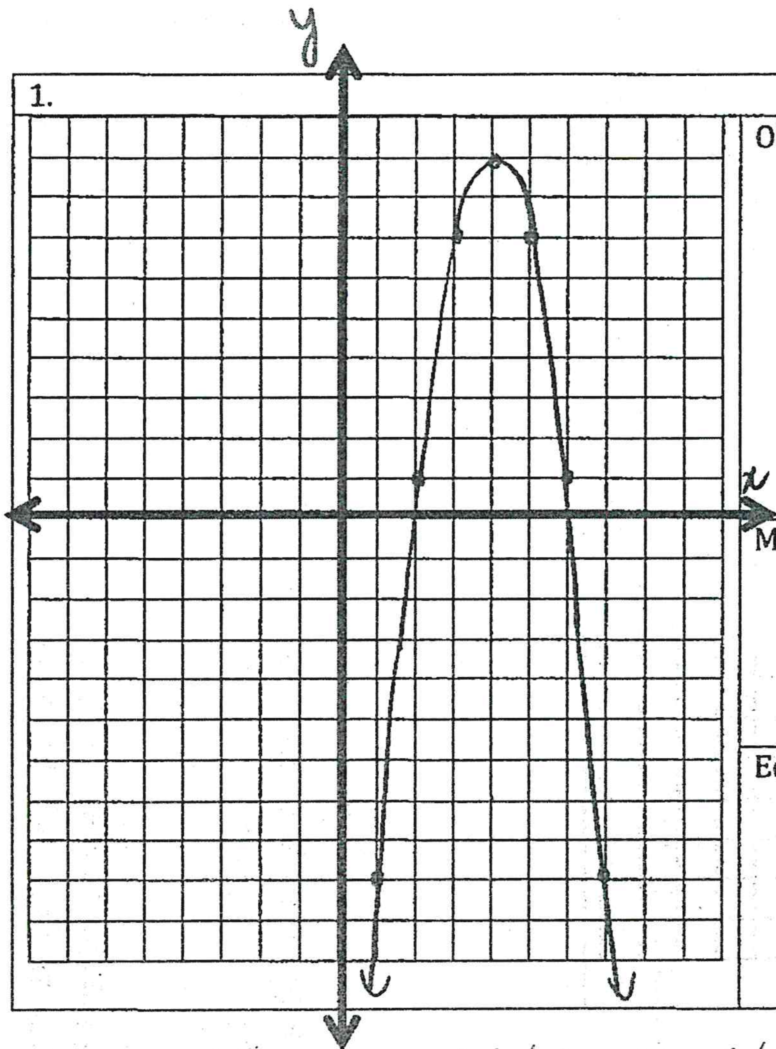
KEY

TRANSFORMATIONS OF A QUADRATIC FUNCTION

1.



Observations:

- vertex $(4,9)$ → HT right by 4 units; → VT up by 9 units
- end behaviour : opens down → R in x-axis
- narrower graph → VSE by a factor of 2 ($a1 \to 2 \Rightarrow a=2$)

Mapping notation:

$(x,y) \to (x+4, -2y+9)$

Equation:

$y = -2(x-4)^2+9$

y-int :
$y=-2(0-4)^2+9$
$y=-2(16)+9$
$y=-32+9$
$y=-23$

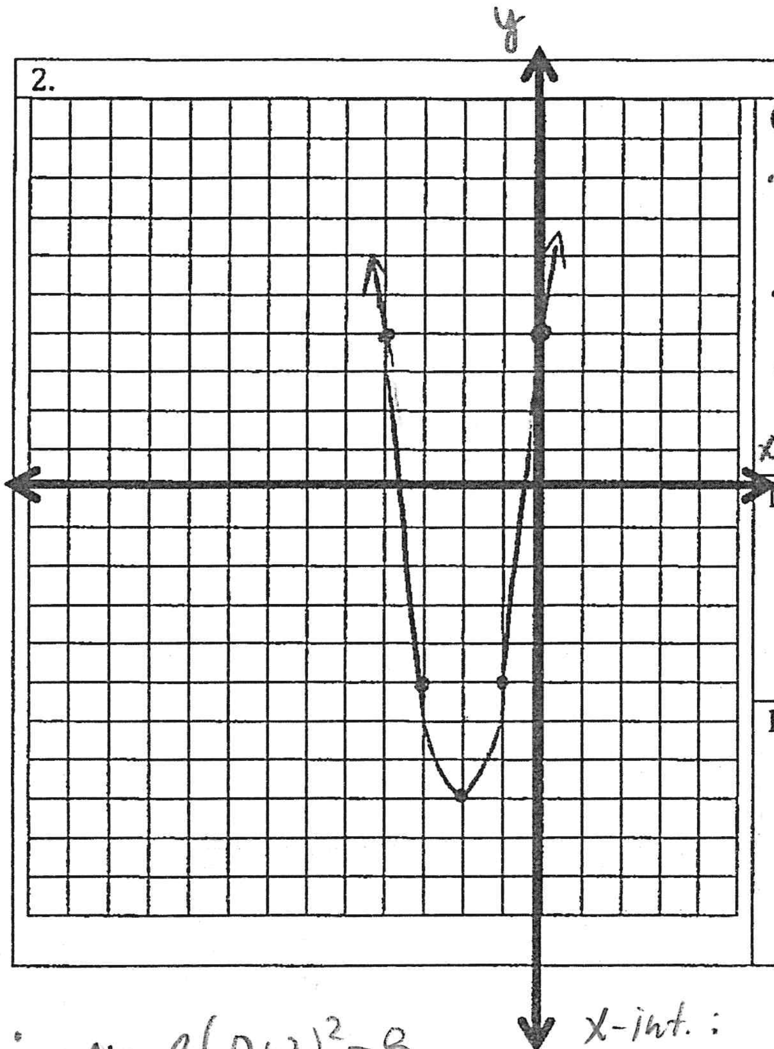
$\therefore (0,-23)$

x-int. :
$0=-2(x-4)^2+9$
$\frac{-9}{-2} = \frac{-2(x-4)^2}{-2}$
$\pm\sqrt{\frac{9}{2}} = \sqrt{(x-4)^2}$

$\frac{\pm 3}{\sqrt{2}} = x-4$
$\frac{\pm 3}{\sqrt{2}} + 4 = x$
$\frac{\pm 3\sqrt{2}}{2} + 4 = x$

$\therefore \left(-\frac{3\sqrt{2}}{2}+4, 0\right)$ and $\left(\frac{3\sqrt{2}}{2}+4, 0\right)$.

2.



Observations:

- vertex : $(-2,-8)$ → HT left by 2 units; → VT down by 8 units
- end behaviour : opens up → no R
- graph is narrower → VSE by a factor of 3

Mapping notation:

$(x,y) \rightarrow (x-2,\ 3y-8)$

Equation:

$y = 3(x+2)^2 - 8$

y-int. :

$y = 3(0+2)^2 - 8$

$y = 3(4) - 8$

$y = 12 - 8$

$y = 4$

$\therefore (0,4)$

x-int. :

