Name:

UNIT 7 LEARNING GUIDE – TRIGONOMETRY

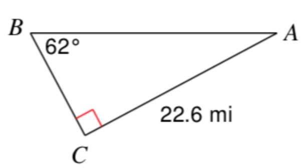
INSTRUCTIONS:

Using a pencil complete the following questions as you work through the related lessons. Show ALL of your work as is explained in the lessons. Do your best and always ask questions if there is anything that you don't understand.

REVIEW

Solve the following triangles. (Find the unknown sides and angles)

1.



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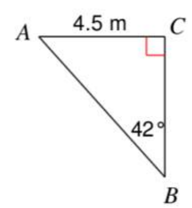

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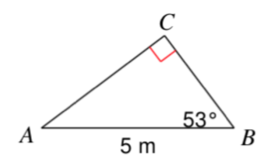

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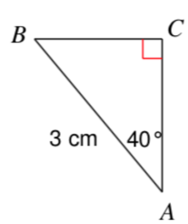

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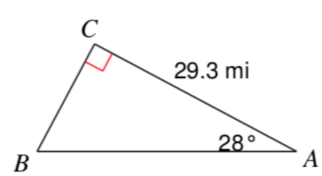

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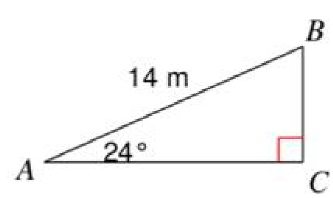


7.1 Angles in Standard Position

1. In your own words define the following:
   Standard Position Angle
   ____

   ____

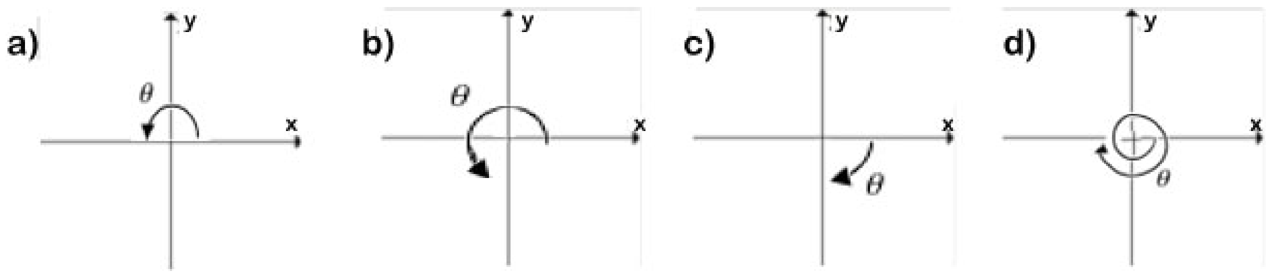
   Terminal Arm:
   ____

   ____

   Reference Angle:
   ____

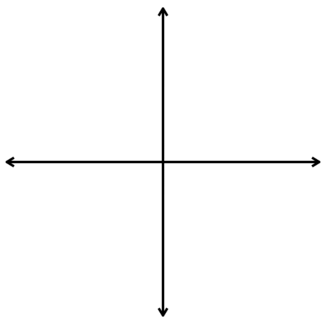
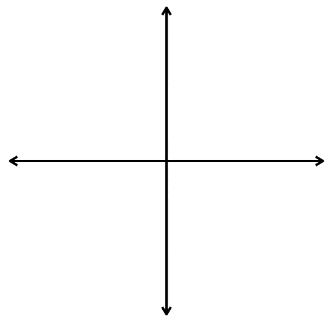
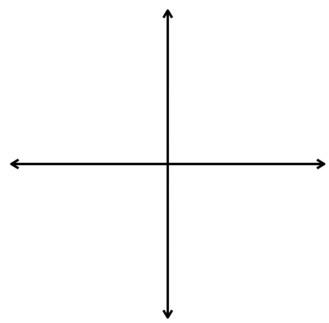
   ____

