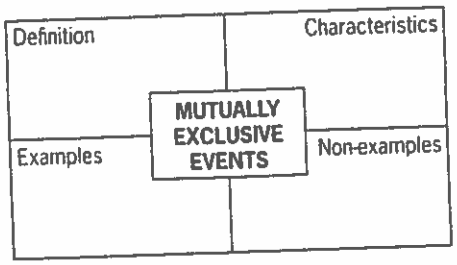
Reflect

R1. Copy and complete the Frayer model shown for the term "mutually exclusive events."



R2. a) What is the principle of inclusion and exclusion?

b) When and why is it important to use it?

R3. Provide an example of non-mutually exclusive events that are different from those already shown.

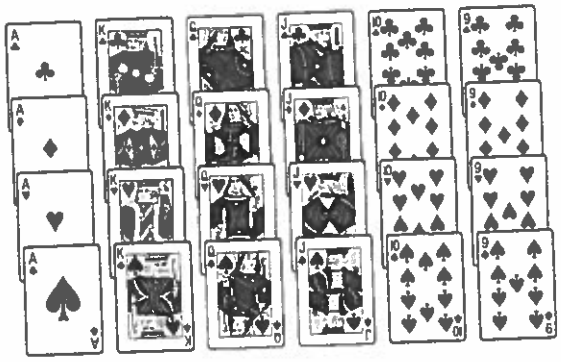
Practise

Choose the best answer for #1 and #2.

1. What is the probability of rolling a 3 or 4 using a standard die?

A $\frac{1}{6}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$

2. The card game euchre uses only the cards shown from a standard deck of playing cards.



What is the probability of randomly drawing an ace or a king from a euchre deck of cards?

A $\frac{5}{12}$ **B** $\frac{1}{2}$ **C** $\frac{7}{12}$ **D** $\frac{1}{3}$

Apply

3. Communication Kara's shirt collection is shown below.

Her shirts are jumbled in a drawer.

a) Determine the probability that Kara randomly draws each of the following:
- a pink shirt or a purple shirt
- a pink shirt or a short-sleeved shirt

b) Which of the scenarios in a) represent:
- a mutually exclusive event?
- a non-mutually exclusive event?

Explain your answers.

4. Application Every Friday night, Rutger's family orders take-out. The table shows their ordering habits for the past several weeks.

Type of Food	Tally
Pizza	~~IIII~~
Mexican	II
Burgers	IIII
Chicken	III

Rutger's favourites are Mexican and chicken. What is the experimental probability that Rutger will get one of his favourites next Friday?

5. What is the probability of rolling a sum that is not a 7 or an 11 with a pair of dice?

6. Refer to the euchre deck of cards in #2.

a) Determine the probability of randomly drawing either an ace or a spade from the deck.

b) What is the probability of randomly drawing a red card or a diamond from the deck?

c) What is the probability of not drawing a face card or a club?

7. Refer to the euchre deck of cards in #2.

a) What is the probability of randomly drawing a 9 or a 10 or a diamond from the deck?

b) Explain how you solved this problem.

8. **Thinking** Deer Button is a game played by people of the Woodland Nations. Players use eight two-colour counters made from deer's antlers, like the ones shown below.

Players take turns throwing all eight deer buttons at the same time. They win beans according to this scoring table:

Number of Buttons of the Same Colour	Beans Awarded
8	10
7	4
6	2
other	0

a) Determine the probability that a player will score 10 points on a given throw.

b) What is the probability of scoring at least 4 points on a throw?

c) Explain how you solved this problem.

✓ Achievement Check

9. Juliette puts these letter tiles into her handbag.

a) If Juliette then reaches into the handbag and randomly takes out one tile, determine the probability of each of the following occurring:
- She chooses an "e" or a "t."
- She chooses a red letter or an "e."
- She chooses a capital letter or a vowel.
- She does not choose a yellow letter or a "t."

b) Draw a Venn diagram to represent each scenario in part a).

c) **Open Question** Create a probability question using these tiles for which the answer is between 25% and 40%.