$0 = 3(x+2)^2 - 8$

$\frac{8}{3} = \frac{3(x+2)^2}{3}$

$\pm\sqrt{\frac{8}{3}} = \sqrt{(x+2)^2}$

$\frac{\pm 2\sqrt{2}}{\sqrt{3}} = x+2$

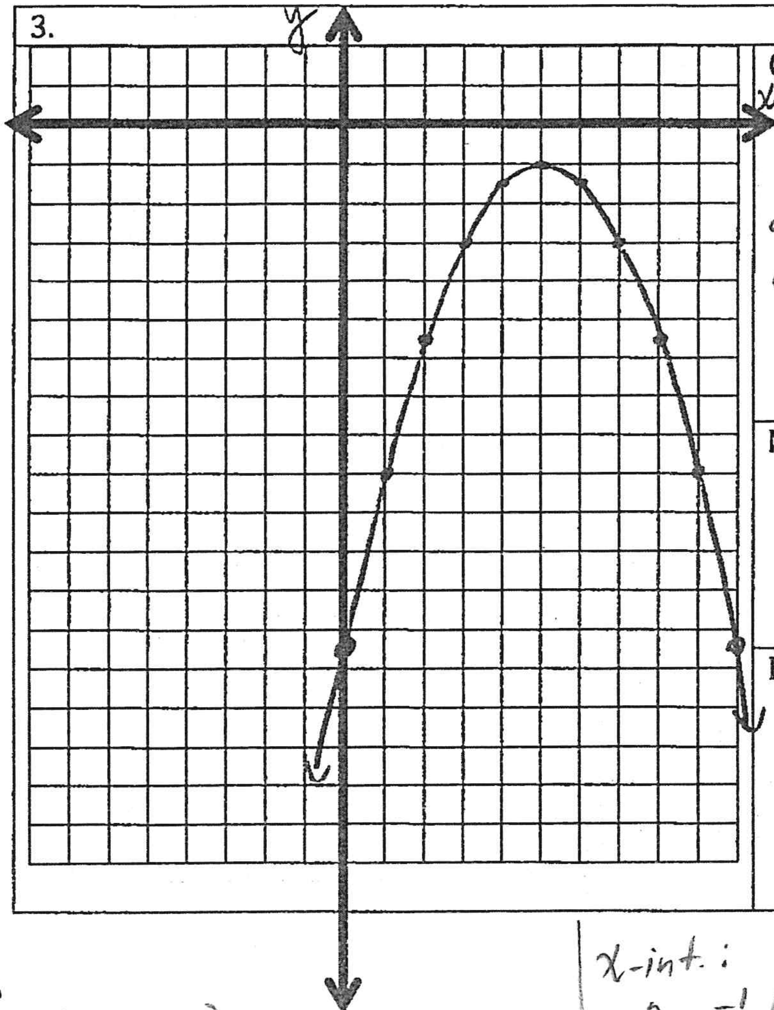
$\frac{\pm 2\sqrt{2}}{\sqrt{3}} - 2 = x$

$\frac{\pm 2\sqrt{2}\cdot\sqrt{3}}{3} - 2 = x$

$\frac{\pm 2\sqrt{6}}{3} - 2 = x$

$\therefore \left(\frac{2\sqrt{6}}{3} - 2, 0\right)$ and $\left(\frac{-2\sqrt{6}}{3} - 2, 0\right)$.

3.



Observations:

- vertex: $(5,-1) <$ HT right by 5 units / VT down by 1 unit
- end behaviour - opens down $\Rightarrow$ R- in x-axis
- wider graph $\rightarrow$ VSC by a factor of $\frac{1}{2}$

$\frac{a4}{4} = \frac{2}{4} \rightarrow a = \frac{1}{2}$

Mapping notation:

$(x,y) \rightarrow (x+5, -\frac{1}{2}y-1)$

$\rightarrow (x+5, \frac{-y}{2}-1)$ OR $(x+5, -0.5y-1)$

Equation:

$y = -\frac{1}{2}(x-5)^2-1$ OR $y = -0.5(x-5)^2-1$

y-int.:

$y = -\frac{1}{2}(0-5)^2-1$

$y = -\frac{1}{2}(25)-1$ $\therefore (0, -13.5)$

$y = -12.5-1$

$y = -13.5$

x-int.:

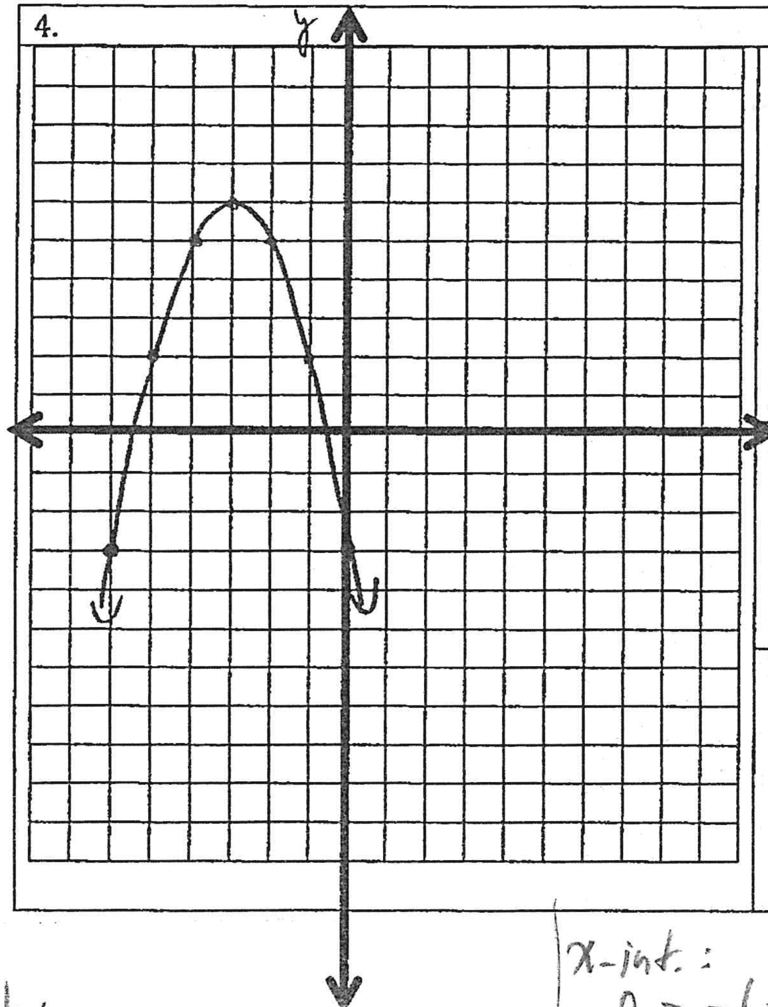
$0 = -\frac{1}{2}(x-5)^2-1$

$\frac{1}{-\frac{1}{2}} = \frac{-\frac{1}{2}(x-5)^2}{-\frac{1}{2}}$

$\sqrt{-2} = \sqrt{(x-5)^2}$

↑ no solutions $\Rightarrow \therefore$ no x-intercepts.

4.



Observations:

- vertex : $(-3, 6)$ → HT left by 3 unit; → VT up by 6 units
- end behaviour : opens down → R in x-axis
- graph has the same shape as the original ⇒ no VS

Mapping notation:

$(x, y) \rightarrow (x-3, -y+6)$

Equation:

$y = -(x+3)^2 + 6$

y-int. :

$y = -(0+3)^2 + 6$

$y = -9 + 6$

$\underline{y = -3}$

$\therefore (0, -3)$

x-int. :

