

# Notes

PC 11

## Transformations of a Quadratic Function

The Language of Transformations:

- HT = horizontal translation ( $\leftrightarrow$  slide) by "h" units left (L) or right (R)
- VT = vertical translation ( $\updownarrow$  slide) by "k" units up (U) or down (D)
- R in x-axis = Reflection in the x-axis = vertical reflection
- VSE = Vertical stretch - expansion by a factor of "a" where  $a > 1$
- VSC = Vertical stretch - compression by a factor of "a" where  $0 < a < 1$

## Vertex Form of the Equation of a Quadratic Function


$$y = \pm a(x-h)^2 + k$$

Annotations for the vertex form equation:

- factor of VSE or VSC (points to  $a$ )
- VT by k units D or U (points to  $+k$ )
- HT by h units R or L (points to  $(x-h)$ )
- R in x-axis (points to  $\pm$ )

$$y = (x-3)^2$$

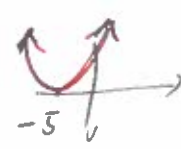
$\downarrow$   
 $h=3$



• HT right by 3 units

$$f(x) = (x+5)^2 = (x-(-5))^2$$

$h=-5$

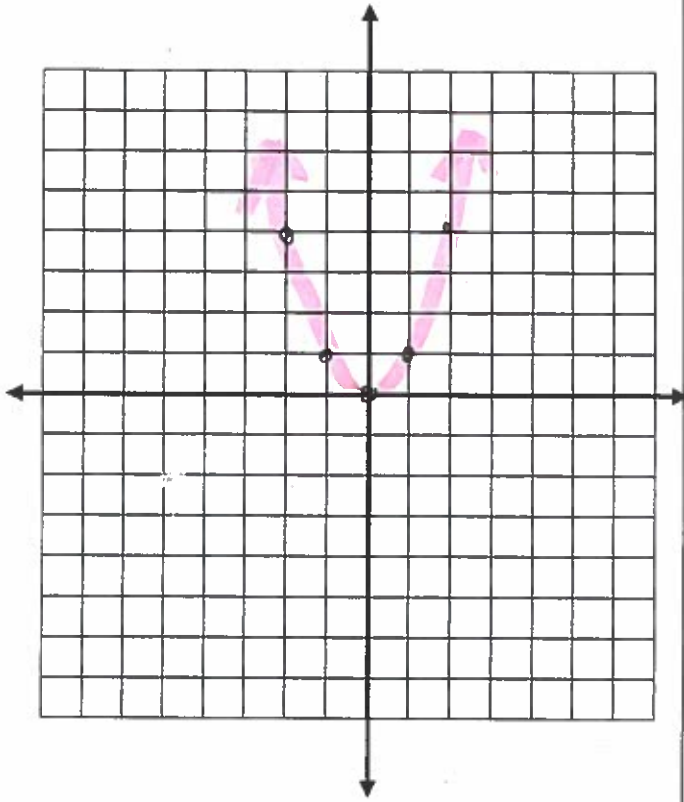


• HT left by 5 units

# The original parabola

Examples:

1.  $y = x^2$



Transformations:

none

Vertex:  $(0,0)$

End behaviour: Opens up

x-intercept(s):  $(0,0)$

y-intercept:  $(0,0)$

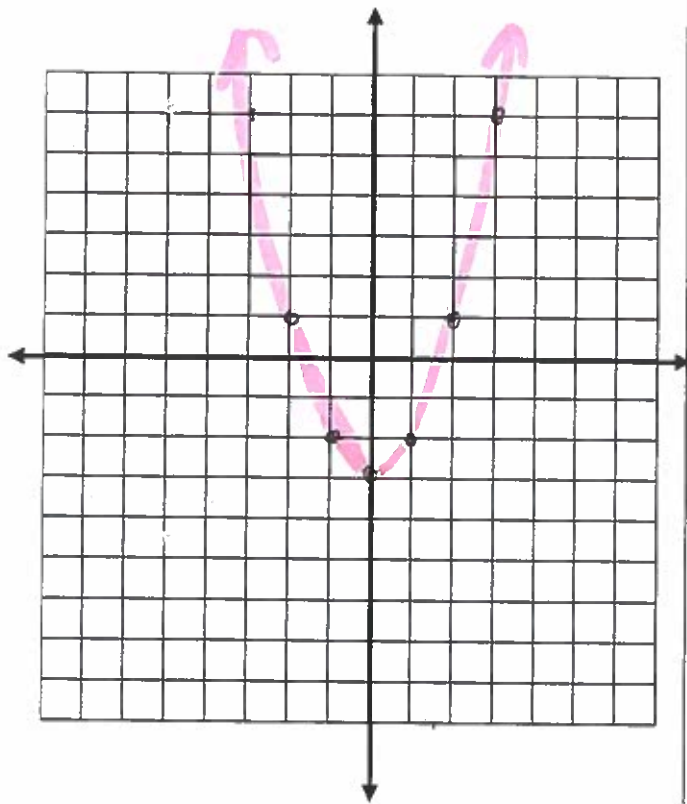
Maximum or Minimum value:  $(0,0)$  or  $y=0$