10. **Open Question** A bag contains three blue marbles and some other marbles. There is a 50% probability that a randomly chosen marble is either green or yellow.

a) What could the contents of the bag be?

b) Provide a different answer that is also correct.

11. **Thinking** Marie is playing a board game and can win if she rolls a sum of either 6 or 8 or doubles with a standard pair of dice. What are the odds against Marie winning on a given throw?

12. **Open Question** Create and solve a probability problem involving mutually exclusive events.

Extend

13. Use algebraic reasoning to prove that the probability of two non-mutually exclusive events, A and B, can be calculated using $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

14. **Application** Renzo knows that his first-semester timetable will include biology, chemistry, English, and a study period, but he does not know when each will occur during the day. Two periods run in the morning and two periods run in the afternoon. The time of day for each course does not change.

a) What is the probability that Renzo will have both science classes in the morning or both in the afternoon?

b) Explain how you solved this problem.

c) Discuss any assumptions you made in your solution.

15. **Thinking**

a) Use algebraic reasoning to develop the additive principle for three non-mutually exclusive events.

b) **Open Question** Design a question that can be solved using the result from part a). Then solve the problem.

Consolidate and Debrief

Key Concepts

- Compound events involve more than one event for a given trial of a probability experiment.
- Independent events have no influence on each other's probability of occurring.
- To calculate the probability of two independent events, A and B, both occurring, multiply the probability of each of them occurring:

  $P(A \text{ and } B) = P(A) \times P(B)$
- When the occurrence or non-occurrence of one event influences the probability of a second event occurring, the events are dependent.
- To calculate the probability of two dependent events, A and B, both occurring, multiply the probability of the first event occurring by the conditional probability of the second event occurring, given that the first event occurred:

  $P(A \text{ and } B) = P(A) \times P(B|A)$

Reflect

R1. **a)** What is the difference between independent events and dependent events?

b) Provide an example of each.

R2. Which of the following scenarios is most likely to occur, and why?
- a coin is flipped three times and comes up heads every time
- after coming up heads four times, a coin comes up heads on the fifth toss

R3. **a)** Explain what is meant by conditional probability.

b) Describe a situation in which conditional probability
- applies
- does not apply

R4. What are some advantages of using a probability tree diagram in solving problems involving dependent events?

Practise

Choose the best answer for #1 to #3.

1. A fair coin is flipped twice. What is the probability that it will come up heads followed by tails?

A 0 **B** $\frac{1}{8}$ **C** $\frac{1}{4}$ **D** $\frac{1}{2}$

2. Hanna forgot to study for her math quiz. In the multiple-choice section there are two questions, each with four answer choices. If Hanna randomly guesses the answer to both questions, what is the probability that she will get them both correct?

A 0% **B** 6.25% **C** 12.5% **D** 25%

3. A fair coin is flipped twice. What is the probability that it will come up once heads and once tails, in either order?

A 0

B $\frac{1}{8}$

C $\frac{1}{4}$

D $\frac{1}{2}$

Apply

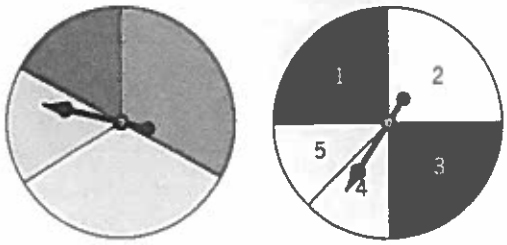
4. Two green tiles, one red tile, and a blue tile are put into a paper bag.

   a) What is the probability that a green tile is drawn, followed by a blue tile, assuming the first tile is replaced before the second tile is drawn?

   b) How does the answer to part a) change if the first tile drawn is not replaced?

   c) Explain why these answers are different.

5. **Thinking** Crazy Spinners is a game in which the two spinners below are spun at the same time.

