Name:

UNIT 5 LEARNING GUIDE – RATIONAL EXPRESSIONS

INSTRUCTIONS:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL of your work as is explained in the lessons. Do your best and always ask questions if there is anything that you don't understand.

5.1 RATIONAL EXPRESSIONS

1. Determine whether each of the following are rational expressions. If the expression is not rational, explain why.

a) $\frac{4x+2}{3}$ b) x^2-x-6 c) $\frac{4m-n}{m+2n}$

d) $\frac{2x-\sqrt{x}}{x}$ e) $\frac{-4}{x^3+1}$ f) $\frac{2^x-1}{3x+7}$

2. Determine what should replace $\otimes$ to make the expressions in each pair equivalent.

a) $\frac{5}{6}, \frac{\otimes}{42}$ b) $\frac{2}{3}, \frac{\otimes}{30a}, \ a \neq 0$ c) $\frac{m+1}{m-3}, \frac{4m+4}{\otimes}, \ m \neq 3$

d) $\frac{3}{x+2}, \frac{\otimes}{x(x+2)}, \ x \neq 0,3$ e) $\frac{a}{a+5}, \frac{\otimes}{(a+5)(a-5)}, \ x \neq \pm 5$

3. Identify the non-permissible values for the variables in each rational expression.

a) $\frac{1}{6w}$ b) $\frac{5x}{x^2+3}$ c) $\frac{a-b}{b-1}$

d) $\frac{m+5}{m^2-1}$ e) $\frac{x^2-1}{x^3-8}$ f) $\frac{6x-5}{2x(x+4)}$

g) $\frac{2x+1}{5}$ h) $\frac{3}{(x+2)(x-1)}$ i) $\frac{4(x+1)}{(x+2)(x+1)}$

j) $\frac{3x}{3x^2+6x}$ k) $\frac{x(x+3)}{x^2+7x+12}$ l) $\frac{x+1}{3x^2+8x+5}$

m) $\frac{x+4}{x^2+16}$ n) $\frac{n+5}{x^2-25}$ o) $\frac{x^2-1}{x^3+1}$

4. Make an **equivalent** rational expression for each of the following and state the non-permissible values.

a) $\frac{3(x+5)}{x+4}$

b) $\frac{4(x+2)}{(x+2)(X-1)}$

5.2 SIMPLIFYING

1. Simplify each rational expression. State restrictions on the variable(s) if they exist.

a) $\frac{3m^4}{6m}$ b) $\frac{-12a^3b^5}{4a^2b^7}$ c) $\frac{15x^2y}{-18x^3y^2z}$

d) $\frac{4x+8}{2x+4}$ e) $\frac{2x-10}{3x-15}$ f) $\frac{3a+12}{6a+24}$

2. Simplify each rational expression for permissible values.

a) $\frac{x-5}{10-2x}$ b) $\frac{9-3x}{x-3}$ c) $\frac{2x^2-10x}{4x-20}$

d) $\frac{3x^2-6x}{14-7x}$ e) $\frac{10xy-15x^2y}{6x^2-4x}$ f) $\frac{60a^2b^2-24ab}{16ab-40a^2b^2}$

3. Simplify each rational expression. State restrictions on the variable(s) if they exist.

a) $\frac{x-3}{x^2+3x-18}$ b) $\frac{m^2-7m+10}{m-2}$ c) $\frac{x+4}{x^2-16}$

d) $\frac{a^2+5a-14}{a^2-6a+8}$ e) $\frac{4a^2-16b^2}{4a^2-8ab}$ f) $\frac{x^2+6x+8}{x^2+7x+12}$

