ANSWER KEY & NOTES

Factoring by GCF (Greatest Common Factor)

GCF (Greatest Common Factor) is the largest term that divides evenly into all given terms without remainders

Examples

a. $8x^2 + 16$
   1. $8x^2 + 8 \times 2$
   2. $8(x^2 + 2)$

b. $-x^3 + x^2 - 3x = \boxed{-x(x^2 - x + 3)}$

c. $12xy^3 + 18x^2y^2$

$6 \cdot 2 \cdot xy^3 + 6 \cdot 3 \cdot x^2y^2 = \boxed{6xy^2(2y + 3x)}$

d. $25a^4b^2 - 15a^3b^3 + 35a^2b^2$

$5(5a^4b^2 - 3a^3b^3 + 7a^2b^2) = \boxed{5a^2b^2(a^2 - 3ab + 7)}$

Factoring the Difference of Two Squares

$a^2 - b^2 = (a - b)(a + b)$

Examples

a. $x^2 - 25$
   1. $x^2 - 5^2$
   2. a = x and b = 5
   3. $x^2 - 5^2 = (x - 5)(x + 5)$

b. $4a^2 - 49b^2 = (2a)^2 - (7b)^2 = \boxed{(2a + 7b)(2a - 7b)}$

$a = 2a$

$b = 7b$

c. $-y^2 + 81 = 81 - y^2 = 9^2 - y^2 = \boxed{(9+y)(9-y)}$

d. $a^2b^2 - 4c^4 = (ab)^2 - (2c)^2 = \boxed{(ab+2c)(ab-2c)}$

Factoring Trinomials: ax^2 + bx + c

AC Method/Grouping Method

1. a x c = ac
2. Find all integer pairs that give the product **ac**
3. Among the pairs above, find the integer pair that gives the sum **b**: p + q = b
4. Rewrite the trinomial as ax^2 + **px** + **qx** + c
5. Split the polynomial into groups
6. Factor GCF from each group
7. Express as a product of two binomials

Examples

a. $2x^2 - 3x - 14$
   1. a x c = 2 x (-14) = -28
   2. -28 = (1, -28) (-1, 28) (2, -14) (-2, 14) (4, -7) (-4, 7)
   3. (4, -7) -> 4 + (-7) = -3
   4. $2x^2 - 3x - 14 = 2x^2 + 4x + (-7x) - 14$
   5. $(2x^2 + 4x) + (-7x - 14)$
   6. 2x(x + 2) + (-7)(x + 2)
   7. (x + 2)[2x + (-7)] = (x + 2)(2x – 7)

Math Behind Step 7: Let's say (x + 2) = A

1. Substitute (x+2) with A: 2x(x + 2) + (-7)(x + 2) = 2x x A + (-7) x A
2. Express as a product of two binomials: 2x x A + (-7) x A = A(2x + 7)
3. Substitute A with (x + 2): A(2x + 7) = (x + 2)(2x + 7)

b. $6a^2 + 8a - 14$ $a=6$ $b=8$ $c=-14$

1. $a \times c = 6 \times (-14) = -84$

2. $-84 = (-1,84)\ (1,-84)\ (-2,42)\ (2,-42)\ (-3,28)\ (3,-28)\ (-4,21)\ (4,-21)$
$(-6,14)\ (6,-14)\ (-7,12)\ (7,-12)$

3. $(-6,14)$

4. $6a^2 - 6a + 14a - 14$

5. $(6a^2 - 6a) + (14a - 14)$

6. $6a(a-1) + 14(a-1)$

7. $\boxed{(a-1)(6a+14)}$

c. $n + 3n^2 - 2$

$= 3n^2 + n - 2$

$= 3n^2 + 3n - 2n - 2$

$= 3n(n+1) - 2(n+1)$

$= \boxed{(n+1)(3n-2)}$

d. $64p^2 + 32pq - 21q^2$ $a=64$ $b=32$ $c=-21$

$\boxed{(8p-3q)(8p+7q)}$

1. $ac = 64 \times (-21) = -2^6 \times 3 \times 7$

2. $-2^6 \cdot 3 \cdot 7 = (1,-1344)\ (-1,1344)\ (2,-672)\ (-2,672)\ (3,-448)\ (-3,448)$
$(4,-336)\ (-4,336)\ (6,-224)\ (-6,224)\ (8,-168)\ (-8,168)$
$(12,-112)\ (-12,112)\ \cdots\cdots\ (56,-24)$

3. $(56,-24)$

4. $64p^2 + 56pq - 24pq - 21q^2$

5, 6 $8p(8p+7q) - 3q(8p+7q)$

7. $\boxed{(8p+7q)(8p-3q)}$