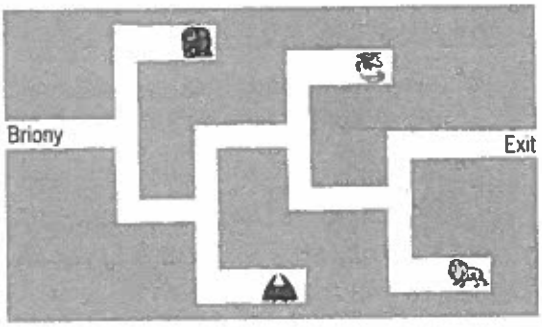


   a) Player A wins a point if the result is Red-1. Player B wins a point if the result is Blue-4. Is this a fair game? Explain.

   b) **Open Question** Change the rules so that this game is almost but not quite fair.

6. Kevin works in car sales. Over a period of time he spoke with 400 customers. The experimental probability of a customer going for a test drive was 20%. If a customer went for a test drive, there was a 5% conditional probability that Kevin made a sale. Assuming that Kevin did not make a sale if there was no test drive, how many sales did Kevin make over this time period?

7. **Application** While playing an online adventure game, Briony finds herself lost in the Maze of Misfortune, as shown below:



Briony is being pursued and has no time to second-guess any of her path decisions.

   a) Assuming she has no knowledge of the maze, what is the probability that Briony will successfully escape the Maze of Misfortune?

   b) What is the conditional probability that Briony will successfully navigate the maze given that she makes
      - her first path decision correctly?
      - her first two path decisions correctly?

8. Refer to Rolly's James Bond movie collection from page 37.

Actor	Number of Movies
Sean Connery	6
George Lazenby	1
Roger Moore	7
Timothy Dalton	2
Pierce Brosnan	4
Daniel Craig	2

Suppose Rolly randomly picks a movie to watch, and then randomly picks a second movie without putting the first movie back on the shelf. Determine the probability of each of the following scenarios:

   a) Rolly will watch a Connery movie followed by a Moore movie.

   b) Rolly will watch two consecutive Dalton movies.

   c) Rolly will watch three consecutive Craig movies.

✓ Achievement Check

9. Petra wants to borrow her dad's car on Saturday, but so does her brother Alek. They decide to play Rock-Paper-Scissors to settle the dispute. Petra decides to go with Rock.
   a) What is the probability that she will win the car on the first trial?
   b) Assuming Petra continues with Rock, how likely is Petra to win the car if she and Alek play until a winner is declared?
   c) Explain why these answers are different.
   d) Is there any single choice that gives an advantage over any other? Explain using probability. What assumptions did you make?

Literacy Link

Rock-Paper-Scissors is a game in which two opponents each make one of the hand signals shown at exactly the same time. The winner is declared as follows:

- Paper covers Rock: Paper wins
- Scissors cuts Paper: Scissors wins
- Rock smashes Scissors: Rock wins

If both players make the same signal, the result is a draw and another trial is conducted.

10. **Thinking** Siko is a contestant on a TV game show called *Win a Million*. Each time she answers a multiple-choice question correctly, she wins money. If she picks a wrong answer, she is eliminated. If Siko does not know the right answer, she can use one of the following Helping Hands:
   - *Quiz the Crowd*: She can poll the audience. The crowd has an experimental probability of being correct 85% of the time.
   - *Double Up*: She can give two answers, instead of just one. If either is correct she stays in the game.
   - *Rule One Out*: One of the incorrect answers is removed, leaving three choices.

   Suppose Siko encounters three questions in a row to which she does not know the answers.
   a) Assuming that she can use each Helping Hand only once during the game, and only once per question, what is the best estimated probability Siko has of staying alive through the three questions? What assumptions did you make.
   b) How many more times is Siko likely to stay in the game if she uses all three Helping Hands than if she simply guesses at random on all three questions?

Extend

11. **Thinking** The Toronto Maple Leafs are facing the Montréal Canadiens in a best of seven playoff series. The first team to win four games wins the series. Ties are broken through sudden decision overtime. Assuming that the teams are evenly matched,
   a) what are the odds in favour of either team sweeping the series, in which one team wins four consecutive games?
   b) what are the odds against the series going a full seven games?

12. **Open Question** Refer to #11. How would your answers change if the teams were not very evenly matched? Pick one team as superior to the other to support your reasoning.

