

* important lesson b/c allows to visualize function

Transformations of Quadratic Function

= changes made to the graph of the original parabola $y = x^2$

3 Ways to Describe Transformations of Quadratic Function

- Equation in vertex form
- Word descriptions
- Mapping notation

Equation in Vertex Form

vertical stretch (VS) by a factor of "a"

$$y = \pm a (x - h)^2 + k$$

reflection (R) in x-axis

horizontal translation (HT) by h units [left right]

vertical translation (VT) by k units [up down]

Word Descriptions

- Horizontal translation: left or right by "h" units
- Vertical translation: up or down by "k" units
- Reflection in x-axis: vertical reflection (flipping upside down)
- Vertical stretch (expansion): by a factor of a where $a > 1$
 - o Graph looks narrower than the original
- Vertical stretch (compression): by a factor of a where $0 < a < 1$
 - o Graph looks wider than the original

Mapping Notation

VS by a factor of "a"

$$(x, y) \rightarrow (x \mp h, \pm ay \pm k)$$

HT by "h" units

R in x-axis

VT by "k" units

Ex-1) Describe all transformations in "Word Descriptions"

a. $y = -x^2 + 3$

- reflection in x-axis
- vertical translation up by 3 units

b. $y = x^2 - 5$

- VT down by 5 units

c. $y = (x + 6)^2$

- HT left by 6 units

d. $y = (x - 8)^2$

- HT right by 8 units

e. $y = (x + 0.5)^2 - 1.5$

- HT left by 0.5 units
- VT down by 1.5 units

f. $y = (x - 2.4)^2 + 3.6$

- HT right by 2.4 units
- VT up by 3.6 units

g. $y = -(x - 2)^2$

- R in x-axis
- HT right by 2 units

h. $y = -x^2 - 12$

- R in x-axis
- VT down by 12 units

Ex-2) Describe all vertical stretches in "Word Descriptions"

