Radicals

Simplifying Radicals

1. Without using a calculator, express each radical as a mixed radical in simplest form.

   a) $\sqrt{24}$

   b) $\sqrt{40}$

   c) $-3\sqrt{50}$

   d) $2\sqrt{160}$

   e) $\sqrt[3]{40}$

   f) $\sqrt[3]{54}$

   g) $\sqrt[3]{375}$

   h) $3\sqrt[4]{32}$

   i) $\sqrt{18x^3}$

   j) $\sqrt{20a^4b}$

   k) $\sqrt{75x^8y^4}$

   l) $\sqrt[3]{16a^5b^6}$

2. Without a calculator, express each radical as an entire radical.

   a) $3\sqrt{2}$

   b) $2\sqrt{5}$

   c) $-4\sqrt{7}$

   d) $5\sqrt{6}$

   e) $10\sqrt{2}$

   f) $2\sqrt[3]{2}$

g) $3\sqrt[3]{3}$

h) $2\sqrt[4]{5}$

i) $10\sqrt[4]{7}$

j) $3a^2\sqrt{5b}$

k) $5x^3y\sqrt{2}$

l) $2a^2b\sqrt[3]{7}$

3. Determine where the following radicals are undefined and then write them as mixed radicals.
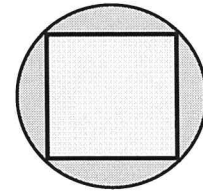
a) $\sqrt{45x^3}$

b) $\sqrt[3]{16x^5}$

c) $\sqrt[4]{-32a^{13}}$

4. A square is inscribed in a circle. The area of the circle is $48\pi \ cm^2$
   a) What is the exact length of the diagonal of the square?

b) Determine the exact perimeter of the square.

Adding and Subtracting Radicals

1. Without using a calculator, simplify by adding or subtracting like terms.

   a) $\sqrt{3} - 3\sqrt{3} + 4\sqrt{3} + 7\sqrt{3}$

   b) $\sqrt{2} + 5\sqrt{2} - \sqrt{8}$

   c) $-\sqrt{12} + \sqrt{3} + \sqrt{75}$

   d) $3\sqrt{2} - \sqrt{50} + 4\sqrt{75} + 2\sqrt{8}$

   e) $\sqrt[3]{81} + \sqrt[3]{24} - 7\sqrt[3]{3}$

   f) $-\sqrt{28} + \sqrt{24} + \sqrt{63} + \sqrt{96}$

2. Simplify.

   a) $3\sqrt{x} + 4\sqrt{x} - 2\sqrt{x}$

   b) $-7\sqrt{a} + \sqrt{9a} - \sqrt{16a}$

   c) $\sqrt[3]{-8b} + 5\sqrt[3]{b} + \sqrt[3]{64b}$

   d) $\sqrt{121xy^2} + 2\sqrt{49xy^2} - 3\sqrt{25xy^2}$

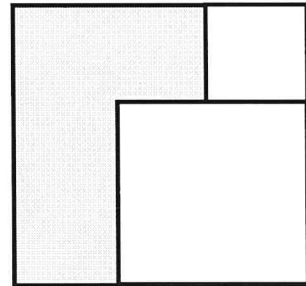3. Identify restrictions on the variable(s), then simplify the radical.

   a) $3\sqrt{-a} + \sqrt{-16a} + 7\sqrt{-a}$

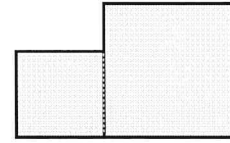   b) $\sqrt[3]{8x^5 y} + 5\sqrt[3]{x^5 y} - x\sqrt[3]{27x^2 y}$

   c) $2\sqrt{3b} + \sqrt[3]{24b^2} + \sqrt{27b} - \sqrt{12b}$

   d) $5\sqrt{a^3} + 3\sqrt{2a} + \sqrt{4a^3}$

4. Two squares are enclosed in a large square as shown. The area of the smallest square is $5cm^2$, and the area of the middle square is $20cm^2$. Determine the area and perimeter of the shaded region.

5. A square with area of $18cm^2$ is placed beside a square with area $28cm^2$ write a simplified expression for the perimeter of the shape.