13. Suppose A and B are two dependent events. In general, will $P(A|B) = P(B|A)$? Use a bag of coloured tokens or an alternate scenario to illustrate your answer.

Processes

Reasoning and Proving
Can you use algebraic reasoning to prove that something is true? Can you use a counter-example to prove that something is not true?

14. **Application** In business, a common planning strategy is to use a decision tree.
   a) Research this topic and write a brief report that addresses:
      - What is a decision tree?
      - What elements can it contain?
      - How is it related to probability?
      - Why is it a useful business strategy tool?
   b) Include a real example of a decision tree and use it to support your answers to a).

Chapter 1 Review

Learning Goals

Section	After this section, I can
1.1	• use probability to describe the likelihood of something occurring • measure and calculate simple probabilities
1.2	• calculate theoretical probability
1.3	• recognize the difference between experimental probability and theoretical probability
1.4	• describe how an event can represent a set of probability outcomes • recognize how different events are related • calculate the probability of an event occurring
1.5	• describe and determine how the probability of one event occurring can affect the probability of another event occurring • solve probability problems involving multiple events

1.1 Simple Probabilities, pages 6–15

1. A mystery spinner is spun several times, producing the results shown in the table.

Colour	Count
Blue	24
Green	48
Yellow	51
Purple	26

   a) Calculate the experimental probability of the spinner landing on each colour.
   b) Sketch what this spinner could look like. Explain your reasoning.
   c) Could the spinner look differently? Explain.

2. A quarterback successfully completed 21 of 35 pass attempts.
   a) What is the experimental probability that the quarterback will complete a pass?
   b) Suppose the quarterback throws 280 pass attempts over the course of a season. How many is he likely to complete, based on your answer to part a)?

3. Match each scenario with its most likely subjective probability. Justify your answers.

	Scenario	Subjective Probability, $P(A)$
a)	Canada will win at least one medal at the next Olympics.	0.1 0.25 0.9
b)	A person selected at random will be left-handed.	
c)	A randomly chosen high school student will be in grade 10.	

1.2 Theoretical Probability, pages 16–25

4. What is the theoretical probability of rolling each of the following sums with a pair of dice?
   a) 2 b) 9
   c) not 7 d) not a perfect square

5. A card is randomly drawn from a standard deck of cards. What is the theoretical probability that it will be
   a) a club? b) an ace?
   c) a face card?

6. A sports analyst predicts that a tennis player has a 25% chance of winning a tournament. What are the odds against winning?

1.3 Compare Experimental and Theoretical Probabilities, pages 26–33

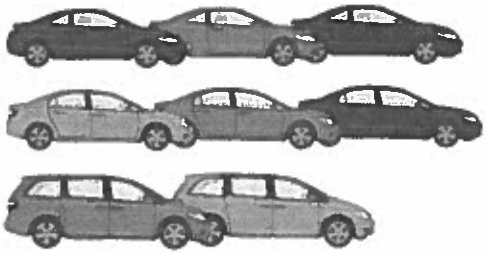
7. A standard die is rolled 24 times and turns up a 3 six times.
   a) What is the experimental probability of rolling a 3 on a given trial?
   b) What is the theoretical probability of rolling a 3?
   c) Explain why these answers are different.

8. Suppose two fair coins are flipped.
   a) Draw a tree diagram to illustrate all possible outcomes.
   b) Sketch a bar graph that shows the predicted relative frequency of each of the following events when a very large number of trials is carried out
      - no heads
      - one head
      - two heads
   c) Explain why your graph has the shape that it does.

1.4 Mutually Exclusive and Non-Mutually Exclusive Events, pages 34–43

9. A graphing calculator is programmed to randomly generate an integer value between 1 and 8. Determine the probability that the number will be
   a) a five or an eight
   b) a prime number or a perfect square
   c) an even number or a seven
   d) not a composite number or an odd number

10. A small vehicle rental company randomly assigns its vehicles to customers based on whatever happens to be available. The fleet is shown below.