$0 = -(x+3)^2 + 6$

$\frac{-6}{-1} = \frac{-(x+3)^2}{-1}$

$\pm\sqrt{6} = \sqrt{(x+3)^2}$

$\pm\sqrt{6} = x+3$

$\pm\sqrt{6} - 3 = x$

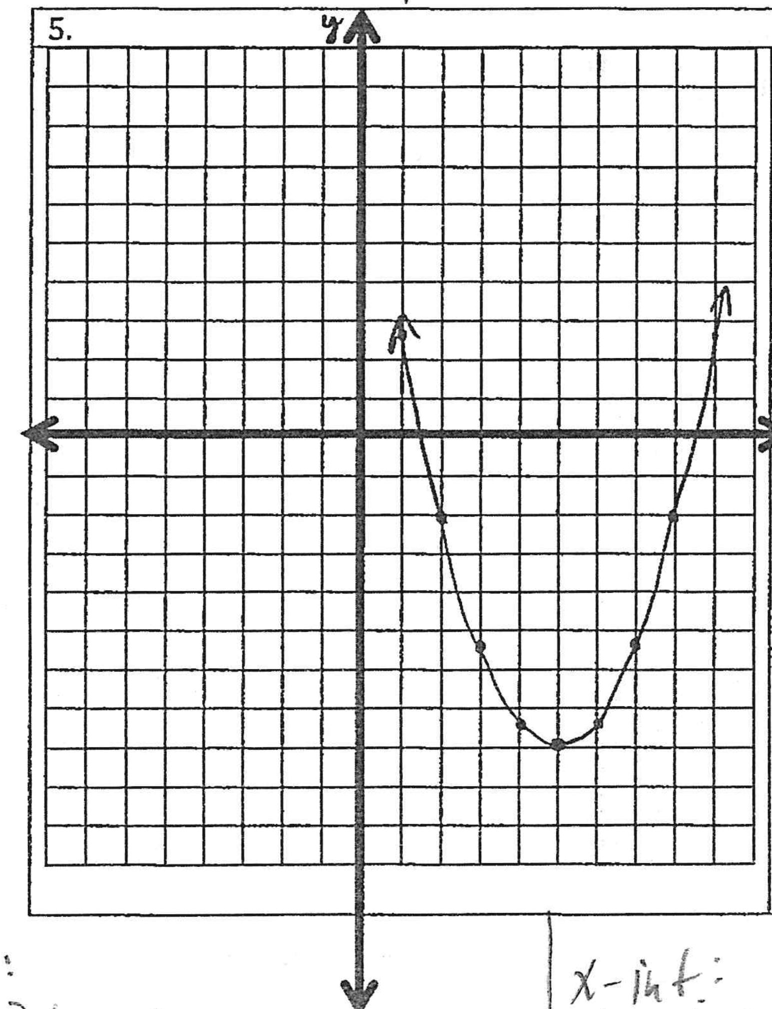
$\therefore (-\sqrt{6}-3, 0)$ and $(\sqrt{6}-3, 0)$.

*

$$\frac{a9}{9} = \frac{6}{9}$$

$$a = \frac{6}{9} = \frac{2}{3}$$

5.



Observations:

- vertex $(5,-8)$ → HT right by 5 units; → VT down by 8 units
- end behaviour: opens up ⇒ no R
- graph is wider *: VSC by a factor of $\frac{2}{3}$

Mapping notation:

$$(x,y) \to \left(x+5,\ \frac{2}{3}y-8\right)$$

OR $\to \left(x+5,\ \frac{2y}{3}-8\right)$

Equation:

$$y = \frac{2}{3}(x-5)^2-8$$

y-int:

$$y = \frac{2}{3}(0-5)^2-8$$

$$y = \frac{2}{3}(25)-8$$

$$y = \frac{50}{3} - \frac{24}{3}$$

$$y = \frac{26}{3}$$

$\therefore \left(0, \frac{26}{3}\right)$

x-int:

$$0 = \frac{2}{3}(x-5)^2-8$$

$$\frac{8}{\frac{2}{3}} = \frac{\frac{2}{3}(x-5)^2}{\frac{2}{3}}$$

$$\pm\sqrt{\frac{24}{2}} = \sqrt{(x-5)^2}$$

$$\frac{\pm\sqrt{24}}{\sqrt{2}} = x-5$$

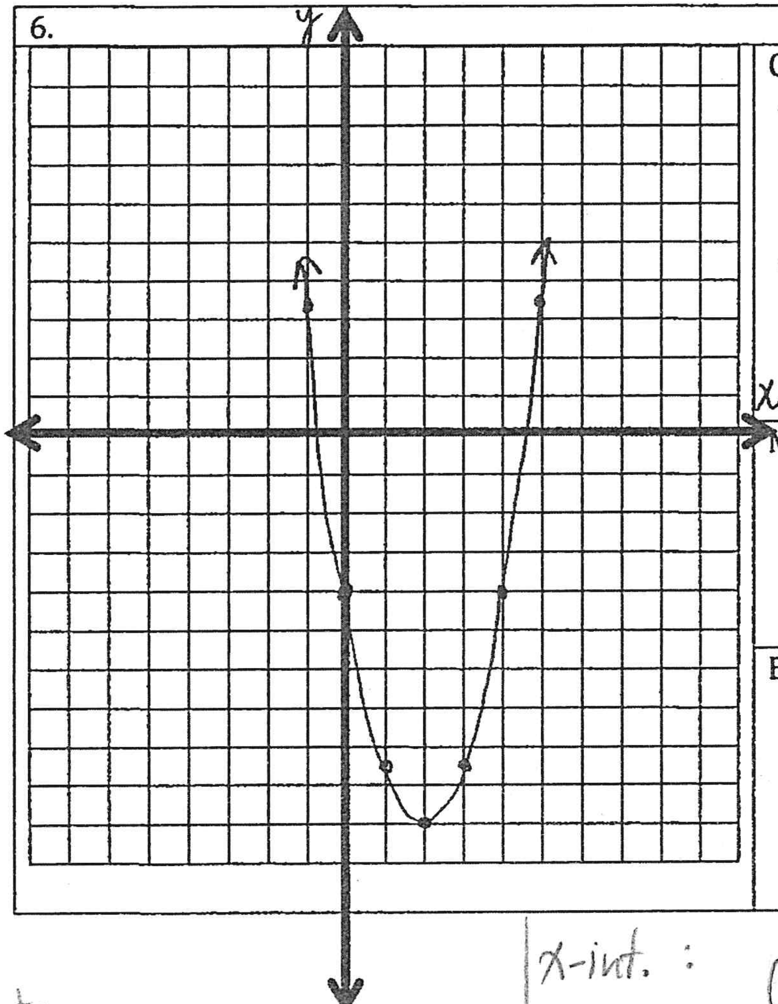
$$\frac{\pm 2\sqrt{6}\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} + 5 = x$$

$$\frac{\pm 2\sqrt{12}}{2} + 5 = x$$

$$\pm 2\sqrt{3}+5 = x$$

$\therefore (-2\sqrt{3}+5, 0)$ and $(2\sqrt{3}+5, 0)$.

6.



Observations:

- vertex $(2,-10)$ → HT right by 2 units; → VT down by 10 units
- end behaviour: opens up → no R
- graph is narrower → VSE by a factor of

$\frac{a4}{4} = \frac{6}{4}$

$a = \frac{3}{2} = 1.5$

Mapping notation:

$(x,y) \rightarrow (x+2,\ 1.5y-10)$

OR $(x+2,\ \frac{3y}{2}-10)$

Equation:

$y = 1.5(x-2)^2-10$

y-int:

$y = 1.5(0-2)^2-10$

$y = 1.5(4)-10$

$y = 6-10$

$y = -4$

$\therefore (0,-4)$

x-int.:

$0 = 1.5(x-2)^2-10$

$\frac{10}{1.5} = \frac{1.5(x-2)^2}{1.5}$

$\frac{10}{1} \times \frac{2}{3} = (x-2)^2$

$\pm\sqrt{\frac{20}{3}} = \sqrt{(x-2)^2}$

$\frac{\pm\sqrt{20}}{\sqrt{3}} = (x-2)$

$\frac{\pm 2\sqrt{5}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = x-2$

$\frac{\pm 2\sqrt{15}}{3} + 2 = x$

$\therefore \left(\frac{-2\sqrt{15}}{3}+2, 0\right)$ and $\left(\frac{2\sqrt{15}}{3}+2, 0\right)$