- When one event has influence on the probability of another event, the events are called **dependent.**

Multiplicative Principle for Independent Events = Fundamental Counting Principle

The probability of two independent events, A and B, occurring is given by: $$P(A \cap B) = P(A) \times P(B)$$

Example: Three green marbles and two yellow marbles are placed into a bag. What is the probability of randomly drawing a green marble followed by a yellow marble, assuming that the first marble is replaced before the second marble is drawn?

Multiplicative Principle for Dependent Events

The probability of two dependent events, A and B, occurring is given by:

$$P(A \cap B) = P(A) \times P(B|A)$$

In words, the probability of two dependent events, A and B, occurring at the same time is given by the product of the probability of occurring and the **conditional probability** that B occurs given that A already occurred.

- **Conditional Probability =** probability of a second event occurring, given that the first event occurred. The Sample space for the second event is reduced from the first event. This happens during sampling **without replacement.**

Example: A fair coin is flipped twice. What is the probability that it will come up once head and once tails, in either order?