Assume that each vehicle has an equal probability of being available at any given time. Determine the probability that a customer will randomly be assigned:
   a) a coupe or a mini-van
   b) a blue vehicle or a mini-van
   c) a grey vehicle or a sedan
   d) not a red vehicle or a coupe

Literacy Link

A *coupe* is a car with two passenger doors.
A *mini-van* is larger than a car but smaller than a van.
A *sedan* is a car with four passenger doors.

1.5 Independent and Dependent Events, pages 44–55

11. A standard die is rolled and a card is drawn from a standard deck of playing cards.
   a) Which of the following is more likely to occur?
      - an even value will be rolled and a heart will be drawn
      - a composite value will be rolled and a face card will be drawn
   b) Justify your answer with mathematical reasoning.

12. A bag has 3 red tiles, 1 yellow tile, and 2 green tiles.
   a) What is the probability that a red tile is drawn, followed by a second red tile, if the first tile is replaced?
   b) How does this value change if the first tile drawn is not replaced?
   c) Explain why these answers are different.

13. Josiah has a 20% experimental probability of hitting the snooze button any morning when his alarm goes off. When he hits the snooze button, there is a 25% conditional probability that he misses his bus. He has never missed the bus when he has not hit the snooze button. If Josiah's alarm woke him 120 times over the course of the semester, how many times did Josiah miss his bus?

Extend

9. Answers may vary.
   a) I chose 5. Draw Cards. You can choose from 1 to 3 decks, with or without replacement, and a 52-card deck or a 32-card deck. Each card is shown in a table along with number and suit.
   b) What is the theoretical probability of drawing a heart from a deck of cards, with replacement? Conduct a large number of trials. How does the experimental probability of drawing a heart from a deck of cards, with replacement, compare?
   The theoretical probability of drawing a heart from a deck of cards, with replacement, is $\frac{1}{4}$.
   For the experimental probability, run repeated trials, save the data, check for the number of hearts (= 1) in the list and divide by the number of trials.
10. Answers may vary.

1.4 Mutually Exclusive and Non-Mutually Exclusive Events, pages 34–43

Example 1 Your Turn

60%

Example 2 Your Turn

50%

Example 3 Your Turn

about 67%

Example 4 Your Turn

50%

Reflect

R1.

Definition cannot occur simultaneously	**Characteristics** events that have different attributes
Examples Coin: either heads or tails, not both. Card: heart, diamond, club or spade, not a combination	**Non-examples** Card: red card and face card

MUTUALLY EXCLUSIVE EVENTS

R2. a) If A and B are non-mutually exclusive events, then the total number of favourable outcomes is: $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.
   b) When events are non-mutually exclusive. It ensures that items are not counted twice.

R3. Answers may vary. Rolling doubles or a sum 6.

Practise

1. C 2. D

Apply

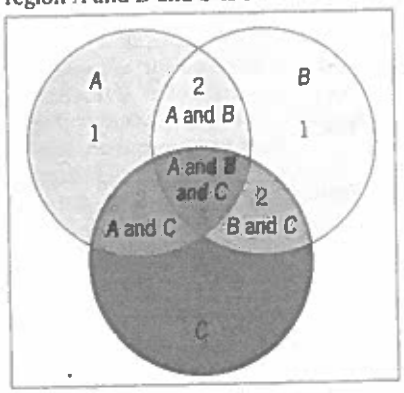
3. a) pink shirt or purple shirt: $\frac{3}{5}$,
   pink shirt or a short-sleeved shirt: $\frac{4}{5}$
   b) first scenario: mutually exclusive events. The shirt cannot be pink and purple.
   second scenario: non-mutually exclusive events. One shirt is pink and short-sleeved.
4. about 36%
5. about 78%
6. a) 37.5% b) 50% c) 62.5%
7. a) 50%
   b) I used the principle of inclusion and exclusion.
8. a) $\frac{1}{128}$ b) $\frac{9}{128}$
   c) There are five possible outcomes to this game: 8 same, 7 same, 6 same, 5 same, or 4 same. If there are 3 the same, then there are 5 of the other colour, and so on. So, the probability of 8 buttons the same colour is 20%. Scoring at least 4 points means 7 or 8 buttons the same colour. So, the probability of 7 or 8 buttons the same colour is 40%.
10. Answers may vary.
   a) Since there are 3 blue marbles and the probability of green or yellow is 50%, there must be at least 3 marbles that belong to the mutually exclusive event of "green or yellow." This could mean that there is 1 green and 2 yellow marbles along with the 3 blue marbles.
   b) Two green and 1 yellow marble along with the 3 blue marbles.
11. 11:7
12. Answer may vary. What is the probability of rolling either doubles or a sum of 5 with a standard pair of dice? $\frac{5}{18}$