2. An angle in standard position is shown. What is the value of θ?

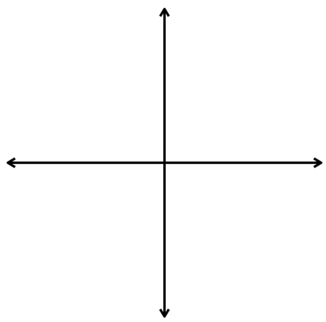


   __________ __________ __________ __________

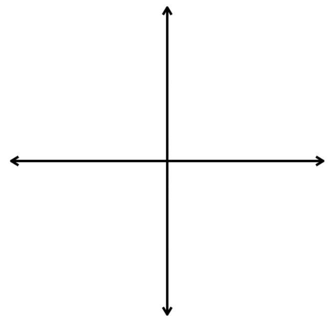
3. Draw each angle in standard position and determine its reference angle.
   a) θ = 200° b) θ = 170° c) θ = 300°

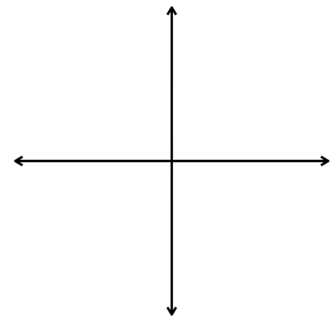
d) θ=45°



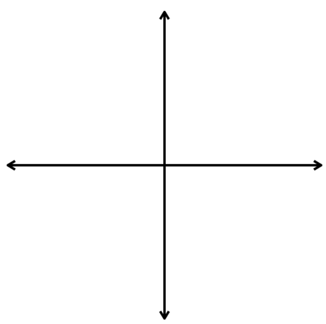
e) θ=510°



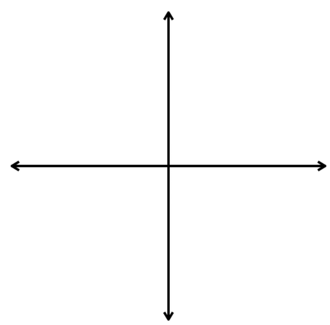
f) θ=−50°



g) θ=−173°



h) θ=−280°



7.2 Trig Ratios

1. Each point *P* is on the terminal arm of a standard position angle θ. Calculate the exact value of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each point. Be sure to rationalize any denominator

   a) P(2,3) b) P(-3,5) c) P(1, -6)

   d) P(-12,-5) e) P(3,-7) f) P(-5,2)

   g) P(a,b)

3. For each angle θ, find the **exact** values of the other two trigonometric ratios.

a) $\sin\theta = \frac{2}{3}$, with θ in Quadrant II

b) $\cos\theta = -\frac{2}{\sqrt{29}}$, with θ in Quadrant III

c) $\tan\theta = \frac{4}{3}$, with $\sin\theta < 0$

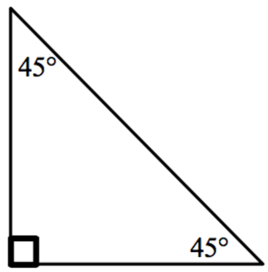
d) $\sin\theta = -\frac{2}{\sqrt{13}}$, with $\tan\theta < 0$

e) $\cos\theta = \frac{-7}{8}$, $180° < \theta < 270°$

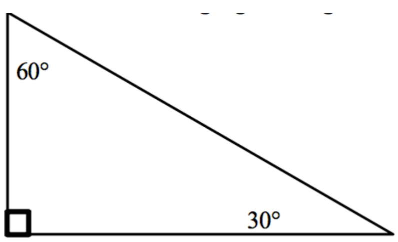
f) $\sin\theta = a$, with θ in Quadrant II
(express in terms of a)