4. Simplify each rational expression.

a) $\frac{m^2-9mn+20n^2}{3m^2-15mn}$

b) $\frac{x^2+9xy+18y^2}{2x^2+12xy}$

c) $\frac{8t^2-32}{2t^2+12t+16}$

d) $\frac{3m^2-15m}{3m^2-16m+5}$

e) $\frac{2x^2+3xy+y^2}{3x^2+2xy-y^2}$

f) $\frac{x^2-5x-6}{36-x^2}$

5. Simplify each rational expression.

a) $\frac{25-x^2}{2x^2-9x-5}$

b) $\frac{16+x^2}{x^4+5x^2+4}$

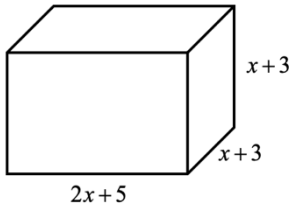
c) $\frac{x^4-5x^2+4}{x^2+x-2}$

d) $\frac{x^2-4xy+4y^2}{x^4-16y^4}$

e) $\frac{x^4-y^4}{(x^2+y^2)(x^2-5xy+4y^2)}$

f) $\frac{16x^4-y^4}{(4x^2+y^2)^2(2x^2+3xy-2y^2)}$

6. Given the following rectangular prism, find an expression for the **ratio** of the volume to the surface area.



7. Two points on a coordinate grid are represented by $A(p,3)$ and $B(2p+1,p-5)$. Write a rational expression for the **slope** of a line passing through *A* and *B*. Write your answer in simplest form.

5.3 MULTIPLYING

1. Simplify the following expressions.

a) $\frac{5}{8} \times \frac{2a}{3}$

b) $\frac{m^2}{4} \times \frac{2}{m}$

c) $\frac{9x^2}{4y^3} \times \frac{8y}{-3x}$

d) $-4 \times \frac{9b^2}{6a}$

e) $\frac{3m^2n}{6mn} \times \frac{4mn}{5m^2n^2}$

f) $\frac{-5x^2y}{(2xy)^3} \times \frac{-12x^2y^2}{-6x^2y}$

2. Simplify the following expressions. State the non-permissible values

a) $\frac{6x}{x+1} \times \frac{2(x+1)}{3x^2}$

b) $\frac{2(4a+3)^2}{4a} \times \frac{12a^3}{3(4a+3)}$

c) $\frac{3(5-c)}{4c} \times \frac{4c+1}{6(c-5)}$

3. Simplify the following expressions

a) $\frac{x^2+x-2}{x^2-x} \times \frac{x^2+x}{x^2-1}$

b) $\frac{x^2-2x-15}{x^2-9} \times \frac{3-x}{x-5}$

c) $\frac{a^2+11ab+30b^2}{a^2-25b^2} \times \frac{3a^2-15ab}{6a^2+36ab}$

d) $\frac{x^2+5xy+6y^2}{x^2+4xy-5y^2} \times \frac{x^2+3xy-10y^2}{x^2+xy-6y^2}$

5.4 DIVIDING

1. Simplify the following expressions.

a) $\frac{x^2}{14} \div \frac{x}{7}$

b) $\frac{-6xy}{15} \div \frac{2x^2}{5}$

c) $\frac{2m}{9n} \div \frac{-4m}{3n^2}$

d) $\frac{5a^2}{12b} \div 25a$

e) $\frac{4c^2d}{8cd} \div \frac{3c^2d^3}{6cd^3}$

f) $\frac{12mn^2}{9mn} \div \frac{(3mn)^2}{6mn^2}$

2. Simplify the following expressions. State the non-permissible values

a) $\frac{3(x-2)}{4(x+5)} \div \frac{6(x-2)}{x+5}$

b) $\frac{(a-3)^2}{14(a-1)} \div \frac{2(a-3)}{7(1-a)}$

c) $\frac{50(x-6)}{24(y-2)} \div \frac{-25(x-6)}{12(2-y)}$

3. Simplify the following expressions

a) $\frac{15m}{2m+6} \div \frac{10m}{3m+9}$

b) $\frac{5x-10}{6x+6} \div \frac{2x-4}{x+1}$

c) $\frac{4a^2-10}{a-3b} \div \frac{6a^2-15}{2a^2-18b^2}$

d) $\frac{y+2}{ay-by} \div \frac{y^2+2y}{ay^2-by^2}$

d) $\frac{a^2-3a-10}{25-a^2} \div \frac{a+2}{a+5}$

e) $\frac{m^2-9mn+14n^2}{m^2+7mn+12n^2} \div \frac{3m^2-21mn}{4m^3+16m^2n}$

4. Simplify the following expressions.

a) $\frac{x+2y}{x-3y} \times \frac{x^2-9y^2}{x^2-4y^2} \div \frac{x+3y}{x-2y}$

b) $\frac{(3a+7b)^2}{2a-5b} \times \frac{4a^2-25b^2}{9a^2-49b^2} \div \frac{2a+5b}{3a-7b}$

c) $\frac{3x+6}{5-x} \div \frac{x^2-4}{x^2-8x+15} \times \frac{x^2-x-2}{x^2+x-12}$