Extend

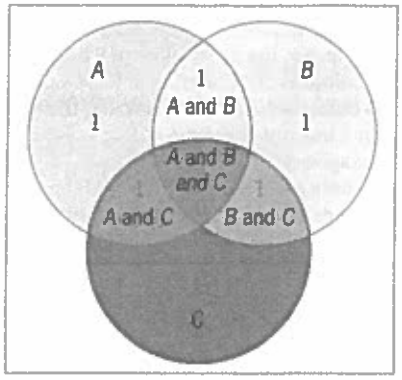
13. From the principle of inclusion and exclusion, $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

$$\begin{aligned} P(A \text{ or } B) &= \frac{n(A \text{ or } B)}{n(S)} \\ &= \frac{n(A) + n(B) - n(A \text{ and } B)}{n(S)} \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \text{ and } B)}{n(S)} \\ &= P(A) + P(B) - P(A \text{ and } B) \end{aligned}$$

14. a) $\frac{1}{3}$
   b) Answers may vary. I solved this using a tree diagram with four time periods and looked for outcomes that included B and C in periods one and two or B and C in periods three and four.
   c) Answers may vary. I assumed that any of Renzo's classes could be in any time period.
15. a) Starting with $n(A) + n(B) + n(C)$, regions A and B, B and C, and A and C will be counted twice, while region A and B and C is counted three times.



Subtract the regions
$n(A \text{ and } B) + n(B \text{ and } C) + n(A \text{ and } C)$:
$n(A) + n(B) + n(C) - n(A \text{ and } B) - n(B \text{ and } C)$
$- n(A \text{ and } C)$



This results in excluding the count for region A and B and C altogether. Add the region A and B and C.
$n(A \text{ or } B \text{ or } C) = n(A) + n(B) + n(C) - n(A \text{ and } B)$
$- n(B \text{ and } C) - n(A \text{ and } C) + n(A \text{ and } B \text{ and } C)$
Then, divide both sides by $n(S)$.
$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$
$- P(A \text{ and } B) - P(B \text{ and } C) - P(A \text{ and } C)$
$+ P(A \text{ and } B \text{ and } C)$

b) Answers may vary. What is the probability of rolling a sum of 6 or doubles or an even sum?
In this case, $n(A) = 5$, $n(B) = 6$, $n(C) = 18$, $n(A \text{ and } B) = 1$, $n(B \text{ and } C) = 6$, $n(A \text{ and } C) = 5$, $n(A \text{ and } B \text{ and } C) = 1$, and $n(S) = 36$.
$P(A \text{ or } B \text{ or } C)$
$= P(A) + P(B) + P(C) - P(A \text{ and } B)$
$- P(B \text{ and } C) - P(A \text{ and } C) + P(A \text{ and } B \text{ and } C)$
$= \frac{5}{36} + \frac{6}{36} + \frac{18}{36} - \frac{1}{36} - \frac{6}{36} - \frac{5}{36} + \frac{1}{36}$
$= \frac{18}{36}$
$= 0.5$
The probability of rolling a sum of 6 or doubles or an even sum is 50%.

1.5 Independent and Dependent Events, pages 44–55

Example 1 Your Turn
6.25%

Example 2 Your Turn
24%

Example 3 Your Turn
25%

Example 4 Your Turn
10%

Example 5 Your Turn
15 sales

Reflect

R1. a) Independent events have no influence on each other's probability of occurring, while dependent events do influence the probability of the other event occurring.

b) Answers may vary. Drawing two cards from a deck with replacement versus drawing two cards from a deck without replacement.