a. $y = 2x^2$

- vertical stretch expansion by a factor of 2

b. $y = 0.4x^2$

- VSE by a factor of 0.4

c. $f(x) = 3.5x^2$

- VSE by a factor of 3.5

d. $y = 0.02x^2$

- VSC by a factor of 0.02

e. $f(x) = 25x^2$

- VSE by a factor of 25

f. $y = 1.01x^2$

- VSE by a factor of 1.01

g. $f(x) = x^2$ = original parabola

- NO VS

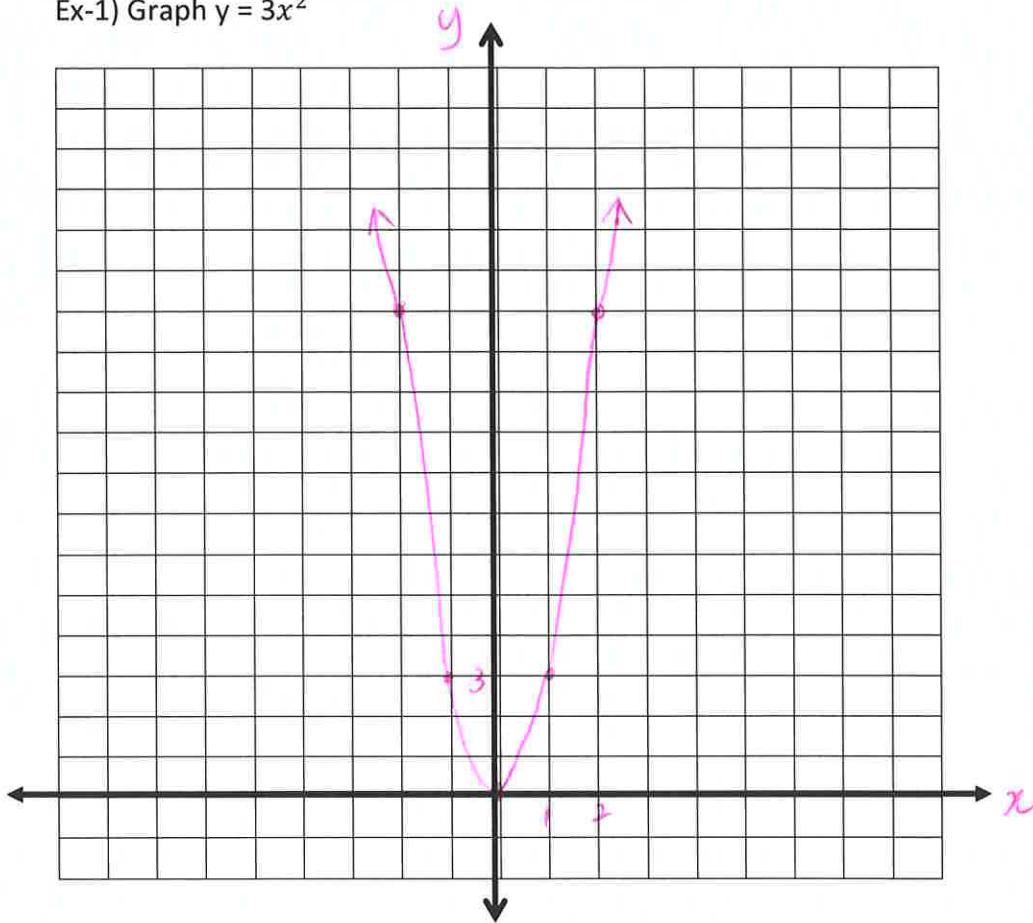
h. $y = 0.2x^2$

- VSC by a factor of 0.2