7.3 EXACT TRIG RATIOS

1. Given the following right triangle, determine the exact values of sine, cosine, and tangent of 45°.



2. Given the following right triangle, determine the exact values of:



a) sine, cosine and tangent of 30°

b) sine, cosine and tangent of 60°

3. Without using a calculator, determine the exact values for the following trigonometric ratios. You may want to draw the angles in standard position and find their reference angles.
   a) Cos 135° b) sin 240° c) tan 210°

d) cos 330° e) sin 315° f) tan 120°

g) cos 240° h) sin 150° i) tan 225°

4. Evaluate the following **without** the use of a calculator.

a) -2 tan 210° b) $\sqrt{2}$ cos 135° + 1

c) cos 135° + sin 240° d) $(\sin(-60°))^2$

e) $(\sin 135° + \cos 315°)^4$ f) $2\cos 225° - \sqrt{2}$

7.4 Solving Trig Equations

1. Find each angle to one decimal place if $0° \leq \theta < 360°$ for each of the following.
   a) $\cos\theta = 0.53$
   b) $\tan\theta = 1.314$
   c) $\sin\theta = 0.65$
   d) $\tan\theta = 0.976$
   e) $\sin\theta = 0.784$
   f) $\cos\theta = -0.235$
   g) $\tan\theta = -0.243$
   h) $\sin\theta = -0.543$
   i) $6\sin\theta + 1 = 0$
   j) $-\cos\theta + 2 = 0$
   k) $\tan\theta - 5 = 0$
   l) $9\tan\theta + 7 = 0$

2. Find the exact value of each angle if $0° \leq \theta < 360°$.

a) $\sin\theta = \frac{1}{\sqrt{2}}$

b) $\sin\theta = -\frac{\sqrt{3}}{2}$

c) $\tan\theta = 1$

d) $\cos\theta = -\frac{1}{2}$

e) $\cos\theta = \frac{\sqrt{3}}{2}$

f) $\cos\theta = \frac{1}{\sqrt{2}}$

g) $\sin\theta = -\frac{1}{\sqrt{2}}$

h) $\tan\theta = -\sqrt{3}$

i) $2\sin\theta - \sqrt{2} = 0$

j) $2\cos\theta + \sqrt{3} = 0$

k) $\tan\theta + 1 = 0$

l) $3\sqrt{2}\cos\theta + 3 = 0$

3. If $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta < 0$ and $0° \leq \theta < 360°$ then determine the exact value of θ.

4. If $\tan\theta = -1$, $\sin\theta < 0$ and $0° \leq \theta < 360°$ then determine the exact value of θ.

5. Solve for θ , if $0\,° \leq \theta < 360°$. Use exact value triangles where appropriate, otherwise leave answers rounded to two decimal spots.

a) $2\sin^2\theta - 1 = 0$

b) $3\tan^2\theta - 1 = 0$

c) $5\cos\theta - \sqrt{3} = 3\cos\theta$

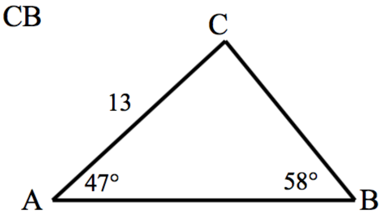
d) $6\cos^2\theta - \cos\theta - 1 = 0$

7.5 SINE LAW

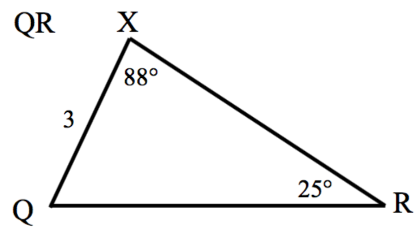
*Note: The diagrams are **not** drawn to scale.

