

# Notes:

S 12

## Permutations and Factorials

- **Permutation** = an arrangement of  $n$  distinct items in a definite order ("order matters" when  $abc \neq bca$ ).
- The number of permutations of  $n$  items is the total number of different arrangements of these distinct items. We write it as:

$$n P_n = n!$$

- **A factorial** = a product of sequential natural numbers written in the form:

$$\begin{aligned} 12! &= (12)(12-1)(12-2) \dots (3)(2)(1) \\ &= 12 \cdot 11 \cdot 10 \dots 3 \cdot 2 \cdot 1 \\ &= (12)(11)(10)! \end{aligned}$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$n! = n(n-1)(n-2)(n-3) \dots (3)(2)(1)$$

- **By agreement**  $0! = 1$

### The Number of permutations of $r$ Items out of $n$ Items

Provided that  $n \geq r$

- **Formula:**

$$n P_r = \frac{n!}{(n-r)!}$$

number of arrangements  
of  $r$  distinct items chosen  
from  $n$  items available

Example 1:

Forty athletes are entered in a triathlon. Medals are presented to the top three finishers. In how many ways could the gold, silver and bronze medals be awarded?

$$n=40$$

$$r=3$$

$$\cdot P_{40}^3 = \underline{\underline{59280}}$$

$$\cdot \text{formula: } \frac{n!}{(n-r)!} = \frac{40!}{(40-37)!} = \frac{40 \cdot 39 \cdot 38 \cdot 37!}{37!}$$

$$\bullet \text{Using the fundamental counting principle: } \underline{\underline{40 \cdot 39 \cdot 38}} = \underline{\underline{59280}}$$

Example 2:

A half-hour TV show has eight 30-second advertisement time slots. In how many ways could the eight advertisements be assigned a time?

$$n=8$$

$r=8 \rightarrow$  all advertisements have to be shown

$$8P_8 = 8! = \underline{\underline{40320}} \quad \text{OR} \quad \frac{n!}{(n-r)!} = \frac{8!}{(8-8)!} = \frac{8!}{1} = \underline{\underline{40320}}$$

Example 3:

Six team photos are hanging on the wall outside a high school gym. Two of the photos are of the junior and senior football teams. In how many ways could they be arranged in a straight line if the two football photos must be beside each other?

$\rightarrow$  consider the 2 football picture as one. They can be arranged in 2 ways ( JF SF ) or ( SF or JF ). So there are  $5P_5 \times 2$  ways

$$= 5! \times 2 = 120 \times 2 = \underline{\underline{240 \text{ ways}}}$$

Example 4:

Write in simplest factorial form:

a)  $(n+5)(n+4)(n+3)!$

$$= (n+5)(n+4)!$$

$$= \underline{\underline{(n+5)!}}$$

B)  $n(n-1)(n-2)!$

$$= n(n-1)!$$

$$= \underline{\underline{n!}}$$

Example 5:

Three players each cut one card from a standard deck. If order is important, in how many ways could they

standard deck 52 cards

13 hearts

4 aces

1 ace is a ♡

a) all be hearts?

$$n = 13$$

$$r = 3$$

$${}_{13}P_3 = \underline{\underline{1716}}$$

b) all aces?

$$n = 4$$

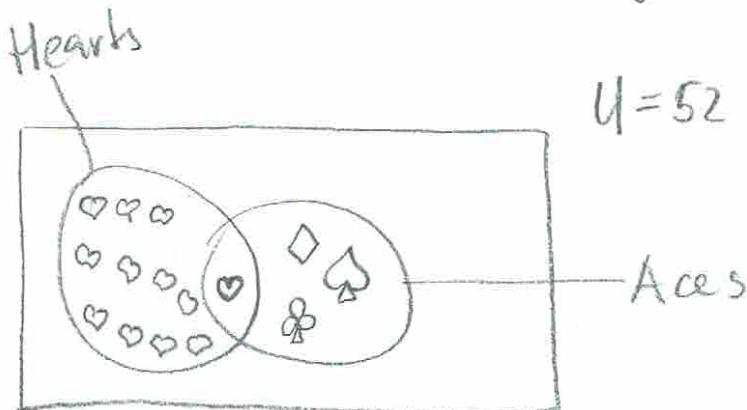
$$n = 3$$

$${}^4P_3 = \underline{\underline{24}}$$

c) all aces or hearts?

= all aces + all hearts - overlap

$$= {}_{13}P_3 + {}^4P_3 - 1$$



$$U = 52$$

$$= \boxed{1739}$$

6 letters 2 vowels

Example 6:

In how many ways could the letters in the word FACTOR be arranged so that the vowels are not together?

- let A denote the event that vowels are not together.

$$n(S) = {}_n P_n = n! = 6! = 720$$

$$n(A^c) = ?$$

- treat AO and OA as one letter

$\Rightarrow$  there are 2 ways to arrange this "letter"

- 4 letters are left

$$- 2 \times {}_5 P_5 = 2 \cdot 5! = 240$$

$$n(A^c) = 240$$

$$\Rightarrow n(A) = 720 - 240 \quad (= n(S) - n(A^c)) \\ = \underline{\underline{480}}$$

$\therefore$  There are 480