Multiplying Radicals

1. Without using a calculator, expand and simplify if possible.

a) $\sqrt{3}\left(\sqrt{5}+4\right)$

b) $\sqrt{6}\left(2-\sqrt{7}\right)$

c) $\sqrt{7}\left(-3+\sqrt{2}\right)$

d) $-\sqrt{11}\left(\sqrt{3}-7\right)$

e) $\sqrt{8}\left(\sqrt{2}+1\right)$

f) $-2\sqrt{3}\left(\sqrt{3}+\sqrt{6}\right)$

g) $\left(\sqrt{2}-3\right)\left(\sqrt{2}+3\right)$

h) $\left(\sqrt{3}-5\right)^2$

i) $\left(\sqrt{7}+\sqrt{3}\right)^2$

j) $\left(2\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+3\sqrt{3}\right)$

k) $\left(\sqrt{2}+3\right)\left(3\sqrt{2}-1\right)-\sqrt{2}\left(4\sqrt{2}-5\right)$

l) $\left(3\sqrt{5}+2\sqrt{3}\right)^2+\left(2\sqrt{5}-\sqrt{3}\right)\left(2\sqrt{5}+\sqrt{3}\right)$

m) $-\left(3\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+4\right)-\left(2\sqrt{3}-\sqrt{2}\right)^2+5$

2. Determine restrictions for each variable, then expand and simplify.

   a) $\sqrt{a}\left(5\sqrt{a}-3\right)$

   b) $\left(2\sqrt{m}+7\right)\left(3\sqrt{m}-1\right)$

c) $\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)$

d) $\left(2\sqrt{a}-\sqrt{b}\right)^2$

e) $\left(\sqrt{x}-3\sqrt{y}\right)\left(3\sqrt{x}-5\sqrt{y}\right)-\left(2\sqrt{x}-\sqrt{y}\right)^2$