* Don't forget to label your axes

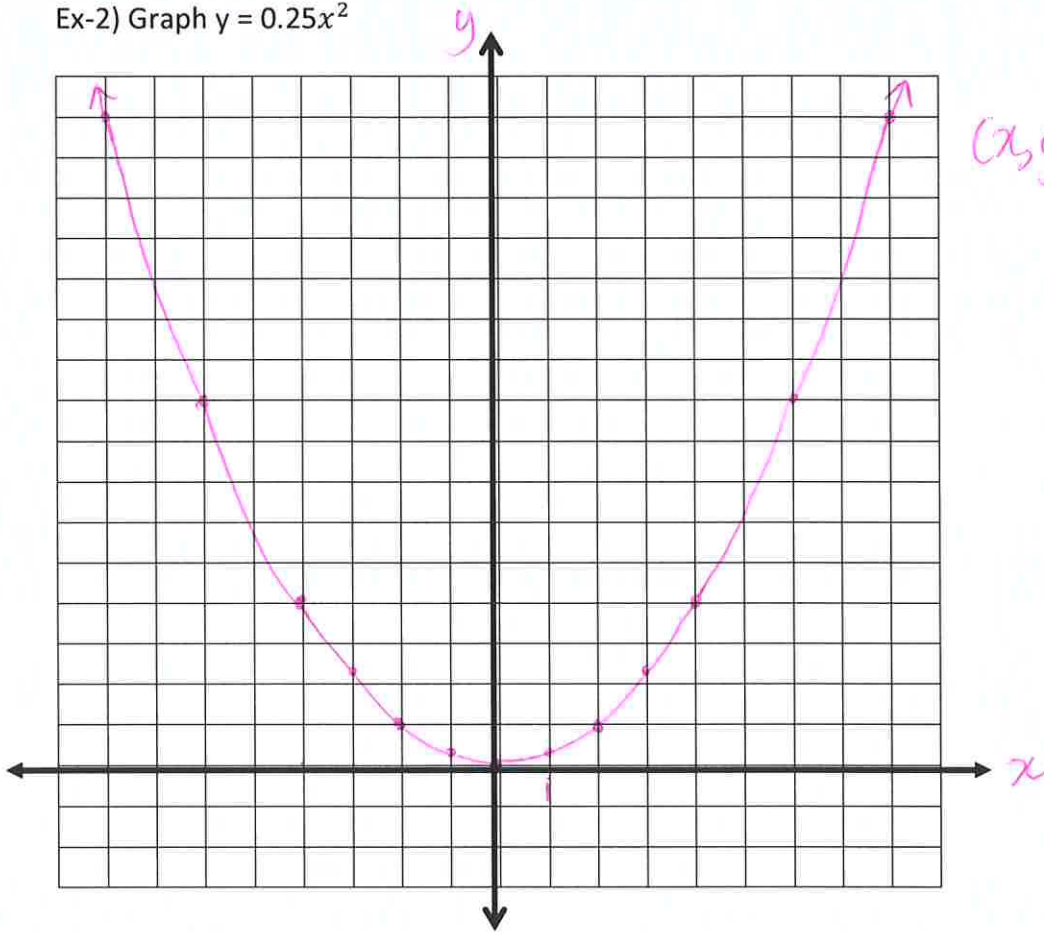
Graphing Quadratic Functions

Ex-1) Graph $y = 3x^2$

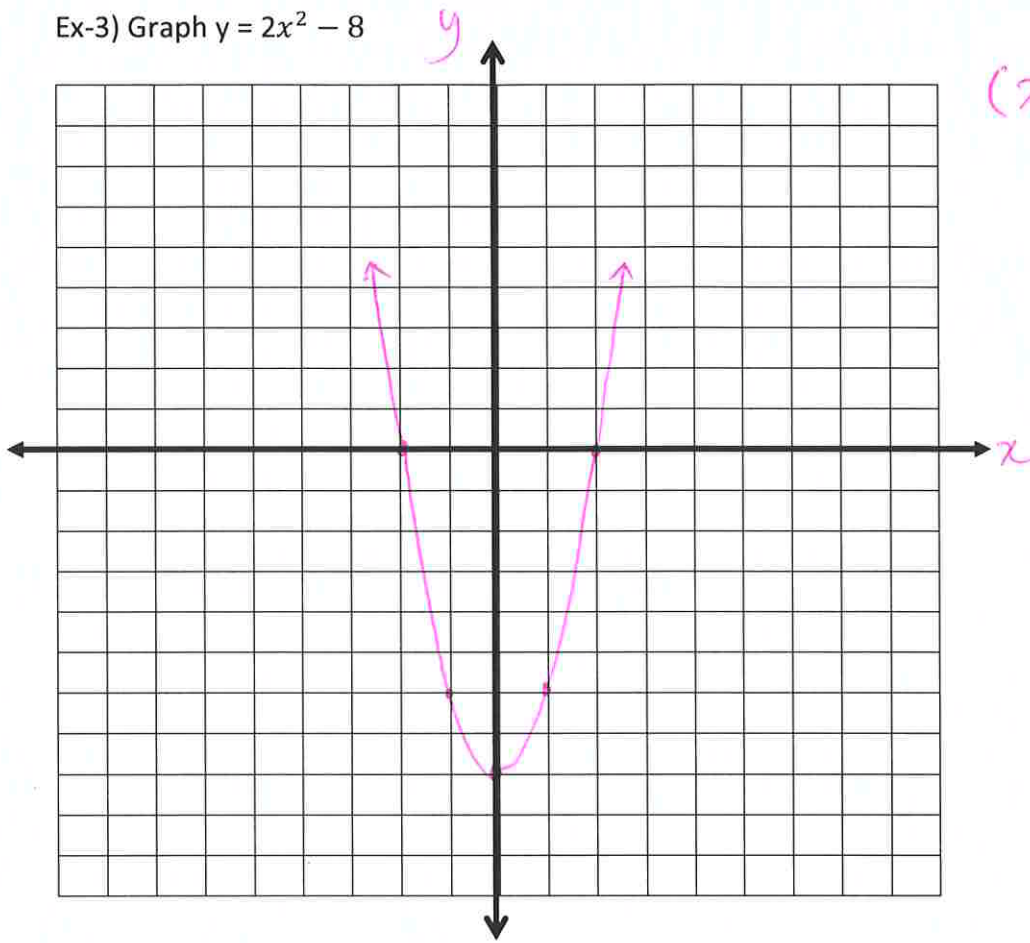


$(x, y) \rightarrow (x, 3y)$

Domain	$\{x x \in \mathbb{R}\}$	Vertex	$(0, 0)$
Range	$\{y y \geq 0, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 0$
x-intercept	$(0, 0)$	End behavior	opens up
y-intercept	$(0, 0)$	Transformations	VSE by a factor of 3

