

Notes

PC 11

Transformations of a Quadratic Function

The Language of Transformations:

- HT = horizontal translation (\leftrightarrow slide) by "h" units left (L) or right (R)
- VT = vertical translation (\updownarrow slide) by "k" units up (U) or down (D)
- R in x-axis = reflection in the x-axis = vertical reflection
- VSE = Vertical stretch - expansion by a factor of "a" where $a > 1$
- VSC = Vertical stretch - compression by a factor of "a" where $0 < a < 1$

Vertex Form of the Equation of a Quadratic Function

$$y = \pm a(x-h)^2 + k$$

factor of VSE or VSC \uparrow

\uparrow VT by k units D or U

\uparrow R in x-axis

\uparrow HT by h units R or L

$$y = (x-3)^2$$

$$\downarrow$$
$$h=3$$

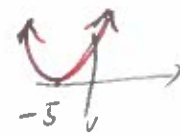


• HT right by 3 units

$$f(x) = (x+5)^2$$

$$= (x - (-5))^2$$

$$h = -5$$

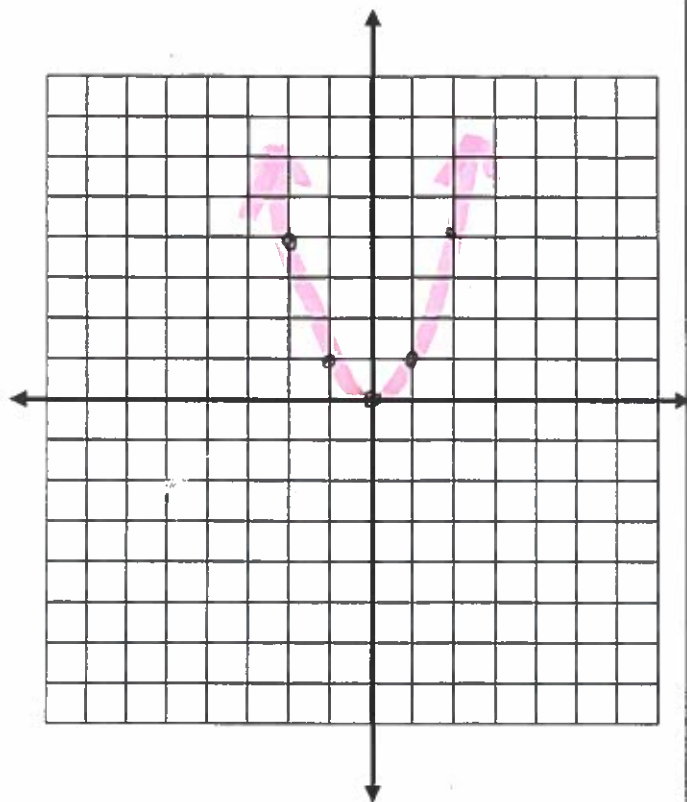


• HT left by 5 units

The original parabola

Examples:

1. $y = x^2$



Transformations:

none

Vertex: $(0,0)$

End behaviour: Opens up

x-intercept(s): $(0,0)$

y-intercept: $(0,0)$

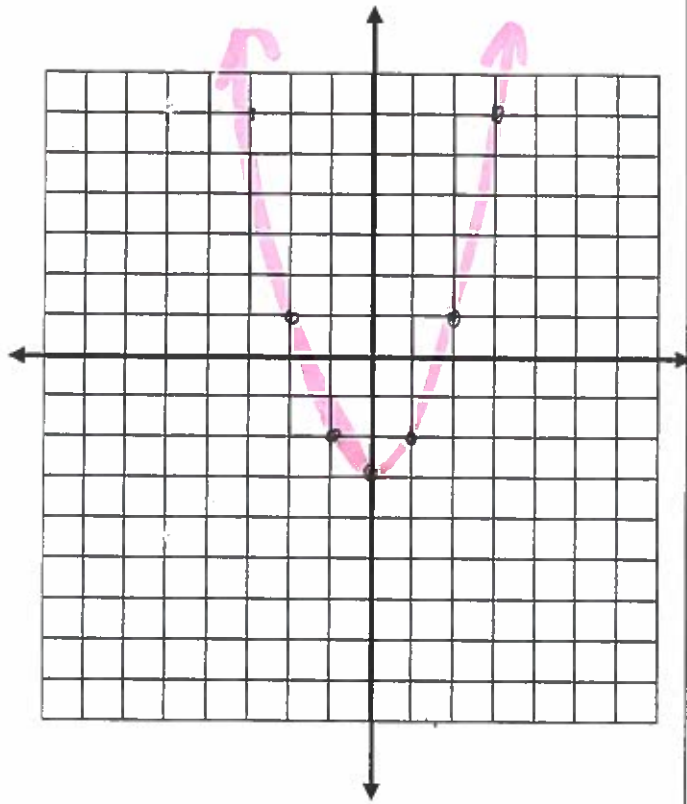
Maximum or Minimum value: $(0,0)$ or $y=0$

