Notes:

S12

Introduction to Sets and Venn Diagrams

1. A **set** is a collection of items (=elements) that share certain properties.
   - Sets are denoted by capital letters.
   - Items that belong into a particular set are enclosed in braces (=curly brackets) and separated by a comma when listed.

Examples: $A = \{$red, white, blue$\}$ = all colours on the USA flag
$B = \{$red, white, blue, yellow$\}$ = all colours on the Filipino flag

2. A **subset** is a collection of none, some or all items in a particular set.
   - Subsets denoted by capital letters.
   - A set A is a subset of a set B if and only if every element in A is also in B.
     - Notation: $\subseteq$ $A \subseteq B$
   - A set A is a proper subset of a set B if and only if every element in A is also in B, and there exists at least one element in B that is not in A.
     - Notation: $\subset$ $A \subset B$
     - Note, by the above definitions, a set can be its own subset.

$\Rightarrow B \subseteq B,\ A \subseteq A$

3. A **universal set** is a collection of everything that has certain properties.
   - Sometimes "…" is used to show that there is a pattern in the list of elements in the set.
   - Notation: U or S ↖ sample set

4. An **empty set = Null set** is a set that contains no elements.
   - An empty set is a subset of every set including an empty set.
   - Notation: $\{\ \}$ or $\emptyset$

5. A **Cardinality = order of a set** is the number of elements in the set (= the size of the set).
   - Order in which elements are listed in a set is not important. For example, given A={1,2, 3} and B = {3, 1, 2}, sets A and B are **equal.**

Example:

$A = \{ \text{red, white} \}$

$\therefore$ the cardinality of A is 2.

6. **Union = OR= "in either set or both"**
   - Symbol: $\cup$
   - Example:

$A = \{ \text{students taking chemistry 12} \}$

$B = \{ \text{students taking physics 12} \}$

- $A \cup B = \{ \text{students in chem 12, or physics 12, or both} \}$

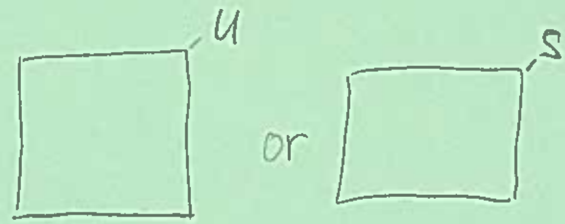
7. **Intersection = AND = "only in both sets"**
   - Symbol: $\cap$
   - Example:

$A \cap B = \{ \text{students who take both chem 12 and physics 12} \}$

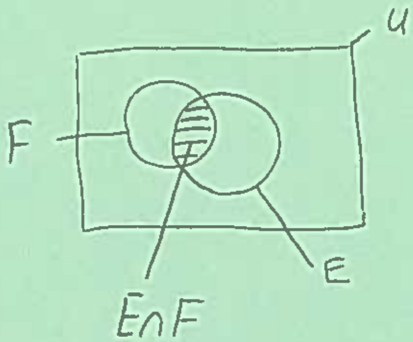
8. **Difference = "in one set bot not in the other"**
   - Symbol: $-$
   - Example:

$A - B = \{ \text{Students that take Chem 12 but not physics 12} \}$.

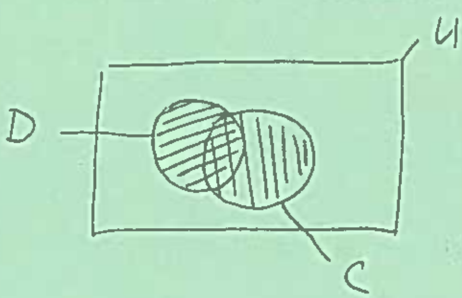
Venn Diagrams



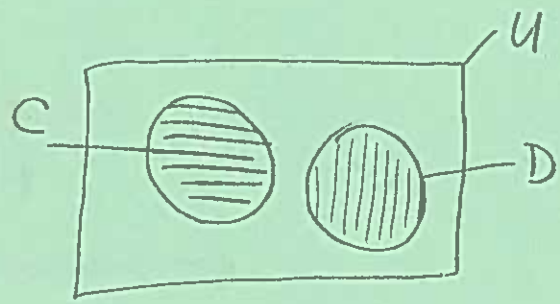
1. Intersection of events E and F



2. Union of events C and D

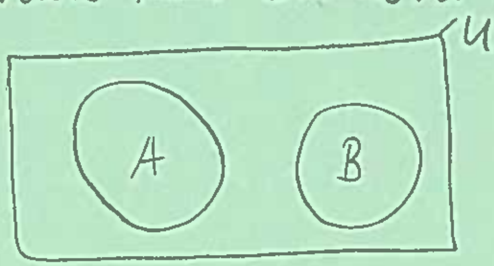


Or

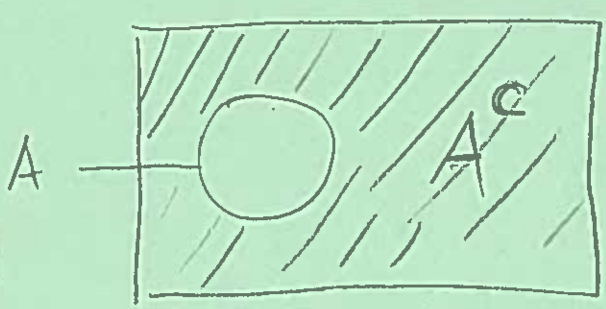


3. Mutually Exclusive Events A and B

- events that can never occur at the same time



4. Complement of event A = A^c $= A' = \overline{A}$



Mutually Exclusive and Non-Mutually Exclusive Events

- Mutually events have different properties = events that can never occur at the same time (=simultaneously).