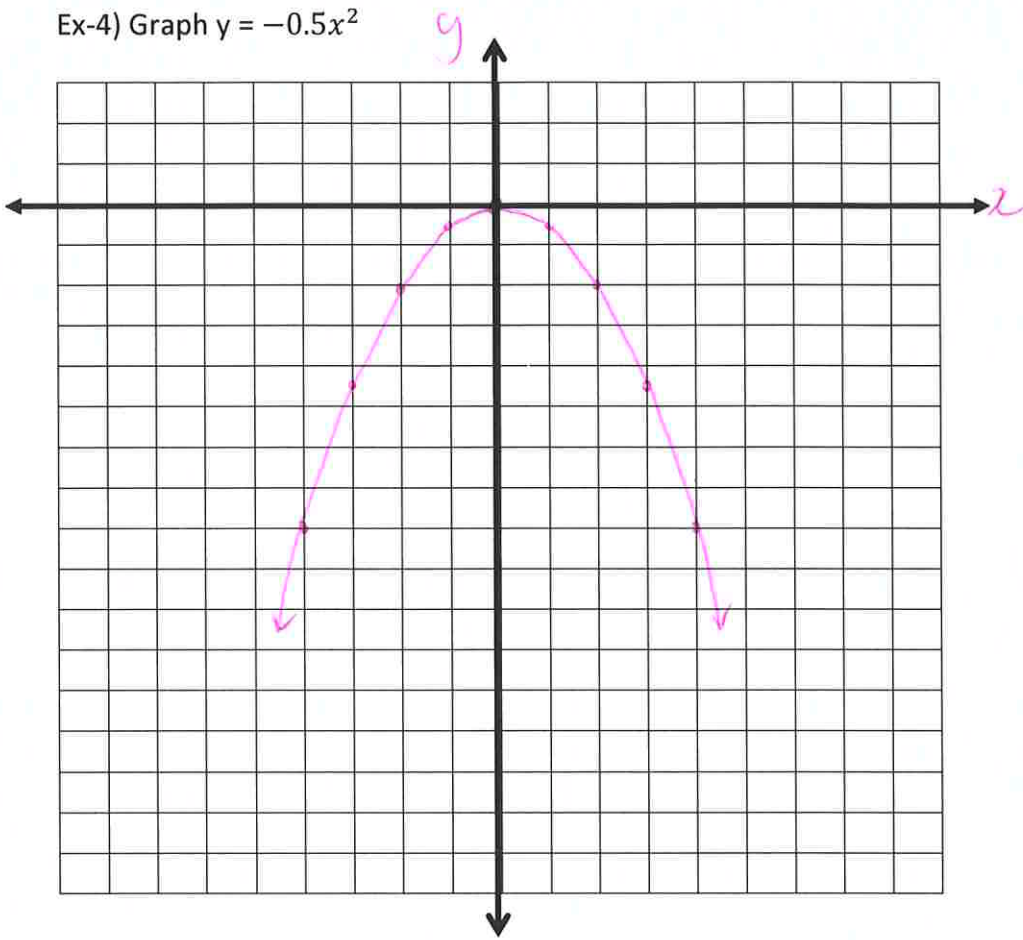
Ex-2) Graph $y = 0.25x^2$ 

Domain	$\{x x \in \mathbb{R}\}$	Vertex	$(0, 0)$
Range	$\{y y \geq 0, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 0$
x-intercept	$(0, 0)$	End behavior	opens up
y-intercept	$(0, 0)$	Transformations	VSC by a factor of 0.25