1. Solve for the length of each indicated side.

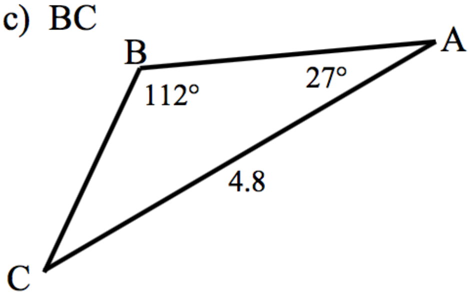
a) CB



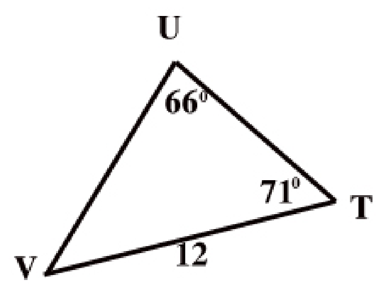
b) QR



c) BC

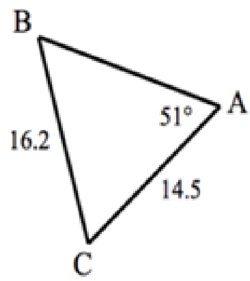


d) UV

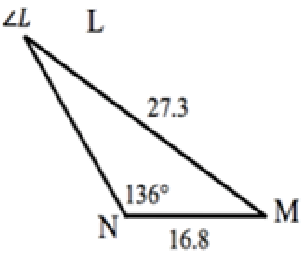


2. For each triangle, determine the indicated angle to the nearest tenth of a degree.

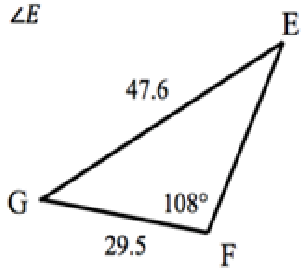
a) $\angle B$



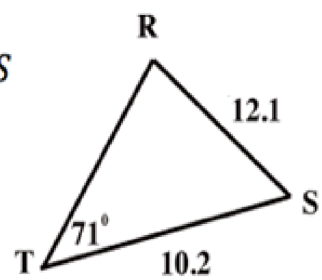
b) $\angle L$



c) $\angle E$

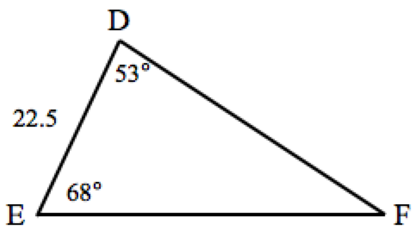


d) $\angle S$

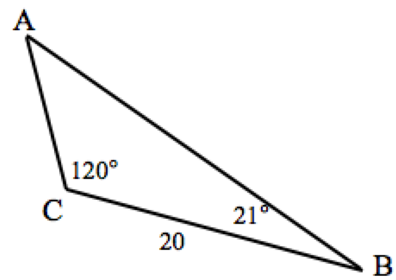


3. Determine the measure of each unknown side to the nearest tenth

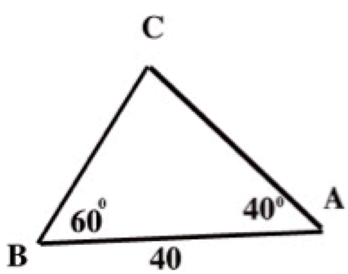
a)



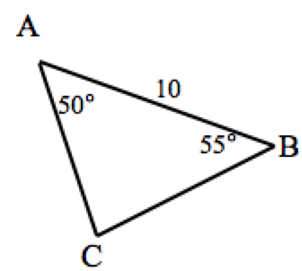
b)



c)



d)



4. Helmut's angle of elevation to the top of a building is 32°. Elli is standing 105*m* closer to the building, directly in front of Helmut. Elli's angle of elevation to the top of the building is 40°. Determine the height of the building.

5. A telephone pole is supported by two wires on opposite sides. At the top of the pole, the wires form an angle of 60°. On the ground, the ends of the wires are 15.0*m* apart. One wire makes a 45° angle with the ground.
   a) How long are the wires to the nearest tenth of a meter?

   b) How tall is the telephone pole?

6. A surveyor in an airplane is flying towards a lake. She observes that the angles of depression to the near side of the lake and the far side of the lake are 45° and 32° respectively. At that moment, her airplane is exactly 9750*m* from the near side of the lake. Determine the width of the lake to the nearest meter.

7. A radar operator on a ship discovers a large sunken vessel lying parallel to the ocean surface, 200*m* directly below the ship. The length of the vessel is a clue to which wreck has been found. The radar operator measures the angles of depression to the front and back of the sunken vessel to be 56° and 62°. How long, to the nearest tenth of a meter, is the sunken vessel?