Examples:

A = happy C = sunny E = even number
B = unhappy D = cloudy F = odd number

Additive Principle = Rule of Sum for Mutually Exclusive Events

The probability of either of two mutually exclusive events, A or B, is given by:

$$P(A \text{ or } B) = P(A) + P(B) = P(A \cup B)$$

Principle of Inclusion and Exclusion

If A and B are non-mutually exclusive events, then the total number of favourable outcomes is given by:

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

Probability of Non-Mutually Exclusive Events

The probability of either of two non-mutually exclusive events, A or B, is number of given by:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 1: What is the probability of selecting either hearts or a face card from a regular deck of cards?

- let A denote the event a heart card is selected.
- let B denote the event a face card is selected.

Find $P(A \cup B)$

- $n(S) = 52$
- $n(A) = 13$
- $n(B) = 12$
- $n(A \cap B) = 3$

- $P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$
- $P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \frac{3}{13}$
- $P(A \cap B) = \frac{3}{52}$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{4} + \frac{3}{13} - \frac{3}{52} = \frac{13+12-3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

∴ the probability of either hearts or a face card is selected is 42.31%.

Example 2: What is the probability of selecting an ace or a face card from a regular deck of cards?

- let A denote the event an ace card is selected.
- let B denote the event a face card is selected.

Find $P(A \cup B)$

- $n(S) = 52$
- $n(A) = 4$
- $n(B) = 12$
- $n(A \cap B) = 0$

$\Rightarrow$ mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$
$$= \frac{4}{52} + \frac{12}{52}$$
$$= \boxed{\frac{16}{52}}$$
$$\doteq 30.77\%$$

∴ the probability of selecting an ace or a face card is 30.77%.

Independent and Dependent Events

- When one event has no effect on the probability of another event, the events are called **independent.**
- When one event has influence on the probability of another event, the events are called **dependent.**

Multiplicative Principle for Independent Events = Fundamental Counting Principle

The probability of two independent events, A and B, occurring is given by:

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

Example: Three green marbles and two yellow marbles are placed into a bag. What is the probability of randomly drawing a green marble followed by a yellow marble, assuming that the first marble is replaced before the second marble is drawn?

- Let G denote the event that a green marble is selected.
- Let Y denote the event that a yellow marble is selected.

Find $P(G \cap Y) = ?$

- Sampling with raplacement
  ⇒ random sampling ⇒ each trial has unchanging probability for event G and event Y regardless of the outcome of previous trials

$$P(G) = \frac{n(G)}{n(S)} = \frac{3}{5}$$

$$P(Y) = \frac{n(Y)}{n(S)} = \frac{2}{5}$$

$$P(G \cap Y) = P(G) \times P(Y)$$
$$= \frac{3}{5} \times \frac{2}{5}$$
$$= \frac{6}{25}$$
$$= 24\%$$

∴ The probability of drawing a green marble followed by a yellow marble is 24%.

Multiplicative Principle for Dependent Events

The probability of two dependent events, A and B, occurring is given by:

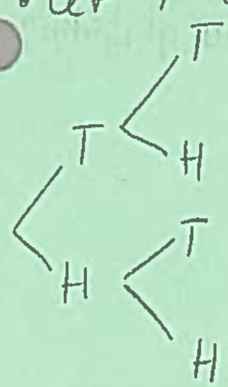
$$P(A \cap B) = P(A) \times P(B|A)$$

In words, the probability of two dependent events, A and B, occurring at the same time is given by the product of the probability of occurring and the **conditional probability** that B occurs given that A already occurred.

- **Conditional Probability** = probability of a second event occurring, given that the first event occurred. The Sample space for the second event is reduced from the first event. This happens during sampling **without replacement.**

Example: A fair coin is flipped twice. What is the probability that it will come up once head and once tails, in either order?

- let H denote the event Heads came up.
- let T denote the event Tails came up.



- $n(S) = 4$
- $n(A) = n(HT) + n(TH) = 2$

- let A denote the event when H happens once and T happens once when a fair coin is flipped twice.

$\rightarrow A = HT$ OR TH (either order).

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2} = 50\%$$

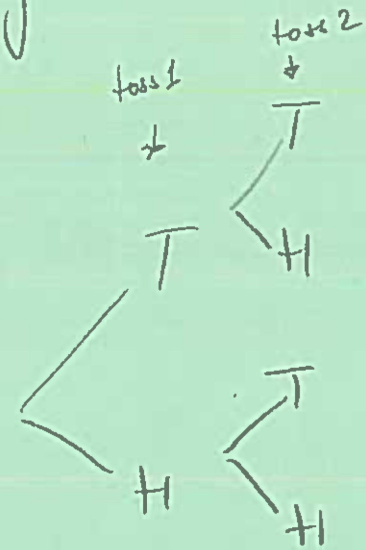
OR

$$P(A) = P(H|T) + P(T|H)$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} = 0.50 = 50\%$$

Why is $P(H|T) = \frac{1}{4}$?



- $n(S) = 4$
- n(getting H on the second toss if T was flipped on the first toss) $= 1$