d) $\frac{3x^2+3x-6}{x^2y-7xy} \div \frac{x^2-6x}{x^2} \times \frac{x^2y-13xy+42y}{6x^2+12x}$

e) $\frac{2m^2-7m-15}{2m^2-10m} \div \frac{4m^2-9}{6} \times (3-2m)$

f) $\frac{8x^2-2x-3}{x^2-1} \div \frac{2x^2-3x-2}{2x-2} \div \frac{3-4x}{x+1}$

5. Simplify the following expressions.

a) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-4}$

b) $\dfrac{x-\frac{1}{4}}{x+\frac{1}{4}}$

5.5 Add / Subtract

1. Simplify the following for all permissible values of *x*.

   a) $\frac{5}{x}+\frac{2}{x}-\frac{3}{x}$ b) $\frac{5x-3}{2x}+\frac{7}{2x}-\frac{3x+1}{2x}$ c) $\frac{3x+1}{x-2}+\frac{2x-5}{2-x}$

2. Simplify. Identify any non-permissible values of the variables.

   a) $\frac{3}{2a}-4$ b) $\frac{7}{y+1}-2$ c) $x-\frac{2}{x+4}$

   d) $\frac{4}{x-1}-(x-2)$ e) $x-5+\frac{2}{x-3}$ f) $\frac{2}{x-4}-x-8$

3. Simplify for all permissible values of the variable

   a) $\frac{3y}{2(y+9)}+\frac{5y}{3(y+9)}$ b) $\frac{5}{3(a-7)}-\frac{2}{3(a+1)}$ c) $\frac{3x}{x-2}-\frac{4x}{x-3}$

4. Simplify. Identify any non-permissible values of the variables.

a) $\frac{5x}{10x-15} - \frac{4x}{16x-24}$

b) $\frac{2x+5}{3x-12} - \frac{2x}{4-x}$

c) $\frac{x-7}{x^2-2x-15} - \frac{3x}{x-5}$

d) $\frac{4x+1}{x+3} + \frac{x-6}{x^2-9}$

e) $\frac{3x}{x-1} - \frac{2x}{x^2+x-2}$

f) $\frac{8x-3}{x^2-7x+12} - \frac{2x+1}{x-4}$

5. Simplify.

a) $\frac{x+3}{x^2+11x+24} - \frac{2x+10}{x^2+11x+30}$

b) $\frac{3x+9}{x^2+5x+6} - \frac{2x-2}{x^2+x-2}$

c) $\frac{5m+25}{2m^2+13m+15} - \frac{10m-20}{m^2-4}$

d) $\frac{2x}{3x^2-11x+6} - \frac{3x-12}{3x^2-14x+8}$

6. Simplify.

a) $\frac{x^2-5xy+6y^2}{x-3y} - \frac{x^2-xy-12y^2}{x-4y}$

b) $\frac{a-b}{a^2+2ab-3b^2} + \frac{a+b}{a^2-2ab-3b^2}$

7. Simplify $\left(\frac{p}{p-x}+\frac{q}{q-x}+\frac{r}{r-x}\right)-\left(\frac{x}{p-x}+\frac{x}{q-x}+\frac{x}{r-x}\right)$

5.6 SOLVING.

1. State the value of the variable for which each equation is undefined, then solve it.

a) $\frac{2}{a}=4$ b) $-15=\frac{-3m}{2}$ c) $\frac{90}{x}=\frac{2x}{5}$ d) $\frac{x}{5}+\frac{1}{2}=\frac{1}{4}$

2. Solve.

a) $\frac{3x-2}{2}+4=\frac{13}{x}-\frac{1-6x}{4}$ b) $\frac{2x-3}{3x-4}=\frac{2x+7}{3x+4}$

c) $\frac{3-x}{x-2}=1-\frac{2x-5}{x+2}$ d) $\frac{2(x-1)}{x-3}=\frac{x-4}{x-5}+1$

e) $1-\frac{x-5}{5x-1}=\frac{4(x-3)}{5x-2}$

f) $\frac{3x^2}{x^2-1}=\frac{x}{x+1}+\frac{x}{1-x}$

3. State the value of the variable for which each equation is undefined, then solve it.

a) $\frac{5}{x+1}+\frac{4}{3}=\frac{x+1}{x-1}$

b) $\frac{a}{a+1}=\frac{1}{3}+\frac{a-1}{a+3}$

c) $\frac{3x+2}{2x+1}=\frac{3x+1}{x-1}-\frac{1}{3}$

d) $\frac{2x-3}{x-1}-\frac{x-1}{x+2}=\frac{2x-5}{x+2}+\frac{2-x}{1-x}$

Solving Problems Involving Rational Equations

1. Dividing thirty by one more than an integer is the same as dividing twenty-four by one less than the integer. Find the integer algebraically using rational expressions.