Domain:  $\mathbb{D}: \{x \mid x \in \mathbb{R}\}$

Range:  $\mathbb{R}: \{y \mid y \geq 0, y \in \mathbb{R}\}$

2.  $f(x) = x^2 - 3$

$\rightarrow f(x) = +1(x+0)^2 - 3 \Rightarrow k = -3$



Transformations:

VT down by 3 units

Vertex:  $(0, -3)$

End behaviour: opens up

x-intercept(s):  $\rightarrow$  algebra \*  
 $(\sqrt{3}, 0)$  and  $(-\sqrt{3}, 0)$

y-intercept:  
 $(0, -3)$

Maximum or Minimum value:  $y = -3$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \geq -3, y \in \mathbb{R}\}$

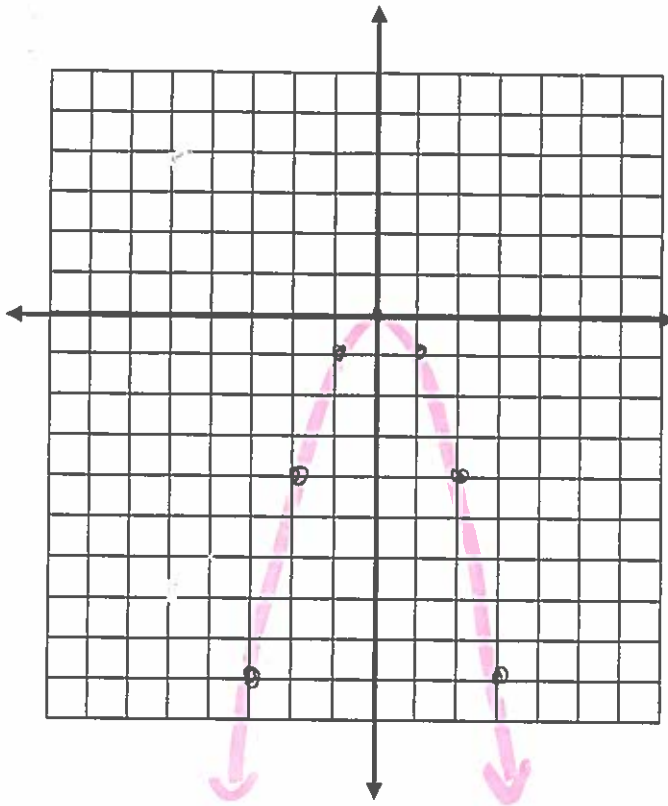
\* x-intercept  $\Rightarrow y = 0$

$$0 = x^2 - 3$$

$$\sqrt{3} = \sqrt{x^2}$$

$$x = \pm\sqrt{3}$$

3.  $y = -x^2 \rightarrow y = -(x-0)^2 + 0$



Transformations:

Reflection in x-axis

Vertex:  $(0,0)$

End behaviour: opens down

x-intercept(s):  $(0,0)$

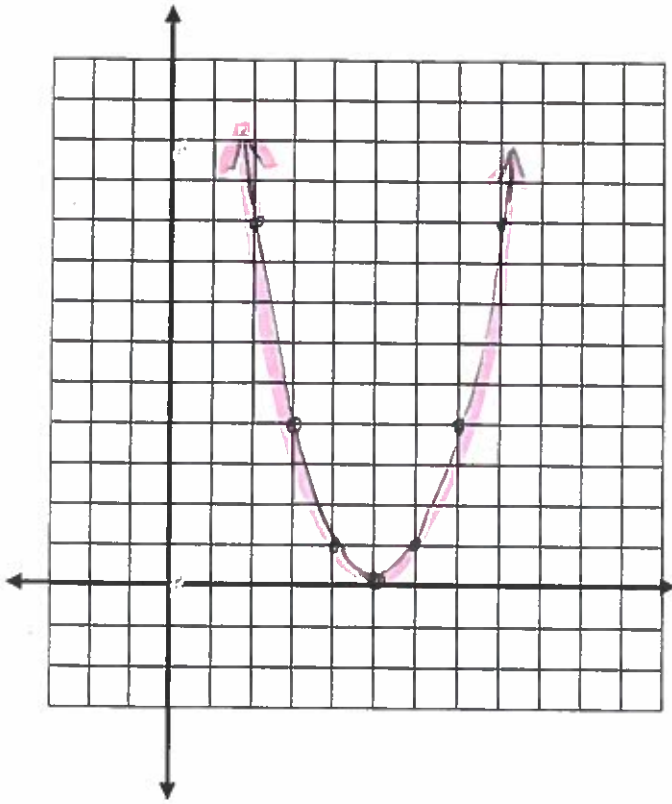
y-intercept:  $(0,0)$

Maximum or Minimum value:  $y=0$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \leq 0, y \in \mathbb{R}\}$

4.  $y = (x - 5)^2 \rightarrow y = +1(x - 5)^2 + 0$



Transformations:

• HT by 5 units Right

Vertex:  $(5, 0)$

End behaviour: opens up

x-intercept(s):  $(5, 0)$

y-intercept:  ~~$(0, 25)$~~   $(0, 25)$

Maximum or Minimum value:  $y = 0$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

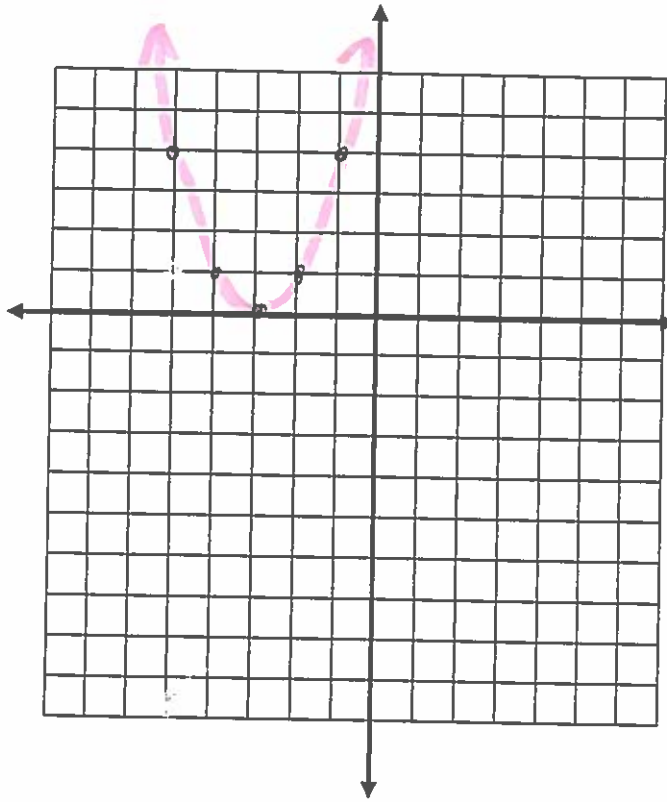
Range:  $R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$(*)$   $x = 0$

$y = (0 - 5)^2$

$y = 25$

5.  $y = (x + 3)^2 \rightarrow y = +1(x + 3)^2 + 0$



Transformations:

• HT left by 3 units

Vertex:  $(-3, 0)$

End behaviour: opens up

x-intercept(s):  $(-3, 0)$

y-intercept:  $(*)$   $(0, 9)$

Maximum or Minimum value:  $y = 0$

Domain:  $\mathbb{D}: \{x \mid x \in \mathbb{R}\}$

Range:  $\mathbb{R}: \{y \mid y \geq 0, y \in \mathbb{R}\}$

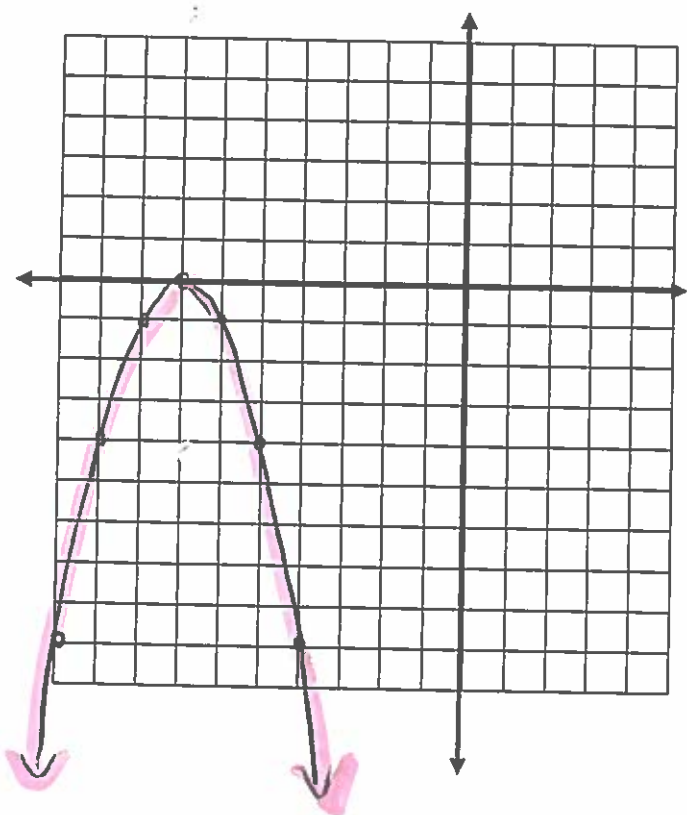
$(*)$

$x = 0$

$y = (0 + 3)^2$

$y = 9$

6.  $f(x) = -(x+7)^2 + 0$



Transformations:

- R in x-axis
- HT left by 7 units

Vertex:  $(-7, 0)$

End behaviour: opens down

x-intercept(s):  $(-7, 0)$

y-intercept:  $(*)$   $(0, -49)$

Maximum or Minimum value:  $y = 0$

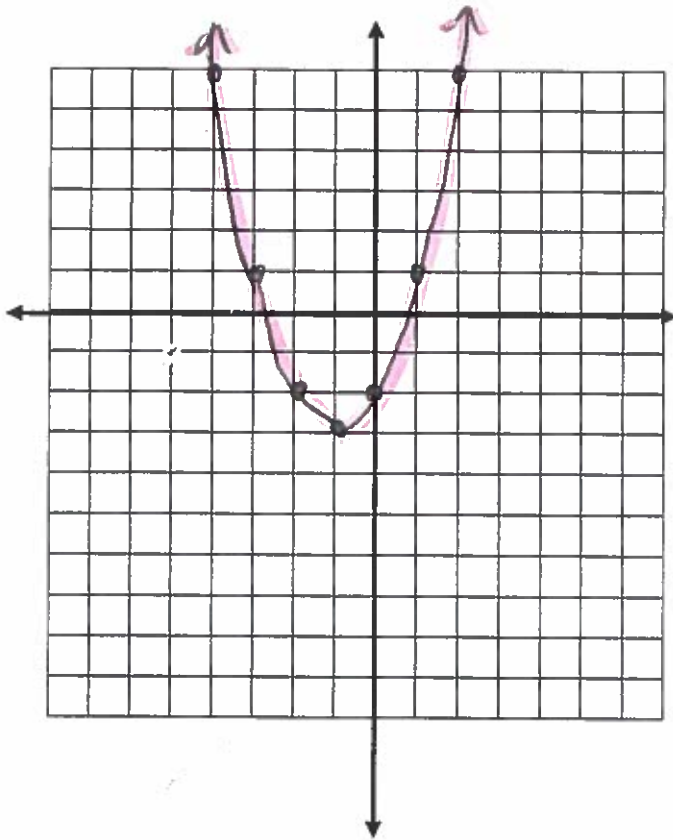
Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \leq 0, y \in \mathbb{R}\}$

$(*)$   $x=0 \rightarrow y = -(0+7)^2$   
 $y = -49$

$$f(x) = y$$

7.  $f(x) = (x + 1)^2 - 3$



Transformations:

- HT left by 1 unit
- VT down by 3 units

Vertex:  $(-1, -3)$

End behaviour: opens up

x-intercept(s):  $(-1, 0)$

y-intercept:  $(0, -2)$

Maximum or Minimum value:  $y = -3$

Domain:  $\mathcal{D}: \{x \mid x \in \mathbb{R}\}$

Range:  $\mathcal{R}: \{y \mid y \geq -3, y \in \mathbb{R}\}$

$\otimes$   $x=0 \rightarrow y = (0+1)^2 - 3$   
 $y = 1 - 3$   
 $y = -2$