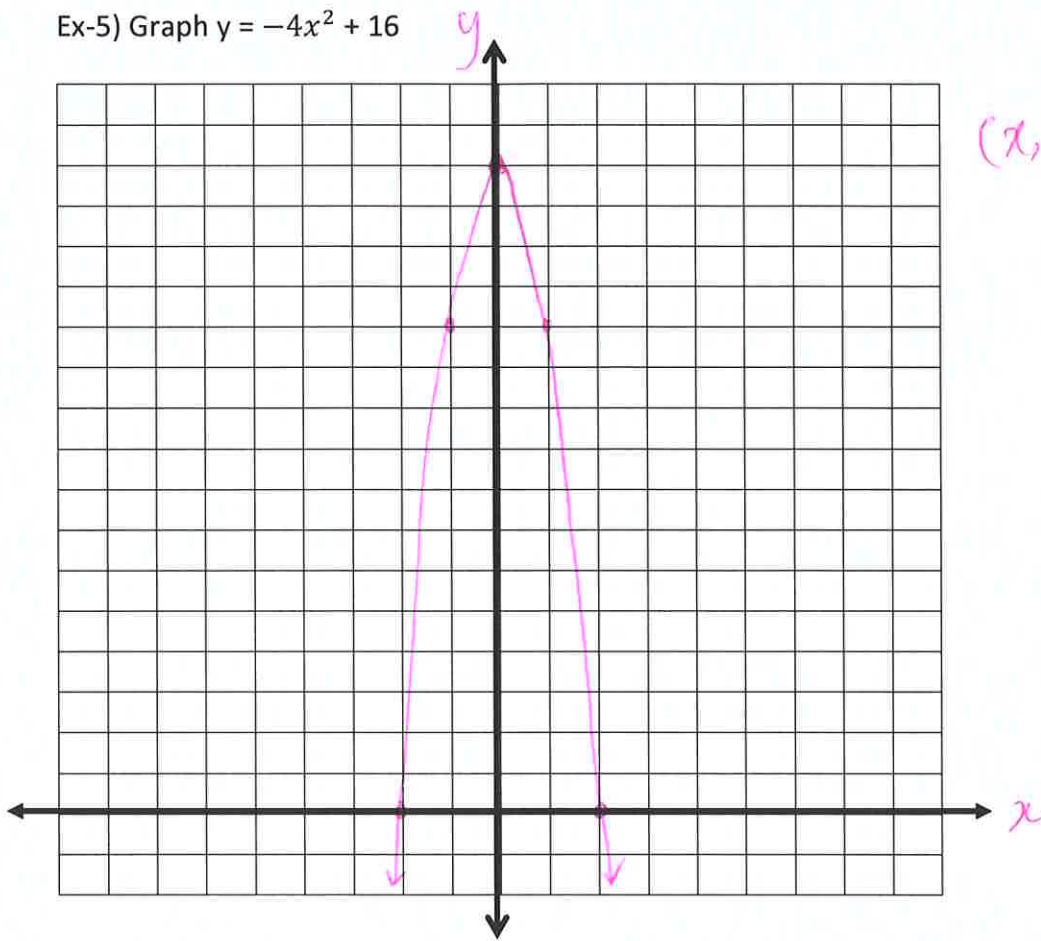
Ex-3) Graph $y = 2x^2 - 8$ 

$$(x, y) \rightarrow (x, 2y - 8)$$

Domain	$\{x x \in \mathbb{R}\}$	Vertex	$(0, -8)$
Range	$\{y y \geq -8, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 0$
x-intercept	$(2, 0), (-2, 0)$	End behavior	opens up
y-intercept	$(0, -8)$	Transformations	<ul style="list-style-type: none"> • VSE by a factor of 2 • VT down by 8 units

Ex-4) Graph $y = -0.5x^2$ 

Domain	$\{x x \in \mathbb{R}\}$	Vertex	$(0, 0)$
Range	$\{y y \leq 0, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 0$
x-intercept	$(0, 0)$	End behavior	opens down
y-intercept	$(0, 0)$	Transformations	<ul style="list-style-type: none"> • R in x-axis • VSC by a factor of 0.5