R2. The second scenario is more likely, because the first scenario involves multiple events while the second only involves a single event.

R3. a) Conditional probability is the probability of a second event occurring, given that a first event occurred.

b) Answers may vary. Three green marbles and two yellow marbles are placed into a bag. What is the probability of randomly drawing a second green marble given that a green marble was already chosen?
What is the probability of randomly drawing a green marble followed by a green marble, assuming that the first marble is replaced before the second marble is drawn?

R4. Answers may vary. A probability tree diagram makes it easier to see the event branch of interest and aids in the calculation of probabilities.

Practise

1. C **2.** B **3.** D

Apply

4. a) 12.5% **b)** about 16.7%

c) Part a) involves independent events, while part b) involves dependent events.

5. a) Yes; both players have the same probability of winning a point on a given trial (about 4.2%).

b) Answers may vary. Player A wins a point if the result is Red-1. Player B wins a point if the result is Green or Blue-4. Then, player A has about a 4.2% probability of winning and player B has a $\frac{1}{16}$, or 6.25% of winning.

6. 4 sales

7. a) 6.25%

b) first path decision is correct: 12.5%, first two path decision are correct: 25%

8. a) about 9.1% **b)** about 0.43%

c) 0%

10. a) about 0.14, assume that the crowd's experimental probability of 85% is accurate

b) about 9 times

Extend

11. a) 1:7 **b)** 11:5

12. Answers may vary. In part a), the superior team would have a higher probability of winning. In part b), the probability of playing seven games would decrease.

13. No. In general, $P(A|B)$ will not equal $P(B|A)$.

14. Answers may vary.

Chapter 1 Review, pages 56–57

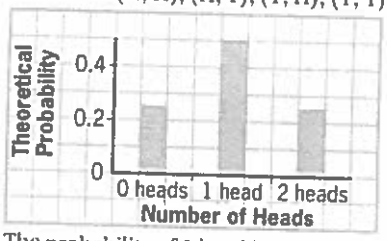
1. a) blue: $\frac{24}{149} \approx 16.11\%$, green: $\frac{48}{149} \approx 32.21\%$, yellow: $\frac{51}{149} \approx 34.23\%$, purple: $\frac{26}{149} \approx 17.45\%$

b) Answers may vary. blue sector: about 58°, green sector: about 116°, yellow sector: about 123°, purple sector: about 63°.

c) Yes; it is based on experimental probability.

2. a) 0.6 **b)** 168 throws

3. a) 0.9; Since Canada has won at least one medal since 1900, the probability is high that we will win at least one medal in the next Olympics.
   b) 0.1; About 10% of the population is left-handed.
   c) 0.25; There are typically four grades in a high school.
4. a) $\frac{1}{36}$ b) $\frac{1}{9}$ c) $\frac{5}{6}$ d) $\frac{29}{36}$
5. a) 25% b) $\frac{1}{13}$ c) $\frac{3}{13}$
6. 3:1
7. a) 25% b) $\frac{1}{6}$
   c) Experimental probability is not a perfect predictor of the outcome of a probability experiment because results of experiments can change. Experimental probability approaches theoretical probability as a very large number of trials are conducted.
8. a) outcomes: (H, H), (H, T), (T, H), (T, T)
   b)



   c) The probability of 1 head is twice that of 0 or 2 heads.
9. a) 25% b) 62.5% c) 62.5% d) 62.5%
10. a) 62.5% b) 50% c) 50% d) 100%
11. a) The first scenario of an even value and a heart will be drawn is more likely to occur.
    b) $P(\text{even number}) = \frac{1}{2}$, $P(\text{heart}) = \frac{1}{4}$.
    So, $P(\text{even number and heart}) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.
    $P(\text{composite number}) = \frac{1}{3}$, $P(\text{face card}) = \frac{3}{13}$.
    So, $P(\text{composite number and face card})$
    $= \frac{1}{3} \times \frac{3}{13} = \frac{1}{13}$
12. a) 25% b) 20%
    c) Part a) involves independent events, while part b) involves dependent events.
13. 6 times