Domain: $D: \{x \mid x \in \mathbb{R}\}$

Range: $R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

2. $f(x) = x^2 - 3$

$\rightarrow f(x) = +1(x+0)^2 - 3 \Rightarrow k = -3$



Transformations: VT down by 3 units

Vertex: $(0, -3)$

End behaviour: opens up

x-intercept(s): \rightarrow algebra \times
 $(\sqrt{3}, 0)$ and $(-\sqrt{3}, 0)$

y-intercept: $(0, -3)$

Maximum or Minimum value: $y = -3$

Domain: $D: \{x \mid x \in \mathbb{R}\}$

Range: $R: \{y \mid y \geq -3, y \in \mathbb{R}\}$

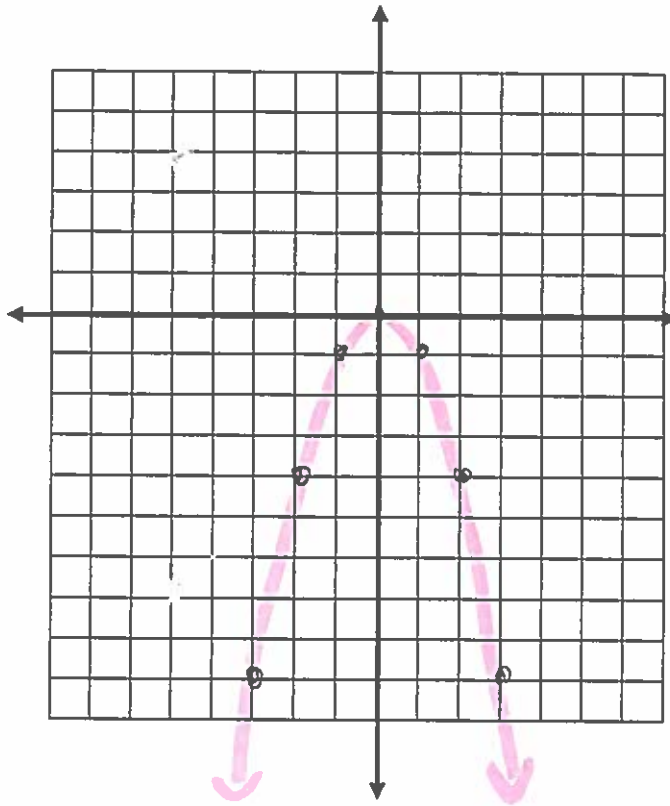
* x-intercept $\Rightarrow y = 0$

$$0 = x^2 - 3$$

$$\sqrt{3} = \sqrt{x^2}$$

$$x = \pm\sqrt{3}$$

$$3. y = -x^2 \rightarrow y = -(x-0)^2 + 0$$



Transformations:

Reflection in x-axis

Vertex: $(0,0)$

End behaviour: opens down

x-intercept(s): $(0,0)$

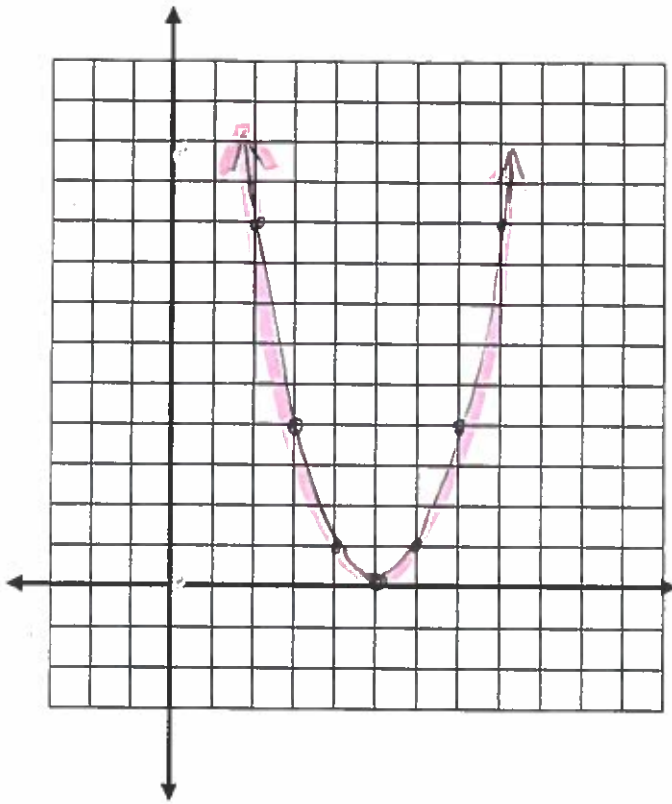
y-intercept: $(0,0)$

Maximum or Minimum value: $y=0$

Domain: $\mathcal{D}: \{x \mid x \in \mathbb{R}\}$

Range: $\mathcal{R}: \{y \mid y \leq 0, y \in \mathbb{R}\}$

4. $y = (x-5)^2 \rightarrow y = +1(x-5)^2 + 0$



Transformations:

• HT by 5 units Right

Vertex: $(5, 0)$

End behaviour: opens up

x-intercept(s): $(5, 0)$

y-intercept: ~~$(0, 25)$~~

Maximum or Minimum value: $y = 0$

Domain: $D: \{x \mid x \in \mathbb{R}\}$

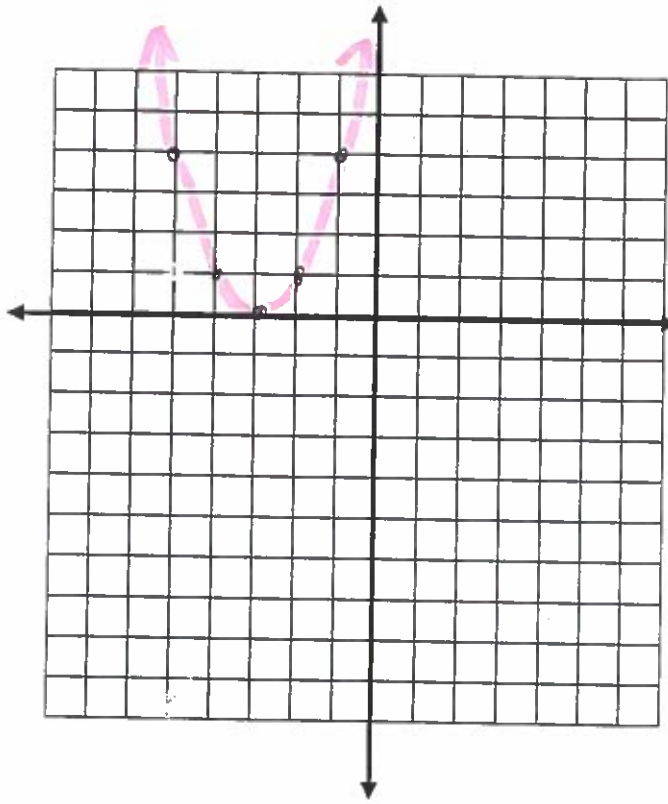
Range: $R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$(*) \quad x = 0$

$y = (0-5)^2$

$y = 25$

5. $y = (x+3)^2 \rightarrow y = +1(x+3)^2 + 0$



Transformations:

• HT left by 3 units

Vertex: $(-3, 0)$

End behaviour: opens up

x-intercept(s): $(-3, 0)$

y-intercept: $(*)$ $(0, 9)$

Maximum or Minimum value: $y = 0$

Domain: $\mathbb{D}: \{x | x \in \mathbb{R}\}$

Range: $\mathbb{R}: \{y | y \geq 0, y \in \mathbb{R}\}$

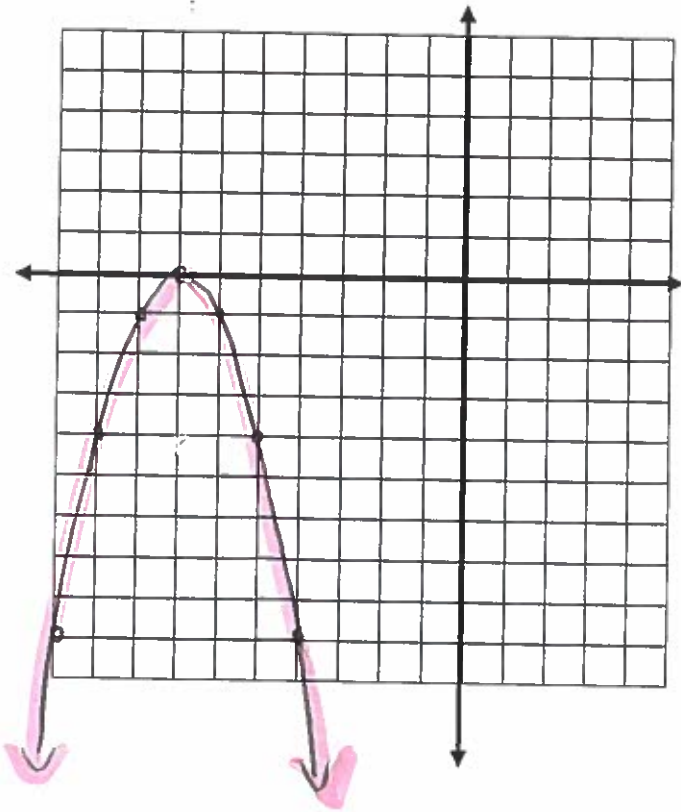
$(*)$

$$x = 0$$

$$y = (0+3)^2$$

$$y = 9$$

6. $f(x) = -(x+7)^2 + 0$



Transformations:

- R in x-axis
- HT left by 7 units

Vertex: $(-7, 0)$

End behaviour: *opens down*

x-intercept(s): $(-7, 0)$

y-intercept: $(*)$ $(0, -49)$

Maximum or Minimum value: $y = 0$

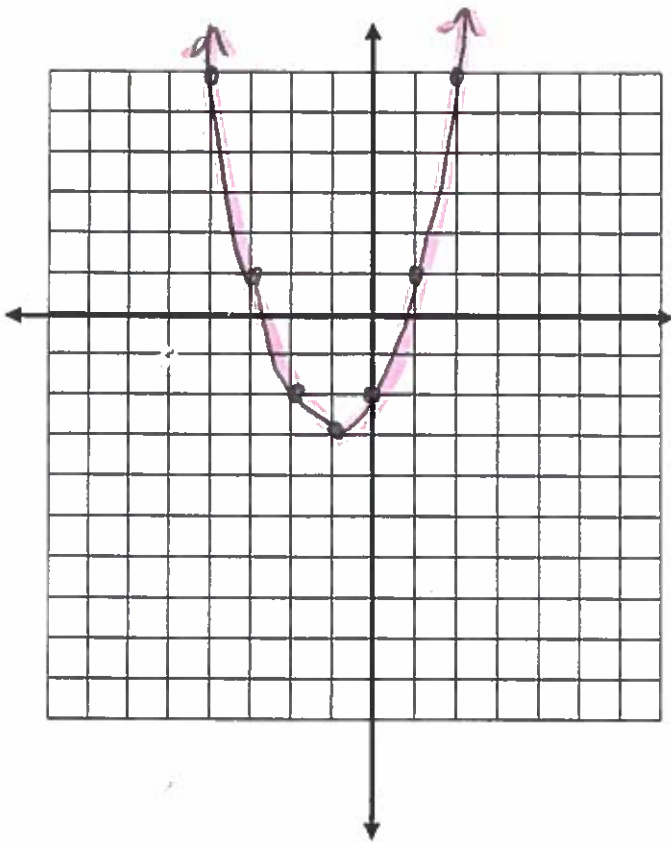
Domain: $D: \{x \mid x \in \mathbb{R}\}$

Range: $R: \{y \mid y \leq 0, y \in \mathbb{R}\}$

$(*)$ $x=0 \rightarrow y = -(0+7)^2$
 $y = -49$

$$f(x) = y$$

7. $f(x) = (x+1)^2 - 3$



Transformations:

- HT left by 1 unit
- VT down by 3 units

Vertex: $(-1, -3)$

End behaviour: opens up

x-intercept(s): $(-1+\sqrt{3}, 0)$
 $(-1-\sqrt{3}, 0)$

y-intercept: $(0, -2)$

Maximum or Minimum value: $y = -3$

Domain: $\mathbb{D}: \{x \mid x \in \mathbb{R}\}$

Range: $\mathbb{R}: \{y \mid y \geq -3, y \in \mathbb{R}\}$

\otimes $x=0 \rightarrow y = (0+1)^2 - 3$
 $y = 1 - 3$
 $y = -2$

$\otimes \otimes$ $y=0$
 $0 = (x+1)^2 - 3$
 $\sqrt{3} = \sqrt{(x+1)^2}$
 $\pm\sqrt{3} = x+1$
 $-1 \pm \sqrt{3} = x$