2. Helmut and Elli agree to meet at the "Big Bird Auction" in Vancouver over the weekend. Helmut travels 120*km* in the same time that Elli travels 114*km*. If Helmut's rate of travel is 5*km/h* more than Elli's, determine algebraically at what speed Helmut travels. (remember that d=vt)

3. The speed of a stream is 4*km/h*. If a boat travels 52*km* downstream in the same time that it takes to travel 36*km* upstream, what is the speed of the boat in still water?

4. The sum of two numbers is 10. The sum of their reciprocals is $\frac{5}{12}$. what are the numbers?

5. Two positive consecutive numbers are represented by x and $x+1$. If four is added to the first number and two is subtracted from the second number, the quotient of the new numbers is $\frac{11}{6}$. Determine the numbers algebraically.

6. The sum of the reciprocals of two consecutive integers is $\frac{9}{20}$. Determine the consecutive integers algebraically.

7. How much alfalfa does Helmut need to add to 30*L* of ostrich pellets to make a feed mix that is 40% alfalfa by volume?

UNIT 5 – ANSWER KEY

SECTION 5.1

1. a) Rational b) Rational c) Rational d) Not Rational → "$\sqrt{x}$" not a polynomial
   e) Rational f) Not Rational → "2^x" not a polynomial
2. a) $w \neq 0$ b) $x \in \mathbb{R}$ c) $b \neq 1$ d) $m \neq -1, 1$ e) $x \neq 2$ f) $x \neq 0, -4$
   g) $x \in \mathbb{R}$ h) $x \neq -2, 1$ i) $x \neq -1, -2$ j) $x \neq 0, -2$ k) $x \neq -3, -4$ l) $x \neq \frac{-5}{3}, -1$
   m) $x \in \mathbb{R}$ n) $x \neq \pm 5$ o) $x \neq -1$

SECTION 5.2

1. a) $\frac{m^3}{2}, m \neq 0$ b) $\frac{-3a}{b^2}, a, b \neq 0$ c) $\frac{5}{-6xyz}, x, y, z \neq 0$ d) $2, x \neq -2$
   e) $\frac{2}{3}, x \neq 5$ f) $\frac{1}{2}, x \neq -4$
2. a) $\frac{-1}{2}$ b) -3 c) $\frac{x}{2}$ d) $\frac{-3x}{7}$ e) $\frac{-5y}{2}$ f) $\frac{-3}{2}$
3. a) 35 b) 20a c) 4m – 12 d) 3x e) a(a-5)
4. a) Answers may vary $x \neq -4$ plus other possible values
   b)) Answers may vary $x \neq -2,$) $x \neq 1$ plus other possible values
5. a) $\frac{1}{x+6}, x \neq -6,3$ b) $m - 5, m \neq 2$ c) $\frac{1}{x-4}, x \neq \pm 4$ d) $\frac{a+7}{a-4}, a \neq 2,4$
   e) $\frac{a+2b}{a}, a \neq 0,2b$ f) $\frac{x+2}{x+3}, x \neq -3, -4$
6. a) $\frac{m-4n}{3m}$ b) $\frac{x+3y}{2x}$ c) $\frac{4(t-2)}{t+4}$ d) $\frac{3m}{3m-1}$ e) $\frac{2x+y}{3x-y}$ f) $-\left(\frac{x+1}{x+6}\right) = \frac{-x-1}{x+6}$
7. a) $-\frac{(x+5)}{(2x+1)} = \frac{-x-5}{2x+1}$ b) $\frac{16+x^2}{(x^2+4)(x^2+1)}$ c) $(x+1)(x-2)$
   d $\frac{x-2y}{(x+2y)(x^2+4y^2)}$ e) $\frac{x+y}{x-4y}$ f) $\frac{2x+y}{(x+2y)(4x^2+y^2)}$
8. $\frac{(x+3)(2x+5)}{2(5x+13)}$ 7. $m = \frac{p-8}{p+1}$