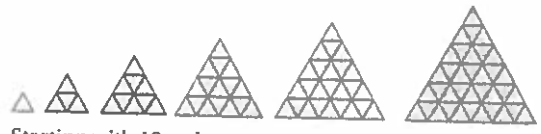
Chapter 1 Test Yourself, pages 58–59

1. C 2. C 3. B 4. 25%
5. a) Since this is Marlis's opinion, it is subjective probability. b) 4:1
6. a) about 8.7% b) about 45.6%
7. a) 11:7
   b) These are mutually exclusive events. There are 6 ways to roll seven, 2 ways to roll 11, and 6 ways to roll doubles. 36 outcomes are possible.
   So, $P(A) = \frac{6}{36} + \frac{2}{36} + \frac{6}{36}$, or $\frac{14}{36}$.
   Then, $P(A') = 1 - \frac{14}{36}$, or $\frac{22}{36}$.
8. a) 25% b) 87.5% c) 62.5% d) 37.5%
9. a) $\frac{1}{12}$ b) $\frac{1}{3}$
10. a) $\frac{1}{3}$ b) $\frac{1}{5}$ c) $\frac{1}{15}$

Chapter 2 Permutations

Prerequisite Skills, pages 62–63

1. a) 0.039, 0.24, 0.5, 0.718 b) 3.0078, 3.078, 3.78
   c) $\frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ d) $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{5}{6}$
2. a) 27.5% b) 490% c) 12 562%
   d) 40% e) 475%
3. $\frac{57}{36}$
4. a) Starting with one triangle, add an increasing number of odd triangles to form a larger triangle.



   b) Starting with 12, subtract 3 continuously.
   12, 9, 6, 3, $3 - 3 = 0$, $0 - 3 = -3$, $-3 - 3 = -6$, ...
   c) Starting with the expression $n - 2$, subtract 1 continuously.
   $n - 2$, $n - 3$, $n - 4$, $n - 4 - 1 = n - 5$, $n - 5 - 1 = n - 6$, $n - 6 - 1 = n - 7$, ...
   d) Starting with $\frac{1}{2}$, multiply the denominator by 2 continuously.
   $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8 \times 2} = \frac{1}{16}, \frac{1}{16 \times 2} = \frac{1}{32}, \frac{1}{32 \times 2} = \frac{1}{64}, \ldots$
5. Answers may vary.
   a) If you view the diagrams as stairs, start with 2 steps then add 1 step continuously to get the next diagram in the pattern. 2, 3, 4, ...
   Starting with a perimeter comprised of 6 line segments, add 2 line segments continuously to get the next diagram in the pattern. 6, 8, 10, ...
   b) 2, 3, 4, 5, 6, ...; 6, 8, 10, 12, 14, ...
6. a) 816 b) 30 c) 15 120
   d) 2850 e) $\frac{65}{81}$ f) $\frac{1}{256}$
7. a) 360 b) 1716
   c) 35 d) approximately 0.005 530
8. a) 60 b) 63 c) 15 d) 722
9. a) $x^3 - 3x^2 + 2x$ b) $2x^2 + 4$
   c) $x + 5$ d) $x^2 - 5x + 6$
10. a) 336 b) 5040 c) 6 d) 2772
11. a) $\frac{1}{6}$ b) $\frac{5}{36}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$ e) $\frac{1}{6}$
12. a) independent, the outcome of flipping a coin does not affect the outcome of rolling a die
    b) dependent, the outcome of dealing a first card affects the second card dealt
    c) independent, the outcome of first die does not affect the outcome of the second die
    d) independent, the outcome of randomly selecting a date does not affect the outcome of randomly selecting someone's name
13. a) $\frac{1}{20}$ b) $\frac{1}{20}$ c) $\frac{2}{20}$ or $\frac{1}{10}$
    d) $\frac{10}{20}$ or $\frac{1}{2}$ e) $\frac{8}{20}$ or $\frac{2}{5}$ f) $\frac{14}{20}$ or $\frac{7}{10}$
14. a) mutually exclusive
    b) non-mutually exclusive
    c) mutually exclusive
    d) non-mutually exclusive
    e) non-mutually exclusive