Dividing and Rationalizing Radicals

1. Divide the following. Leave answers in simplest radical form.

a) $\dfrac{\sqrt{40}}{\sqrt{5}}$

b) $\dfrac{8\sqrt{24}}{2\sqrt{6}}$

c) $\dfrac{3\sqrt{60}}{\sqrt{27}}$

2. Rationalize the denominator.

a) $\dfrac{30}{\sqrt{10}}$

b) $\dfrac{7}{2\sqrt{5}}$

c) $6\sqrt{\dfrac{5a}{3}}$

3. Simplify and rationalize the denominator.

a) $\dfrac{7\sqrt{3}+2}{\sqrt{3}}$

b) $\dfrac{2\sqrt{5}-5\sqrt{3}}{\sqrt{6}}$

c) $\dfrac{-2\sqrt{7}+3\sqrt{3}}{2\sqrt{10}}$

d) $\dfrac{-2\sqrt{3}-3\sqrt{2}}{5\sqrt{6}}$

e) $\dfrac{-3}{5+\sqrt{7}}$

f) $\dfrac{\sqrt{6}}{-\sqrt{8}-\sqrt{10}}$

g) $\dfrac{7+\sqrt{3}}{1-\sqrt{3}}$

h) $\dfrac{\sqrt{2}+\sqrt{5}}{-\sqrt{7}+\sqrt{2}}$

i) $\dfrac{-\sqrt{7}+2\sqrt{6}}{2\sqrt{3}+\sqrt{2}}$

4. Simplify and rationalize the denominator.

a) $2 + \dfrac{1}{\sqrt{11}}$

b) $\dfrac{1}{\sqrt{3}} - \dfrac{2}{\sqrt{5}}$

c) $\dfrac{3}{\sqrt{8}} + \dfrac{\sqrt{5}}{\sqrt{12}}$

d) $\dfrac{5}{\sqrt{2} - \sqrt{3}} + \dfrac{1}{2}$

e) $\dfrac{2}{\sqrt{5} + \sqrt{2}} + \dfrac{3}{\sqrt{5} - \sqrt{2}}$

f) $\dfrac{\sqrt{2}}{\sqrt{3} + 2\sqrt{6}} + \dfrac{4}{\sqrt{3} - 2\sqrt{6}}$

5. Rationalize the denominator and simplify.

   a) $\dfrac{2}{\sqrt[3]{5}}$

   b) $\dfrac{21}{\sqrt[4]{3}}$

   c) $\dfrac{10+\sqrt[3]{4}}{\sqrt[3]{2}}$

Solving Radical Equations

1. Determine the restrictions on x for the following equations.

   a) $\sqrt{x}+2=9$

   b) $3\sqrt{-x}=12$

   c) $\sqrt{x-5}=7$

   d) $\sqrt{3-x}=2$

   e) $\sqrt{3x+12}=2$

   f) $\sqrt{x+7}=\sqrt{4-x}$

2. Solve each equation algebraically, and verify the solution.

   a) $\sqrt{2x}=8$

   b) $\sqrt{-x}+5=7$

   c) $2\sqrt{3x}=12$

   d) $30=5\sqrt{2x}$

e) $7 = 2\sqrt{3x} - 1$

f) $\sqrt{2x-1} = 3$

g) $\sqrt{x+3} = 5$

h) $\sqrt{3-x} + 1 = 4$

i) $2\sqrt{3x} + 5 = 6$

j) $2\sqrt{x} + 5 = 3$

k) $\sqrt{3x+4} - 5 = -1$

l) $-4 = -\sqrt{-x+2}$

m) $3\sqrt{x+1}=1$

n) $2\sqrt{1-x}=3$

o) $5=2\sqrt{x}+9$

p) $2\sqrt{3x-4}+5=6$

q) $1-3\sqrt{2-x}=-3$

r) $\dfrac{\sqrt{3x}}{2}+1=5$

3. Solve each equation algebraically.

a) $\sqrt{\dfrac{1}{2}x} = \sqrt{x-3}$

b) $\sqrt{5-x} = \sqrt{x+2}$

c) $2\sqrt{x-2} = \sqrt{3x+5}$

4. The Sun approximates a sphere of radius 695,800 km.

   a) The formula for the surface area of a sphere is $SA = 4\pi r^2$. Determine the SA of the sun.

   b) To the nearest kilometer, determine the edge length of a cube ($SA=6s^2$) that would have the same surface area as the Sun.

   c) The formula for the volume of a sphere is $V = \dfrac{4}{3}\pi r^3$. Determine the V of the sun.

   d) To the nearest kilometer, determine the edge length of a cube ($V = s^3$) that would have the same volume as the Sun.

5. Determine the root of each equation algebraically.

a) $\sqrt[3]{x+1}=3$

b) $\sqrt[3]{2-x}+3=5$

c) $2\sqrt[4]{3x}=6$

d) $\sqrt[4]{5x+1}+7=9$

e)
$\sqrt[3]{1+7x}+9=6$

f) $\dfrac{\sqrt[4]{x+40}}{3}=1$

6. Solve the following equation exactly.

    a) $\sqrt{x}+1=\sqrt{x+2}$

    b) $3-\sqrt{x-8}=\sqrt{x-5}$

7. Five more than two times the square root of a number is fifteen. Find the number.

8. The square root of four more than three times a number is seven. Find the number.

ANSWERS

Real Numbers and Absolute Values

1. Answers will vary.   2. a) 7   b) 3   c) 2.17   d) $\frac{2}{3}$

3. a) $|2.2-(-2)|=4.2$ b) $|-0.8-1.1|=1.9$ c) $|-2-1.1|=3.1$

4. a) 5.3   b) $5\frac{1}{4}$ c) 9   5. Answers will vary.

6. a) 4   b) 1   c) -45 d) 7   e) 9   f) 22   g) $\frac{7}{24}$

h) $\frac{19}{20}$ i) $\frac{17}{12}$ j) $\frac{5}{3}$ k) $\frac{2}{5}$ l) $\frac{15}{8}$

Simplifying Radicals

1. a) $2\sqrt{6}$ b) $2\sqrt{10}$ c) $-15\sqrt{2}$ d) $8\sqrt{10}$ e) $2\sqrt[3]{5}$ f) $3\sqrt[3]{2}$ g) $5\sqrt[3]{3}$

h) $6\sqrt[4]{2}$ i) $3|x|\sqrt{2x}$ j) $2a^2\sqrt{5b}$ k) $5x^4y^2\sqrt{3}$ l) $2ab^2\sqrt[3]{2a^2}$

2. a) $\sqrt{18}$ b) $\sqrt{20}$ c) $-\sqrt{112}$ d) $\sqrt{150}$ e) $\sqrt{200}$ f) $\sqrt[3]{16}$ g) $\sqrt[3]{81}$

h) $\sqrt[4]{80}$ i) $\sqrt[4]{70000}$ j) $\sqrt{40a^4b}$ k) $\sqrt{50x^6y^2}$ l) $\sqrt[3]{56a^6b^3}$

3. a) $0\leq x,\ 3x\sqrt{5x}$ b) $x\in\square,\ 2x\sqrt[3]{2x^2}$ c) $a\leq 0,\ 2|a^3|\sqrt[4]{-2a}$ 4. a) $8\sqrt{3}\,cm$ b) $16\sqrt{6}\,cm$

Adding and Subtracting Radicals

1. a) $9\sqrt{3}$ b) $4\sqrt{2}$ c) $4\sqrt{3}$ d) $2\sqrt{2}+20\sqrt{3}$ e) $-2\sqrt[3]{3}$ f) $\sqrt{7}+6\sqrt{6}$