8. A hot air balloon is flying above the mall. Elli is standing due north of the mall and can see the balloon at an angle of inclination of 64°. Helmut is due south of the mall and can see the balloon at an angle of inclination of 49°. The horizontal distance between Helmut and Elli is 500*m*. Determine the distance that the hot air balloon is from Elli.

9. Elli decided to ski to a friend's cabin. She skied 10.0*km* in the direction N40°E. She rested, then skied S45°E and arrived at the cabin. The cabin is 14.5*km* from her home, as the crow flies. Determine, to the nearest tenth of a kilometer, the distance she travelled on the second leg of her trip.

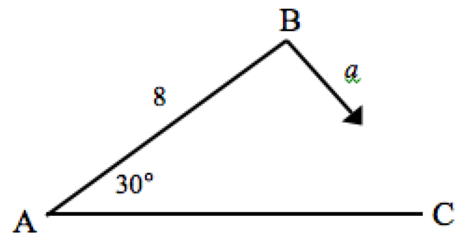
10. Elli is sailing from Bear Creek Provincial Park on Okanagan Lake to Rattlesnake Island. She had planned to sail 26.0*km* in the direction S71°E; however, the wind and current pushed her off course. After several hours, she discovered that she had actually been sailing S79°E. She checked her map and saw that she must sail S18°W to reach Rattlesnake Island. Determine, to the nearest tenth of a kilometer, the distance remaining to Rattlesnake Island.

11. The interior angles of a triangle measure 120°, 40°, and 20°. The longest side of the triangle is 10*cm* longer than the shortest side. Determine the perimeter of the triangle, to the nearest centimeter.

7.6 THE AMBIGUOUS CASE

1. In ΔABC, determine the range of values of a that will give:
   a) 0 triangles

   b) 1 triangle



   c) 2 triangles

2. Determine the number of possible triangles that could be drawn with the given measures. Draw the triangle(s). Then, find the measures of the other angles and sides in each possible triangle. Show your work clearly.
   a) ΔABC, where $\angle A=42°$, $a=30$ cm, and $b=25$ cm

   b) ΔABC, where $\angle B=27°$, $b=25$ cm, and $c=30$ cm

c) ΔPQR, where $\angle P=30°$, $p=24$ cm, and $q=48$ cm

d) ΔKLM, where $\angle M=37.3°$, $m=85$ cm, and $l=90$ cm

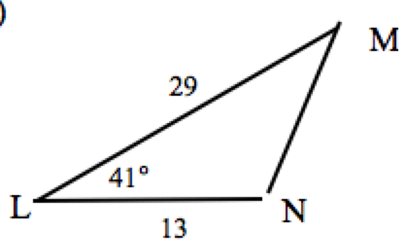
e) ΔUVW, where $\angle W=38.7°$, $w=10$ cm, and $v=25$ cm

7.7 Cosine Law

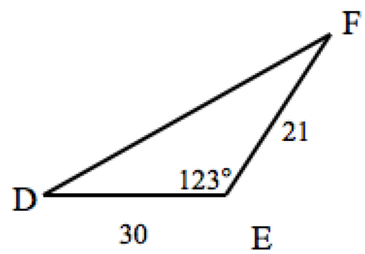
*Note: The diagrams are **not** drawn to scale.

1. Determine the measure of the unknown side length.

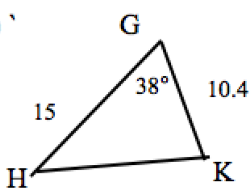
a)



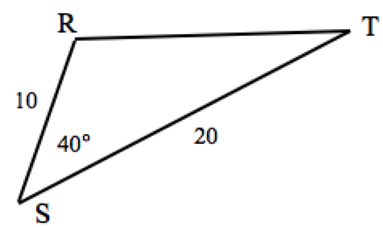
b)



c)

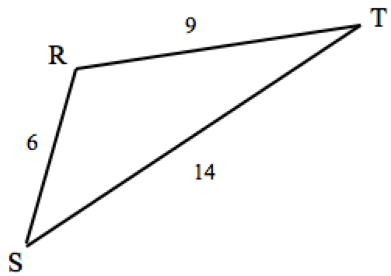


d)