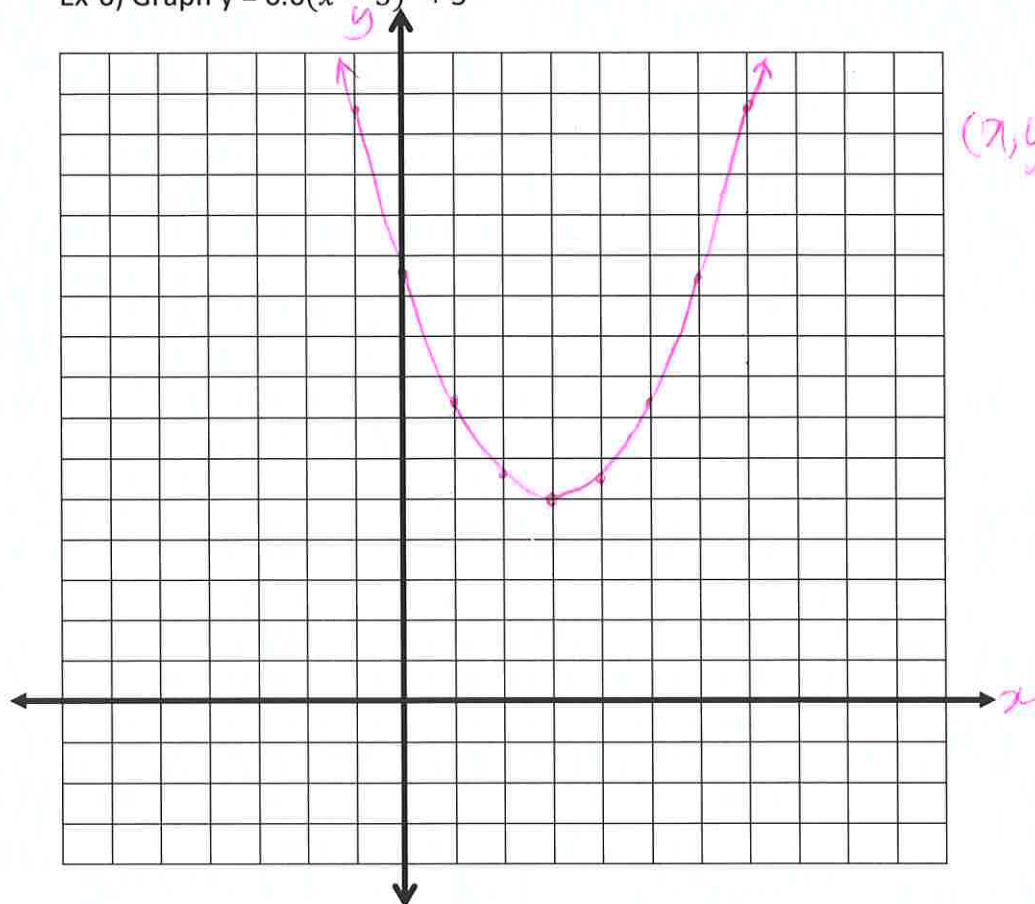
Ex-5) Graph $y = -4x^2 + 16$ 

$$(x, y) \rightarrow (x, -4y + 16)$$

Domain	$\{x \mid x \in \mathbb{R}\}$	Vertex	$(0, 16)$
Range	$\{y \mid y \leq 16, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 0$
x-intercept	$(2, 0), (-2, 0)$	End behavior	opens down
y-intercept	$(0, 16)$	Transformations	<ul style="list-style-type: none"> • R in x-axis • VSE by a factor of 4

• VT up by 16

Ex-6) Graph $y = 0.6(x - 3)^2 + 5$



$$(x, y) \rightarrow (x+3, 0.6y+5)$$

Domain	$\{x x \in \mathbb{R}\}$	Vertex	$(3, 5)$
Range	$\{y y \geq 5, y \in \mathbb{R}\}$	Axis of Symmetry	$x = 3$
x-intercept	none	End behavior	opens up
y-intercept	$(0, 10.4)$	Transformations	<ul style="list-style-type: none"> • VSC by a factor of 0.6 • HT right by 3 units • VT up by 5 units

$$\begin{aligned}
 &0.6(0-3)^2 + 5 \\
 &= 0.6(3)^2 + 5 \\
 &= 0.6(9) + 5 \\
 &= 10.4
 \end{aligned}$$