2. a) $5\sqrt{x}$ b) $-8\sqrt{a}$ c) $7\sqrt[3]{b}$ d) $10|y|\sqrt{x}$

3. a) $a\leq 0,\ 14\sqrt{-a}$ b) $x,y\in\square,\ 4x\sqrt[3]{x^2y}$ c) $0\leq b,\ 2\sqrt[3]{3b^2}+3\sqrt{3b}$ d) $0\leq a,\ 7|a|\sqrt{a}+3\sqrt{2a}$

4. $A=20cm^2,\ P=10\sqrt{5}\ cm$ 5. $P=6\sqrt{2}+8\sqrt{7}\ cm$

Multiplying Radicals

1. a) $\sqrt{15}+4\sqrt{3}$ b) $2\sqrt{6}-\sqrt{42}$ c) $-3\sqrt{7}+\sqrt{14}$ d) $-\sqrt{33}+7\sqrt{11}$

e) $4+2\sqrt{2}$ f) $-6-6\sqrt{2}$ g) -7 h) $28-10\sqrt{3}$

i) $10+2\sqrt{21}$ j) $1+5\sqrt{15}$ k) $-5+13\sqrt{2}$ l) $74+12\sqrt{15}$

m) $-18+5\sqrt{6}-12\sqrt{3}+4\sqrt{2}$

2. a) $0\leq a,\ 5a-3\sqrt{a}$ b) $0\leq m,\ 6m+19\sqrt{m}-7$ c) $0\leq x, 0\leq y,\ x-y$

d) $0\leq a, 0\leq b,\ 4a-4\sqrt{ab}+b$ e) $0\leq x, 0\leq y,\ -x-10\sqrt{xy}+14y$

Dividing and Rationalizing Radicals

1. a) $2\sqrt{2}$ b) 8   c) $2\sqrt{5}$ 2. a) $3\sqrt{10}$ b) $\frac{7\sqrt{5}}{10}$ c) $2\sqrt{15a}$

3. a) $\frac{21+2\sqrt{3}}{3}$ b) $\frac{2\sqrt{30}-15\sqrt{2}}{6}$ c) $\frac{-2\sqrt{70}+3\sqrt{30}}{20}$ d) $\frac{-\sqrt{2}-\sqrt{3}}{5}$ e) $\frac{-5+\sqrt{7}}{6}$

f) $2\sqrt{3} - \sqrt{15}$ g) $-5 - 4\sqrt{3}$ h) $\dfrac{2 + \sqrt{14} + \sqrt{35} + \sqrt{10}}{-5}$ i) $\dfrac{12\sqrt{2} - 4\sqrt{3} + \sqrt{14} - 2\sqrt{21}}{10}$

4. a) $\dfrac{22 + \sqrt{11}}{11}$ b) $\dfrac{5\sqrt{3} - 6\sqrt{5}}{15}$ c) $\dfrac{9\sqrt{2} + 2\sqrt{15}}{12}$ d) $\dfrac{-10\sqrt{2} - 10\sqrt{3} + 1}{2}$

   e) $\dfrac{5\sqrt{5} + \sqrt{2}}{3}$ f) $-\dfrac{3\sqrt{6}}{7}$

5. a) $\dfrac{2\sqrt[3]{25}}{5}$ b) $7\sqrt[4]{27}$ c) $5\sqrt[3]{4} + \sqrt[3]{2}$

Solving Radical Equations

1. a) $0 \le x$ b) $x \le 0$ c) $5 \le x$ d) $x \le 3$ e) $-4 \le x$ f) $-7 \le x \le 4$

2. a) 32   b) -4 c) 12   d) 18   e) $\dfrac{16}{3}$ f) 5   g) 22

h) -6 i) $\dfrac{1}{12}$ j) $x = 1$, but extraneous   k) 4   l) -14 m) $-\dfrac{8}{9}$

n) $-\dfrac{5}{4}$ o) $x = 4$, but extraneous   p) $\dfrac{17}{12}$ q) $\dfrac{2}{9}$ r) $\dfrac{64}{3}$

3. a) 6   b) $\dfrac{3}{2}$ c) 13

4. a) $6.08 \times 10^{12} \text{km}^2$ b) $1{,}006{,}964 \text{km}$ c) $1.41 \times 10^{18} \text{km}^3$ d) $1{,}121{,}624 km$

5. a) 26   b) -6 c) 27   d) 3   e) -4 f) 41

6. a) $\dfrac{1}{4}$ b) 9   7. 25   8. 15