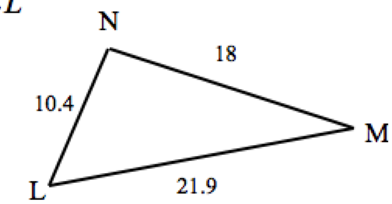


2. Determine the measure of the indicated angle.

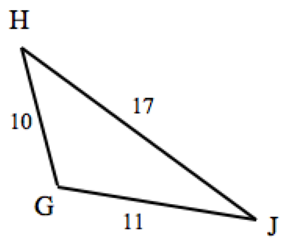
a) $\angle R$



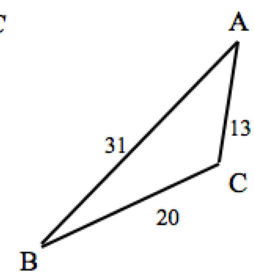
b) $\angle L$



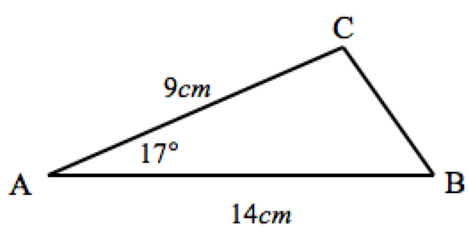
c) $\angle J$



d) $\angle C$



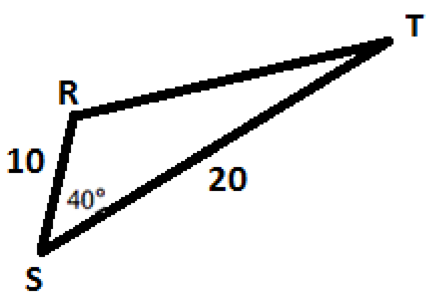
3. Determine the area of the following triangle.



4. For a triangle with lengths 14mm, 16mm, and 17mm
   a) Find the measure of the smallest angle.

   b) Find the area of the triangle.

5. Determine the measure of ∠T



6. Helmut sails on a bearing of N20°E for 18km, then S85°E for 6km. How far is Charlie from his starting point?

7. Elli sails at a speed of 16km/h in the direction N83°W. Helmut sails at a speed of 14km/h in the direction S40°W. How far apart were Elli and Helmut after two hours, to the nearest tenth of a kilometre assuming they start from the same location?

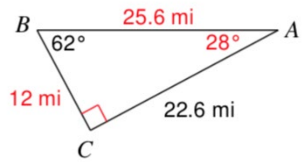
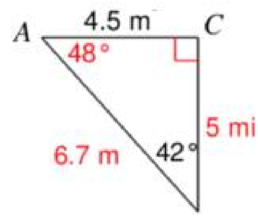
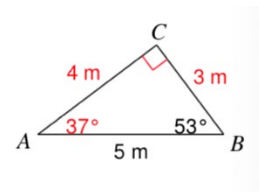
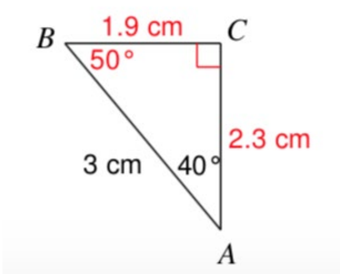
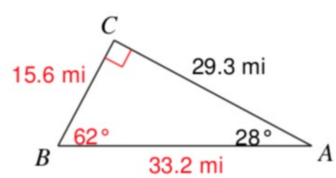
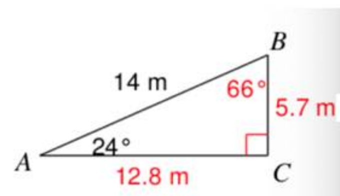
8. The posts of a hockey goal are 6 feet apart. Elli attempts to score by shooting the puck along the ice from a point that is 21 feet from one post and 26 feet from the other post. Within what angle, θ, must the shot be made?