SECTION 5.3

1. a) $\frac{5a}{12}$ b) $\frac{m}{2}$ c) $\frac{-6x}{y^2}$ d) $\frac{-6b^2}{a}$ e) $\frac{2}{5n}$ f) $\frac{-5}{4xy}$ 2.a) $\frac{4}{x}, x \neq 0, x \neq -1$ b) $2a^2(4a+3), a \neq 0, a \neq -\frac{3}{4}$ c) $\frac{-4c-1}{8c}; c \neq 0, c \neq 5$ 3. a) $\frac{x+2}{x-1}$ b) -1
   c) $\frac{1}{2}$ d) $\frac{x+2y}{x-y}$

SECTION 5.4

1. a) $\frac{x}{2}$ b) $-\frac{y}{x}$ c) $\frac{-1n}{6}$ d) $\frac{a}{60b}$ e) 1 f) $\frac{8n}{9m}$ 2. a) $\frac{1}{8}$, $x \neq -5$, $x \neq 2$ b) $\frac{-(a-3)}{4} = \frac{-a+3}{4}$, $a \neq 1$, $a \neq 3$ c) $1, y \neq 2, x \neq 6$ 3. a) $\frac{9}{4}$ b) $\frac{5}{12}$ c) $\frac{4a+12b}{3}$ d) 1 e) -1 f) $\frac{4m(m-2n)}{3(m+3n)}$ 4. a) 1 b) $3a+7b$ c) $\frac{-3(x+1)}{x+4}$ d) $\frac{x-1}{2x}$ e) $\frac{-3}{m}$ f) $\frac{-2}{x-2}$
5. a) $\frac{1+4x}{1-4x}$ b) $\frac{4x-1}{4x+1}$

SECTION 5.5

1. a) $\frac{4}{x}$ b) $\frac{2x+3}{2x}$ c) $\frac{x+6}{x-2}$
2. a) $\frac{3-8a}{2a}$, $a \neq 0$ b) $\frac{5-2y}{y+1}$, $y \neq -1$ c) $\frac{x^2+4x-2}{x+4}$, $x \neq -4$ d) $\frac{-x^2+3x+2}{x-1}$, $x \neq 1$ e) $\frac{x^2-8x+17}{x-3}$, $x \neq 3$ f) $\frac{-x^2-4x+34}{x-4}$, $x \neq 4$
3. a) $\frac{19y}{6(y+9)}$ b) $\frac{3a+19}{3(a-7)(a+1)}$ c) $\frac{-x^2-1x}{(x-2)(x-3)}$
4. a) $\frac{x}{2(2x-3)}$, $x \neq \frac{3}{2}$ b) $\frac{8x+5}{3(x-4)}$, $x \neq 4$ c) $\frac{-3x^2-8x-7}{(x-5)(x+3)}$, $x \neq 5, -3$ d) $\frac{4x^2-10x-9}{(x+3)(x-3)}$, $x \neq -3,3$ e) $\frac{3x^2+4x}{(x-1)(x+2)}$, $x \neq -2,1$ f) $\frac{-2x^2+13x}{(x-4)(x-3)}$, $x \neq 4,3$
5. a) $\frac{-x-10}{(x+8)(x+6)}$ b) $\frac{1}{x+2}$ c) $\frac{-15m-20}{(2m+3)(m+2)}$ d) $\frac{-x+9}{(3x-2)(x-3)}$
6. a) $-5y$ b) $\frac{2a}{(a+3b)(a-3b)}$
7. 3

SECTION 5.6

1. a) $a = \frac{1}{2}$, $a \neq 0$ b) $m = 10$, $m \in \mathbb{R}$ c) $x = \pm 15$, $x \neq 0$ d) $x = \frac{-5}{4}$, $x \in \mathbb{R}$
2. a) $x = 4$ b) $x = \frac{8}{7}$ c) $x = \frac{5}{2}$ d) $x = \frac{17}{3}$ e) $x = \frac{5}{19}$ f) $x = 0, \frac{-2}{3}$
3. a) $x = -11, 2$, $x \neq -1, 1$ b) $a = 0, 5$, $a \neq -1, 3$ c) $x = -2, -\frac{5}{7}$, $x \neq -\frac{1}{2}, 1$ d) $x = 4$, $x \neq 1, -2$

Lesson Six Solving Problems Involving Rational Equations

1. $x = 9$ 2. Helmut = 100*km/h* 3. Boat = 22*km/h* 4. 4 and 6 5. 7 and 8 6. 4 and 5 7. 20*L*