

Key

Pre-Calculus 11

Word Problems with Rational Equations

Example 1:

The difference of two integers is 5. The sum of their triple is -27. What are the two integers?

• $x \in \mathbb{Z}$ • $x > 5$
• $y \in \mathbb{Z}$

$$x - y = 5 \rightarrow x = 5 + y$$

$$3x + 3y = -27$$

$$3(5 + y) + 3y = -27$$

$$15 + 3y + 3y = -27$$

$$15 + 6y = -27$$

$$\frac{6y}{6} = \frac{-42}{6}$$

$$\boxed{y = -7}$$

$$-7 \in \mathbb{Z} \checkmark$$

$$x = 5 + (-7)$$

$$\boxed{x = -2}$$

$$-2 \in \mathbb{Z} \checkmark$$

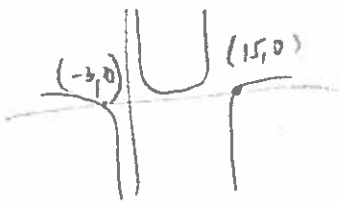
$$-2 > -7 \checkmark$$

∴ The integers are -2 and -7.

Example 2:

The sum of two integers is twelve. The sum of their reciprocals is $\frac{-4}{15}$. What are the two integers?

• $x \in \mathbb{Z}$
• $y \in \mathbb{Z}$



$$x + y = 12 \rightarrow x = 12 - y$$

$$\frac{1}{x} + \frac{1}{y} = \frac{-4}{15}$$

$$\frac{1}{12 - y} + \frac{1}{y} = \frac{-4}{15}$$

$$15y + 15(12 - y) = -4y(12 - y)$$

$$15y + 180 - 15y = -48y + 4y^2$$

$$180 = 4y^2 - 48y$$

$$0 = 4y^2 - 48y - 180$$

$$0 = 4(y^2 - 12y - 45)$$

$$0 = 4(y - 15)(y + 3)$$

$$y - 15 = 0$$

$$\boxed{y = 15}$$

$$y + 3 = 0$$

$$\boxed{y = -3}$$

$$x = 12 - y$$

$$\boxed{x = -3}$$

$$x = 12 - (-3)$$

$$\boxed{x = 15}$$

Even number: $x = 2n$

Consecutive even number: $y = 2n + 2$

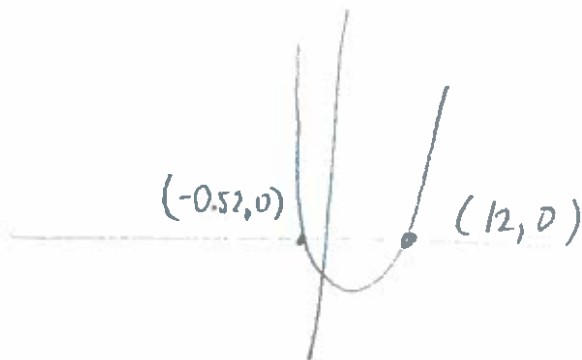
Example 3:

The sum of the reciprocals of two consecutive even numbers is $\frac{25}{312}$. Determine the two numbers algebraically and graphically. Sketch the graph of your solution.

- $x \in \mathbb{Z}$
- $y \in \mathbb{Z}$
- $n \in \mathbb{Z}$

$$\frac{1}{2n} + \frac{1}{2n+2} = \frac{25}{312}$$

$$\frac{1}{2n} + \frac{1}{2(n+1)} = \frac{25}{312}$$



$$\frac{(n+1)(312) + 312n}{2n(n+1)(312)} = \frac{25(2n)(n+1)}{2n(n+1)(312)}$$

$$312n + 312 + 312n = 50n^2 + 50n$$

$$0 = 50n^2 - 574n - 312$$

$$0 = 25n^2 - 287n - 156$$

$$n = 12 \quad \checkmark$$

$$n = -0.52 \quad \times$$

\therefore The integers are 24 and 26

$$x = (12)(2)$$

$$\boxed{x = 24}$$

$$y = 2(12) + 2$$

$$\boxed{y = 26}$$