9. Helmut gone for a hike in Bertram Creek Park. He has a walkie-talkie so that he can keep in touch with his friends at the camp. The walkie-talkie has a range of 6km. Helmut hikes 5km in a S60°E direction. He then hikes 2km in a N30°E direction. How far is Helmut from the camp? Can he still communicate with his friends at the camp?

10. The radar screen in an air-traffic control tower shows that two airplanes are at the same altitude. According to radar, one airplane is 100km away, in the direction N60°E. The other airplane is 160km away, in the direction S50°E. How far apart are the airplanes, to the nearest tenth of a kilometer?

UNIT 7 – ANSWER KEY

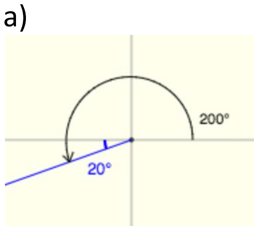
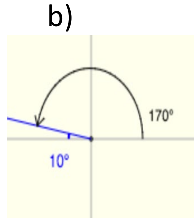
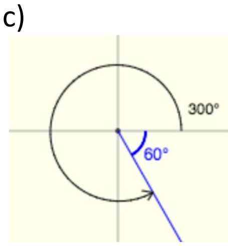
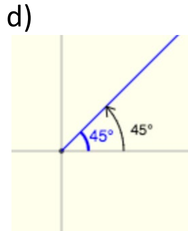
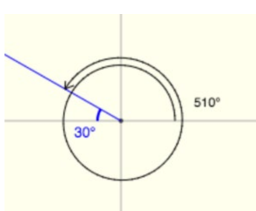
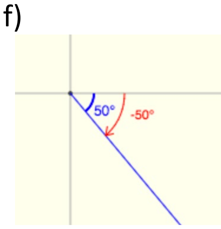
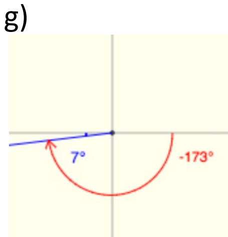
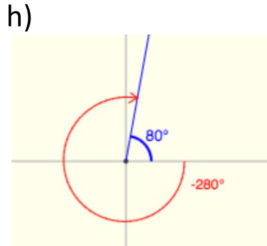
REVIEW

1. 	2. 	3. 
4. 	5. 	6. 

SECTION 7.1

1. Answers will vary 2. a) 180° b) 270°c) –90° d) –540°

4.

a) 	b) 	c) 	d) 
e) 	f) 	g) 	h) 

SECTION 7.2

1. a) sin θ = $\frac{3\sqrt{13}}{13}$, cos θ = $\frac{2\sqrt{13}}{13}$, tan θ = $\frac{3}{2}$, b) sin θ = $\frac{5\sqrt{34}}{34}$, cos θ = $\frac{-3\sqrt{34}}{34}$, tan θ = $-\frac{5}{3}$
c) sin θ = $\frac{-6\sqrt{37}}{37}$, cos θ = $\frac{\sqrt{37}}{37}$, tan θ = -6 d) sin θ = $\frac{-5}{13}$, cos θ = $\frac{-12}{13}$, tan θ = $\frac{5}{12}$
e) sin θ = $\frac{-7\sqrt{58}}{58}$, cos θ $\frac{3\sqrt{58}}{58}$, tan θ $= -\frac{7}{3}$ f) sin θ = $\frac{2\sqrt{29}}{29}$, cos θ = $\frac{-5\sqrt{29}}{29}$, tan θ = $\frac{-2}{5}$-
f) sin θ = $\frac{b(\sqrt{a^2+b^2})}{a^2+b^2}$, cos θ = $\frac{a(\sqrt{a^2+b^2})}{a^2+b^2}$, tan θ = $\frac{b}{a}$
2. a) $cos\ \theta = \frac{\sqrt{5}}{3}$, $tan\ \theta = \frac{2\sqrt{5}}{5}$ b) $sin\ \theta = \frac{-5\sqrt{29}}{29}$, $tan\ \theta = \frac{5}{2}$ c) $cos\ \theta = \frac{-4}{5}$, $sin\ \theta = \frac{-3}{5}$,
d) $cos\ \theta = \frac{3\sqrt{13}}{13}$, $tan\ \theta = \frac{-2}{3}$ e) $sin\ \theta = \frac{-\sqrt{15}}{8}$, $tan\ \theta = \frac{\sqrt{15}}{7}$ f) $cos\ \theta = -\sqrt{1-a^2}$,
$tan\ \theta = \frac{-a\sqrt{1-a^2}}{1-a^2}$

SECTION 7.3

1. $\sin 45 = \frac{1}{\sqrt{2}}$ $\cos 45 = \frac{1}{\sqrt{2}}$, $\tan 45 = 1$ 2. a) $\sin 30 = \frac{1}{2}$, $\cos 30 = \frac{\sqrt{3}}{2}$, $\tan 30 = \frac{1}{\sqrt{3}}$
b) $\sin 60 = \frac{\sqrt{3}}{2}$, $\cos 60 = \frac{1}{2}$, $\tan 60 = \sqrt{3}$ 3. a) $\frac{-1}{\sqrt{2}}$ b) $\frac{-\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$ e) $\frac{-1}{\sqrt{2}}$ f) $-\sqrt{3}$ g) $\frac{-1}{2}$ h) $\frac{1}{2}$ I) 1
4. a) $\frac{-2}{\sqrt{3}}$ b) 0 c) $\frac{-\sqrt{2}-\sqrt{3}}{2}$ d) $\frac{3}{4}$ e) 4 f) $-2\sqrt{2}$

SECTION 7.4

1. a) 58.0°, 302.0° b) 52.7°, 232.7° c) 40.5°, 139.5° d) 44.3°, 224.3° e) 51.6°, 128.4° f) 103.6°, 256.4° g) 166.3°, 346.3° h) 212.9°, 327.1° i) 189.6°, 350.4° j) no solution k) 78.7°, 258.7 l) 142.1°, 322.1° 2 a) 45°, 135° b) 240°, 300° c) 45°, 225° d) 120°, 240° e) 30°, 330° f) 45°, 315° g) 225°, 315° h) 120°, 300° i) 45°, 135° j) 150°, 210° k) 135°, 315° l) 135°, 225° 3. 135°
4. 315° 5. a) 45°, 135°, 225°, 315° b) 30°, 150°,210° ,330° c) 30°, 330° d) 60°, 109.47°, 250.53°, 300°

SECTION 7.5

1. a) 11.2 b) 7.1 c) 2.35 d) 12.4 2. a) 44.1° b) 25.3° c) 36.1° d) 56.2° 3. a) 24.3, 21.0 b) 11.4, 27.5 c) 26.1, 35.2 d) 7.9, 8.5 4.256.99*m* 5. a) 12.2*m*, 16.7*m* b) 11.8*m* 6. 4139*m* 7. 241.2*m* 8. 409.9 *m* 9. 11.4 k*m* 10. 3.6*km* 11. 35 *cm*

Section 7.6

1. a) $a < 4$ b) $a = 4$, $a > 8$ c) $4 < a < 8$ 2. a) one; $\angle B$=33.9°, $\angle C$=104.1°, c=43.5 b) two; $\angle C$=33.0°, $\angle A$=120° BC = 51.2; or $\angle C$=147.0°, $\angle A$=6.0° BC = 9.3 c) one; $\angle Q$=90°, $\angle R$=30° d) two; $\angle L$=39.9°, $\angle K$=102.8°, ML=136.8 or $\angle L$=140.1°, $\angle K$=2.6°, ML = 6.4 e) no triangle

Section 7.7

1. a) 21 b) 45 c) 9.3 d) 13.9 2. a) $\angle R$=137° b) $\angle L$=54.6° c) $\angle J$=34° d) $\angle C$=138.9°
3. 18.4cm^2 4. a) $\angle A$=50° b) 104mm^2 5. $\angle T$=27.5° 6. 20.3*km* 7. 28.8*km* 8. *8.1*°
9. 5.4*km*, Yes 10. 157*km*