Notes

PC 11

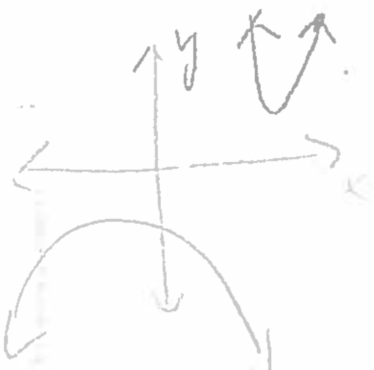
Solving a System of Quadratic Equations

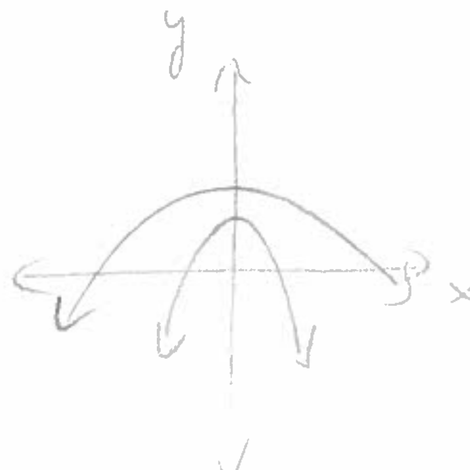
- There are 4 possible scenarios when solving a quadratic-quadratic system

1. There are no real solutions to the system

- When graphed, the parabolas never intersect
- When solving algebraically, the variables cancel out and the remaining statement is false OR the resulting equation does not have real solutions.

Examples:





Solve:

$$3x^2 = y$$
$$-x^2 - 2x - 35 = y$$

$$3x^2 = -x^2 - 2x - 35$$

$$0 = -4x^2 - 2x - 35$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-4)(-35)}}{2(-4)}$$

$$x = \frac{+4 \pm \sqrt{-556}}{-8} \rightarrow$$ $D < 0 \Rightarrow$ no $\mathbb{R}$ solutions

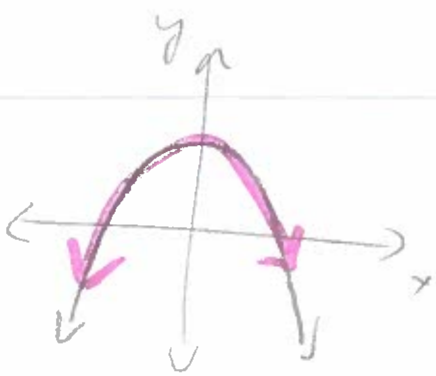
$\therefore$ The system has no $\mathbb{R}$ solutions.

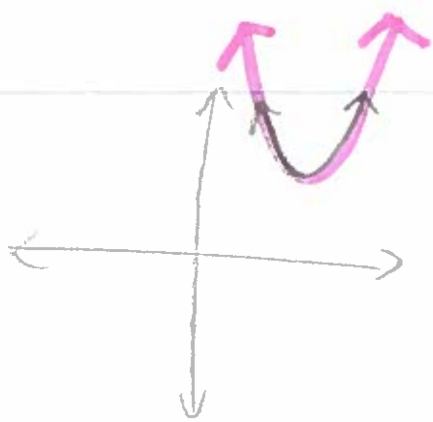
2. **There are infinitely many solutions to the system**

- When graphed, the parabolas coincide (overlap)
- When solving algebraically, the variables cancel out and the remaining statement is true

[redacted]

Examples:





Solve:

$$(x-1)^2-9 = y$$

$$x^2 - 2x - 8 = y$$

$(x-1)^2-9 = x^2-2x-8$

$\cancel{x^2}-\cancel{2x}+1-9 = \cancel{x^2}-\cancel{2x}-8$

$-8 = -8$

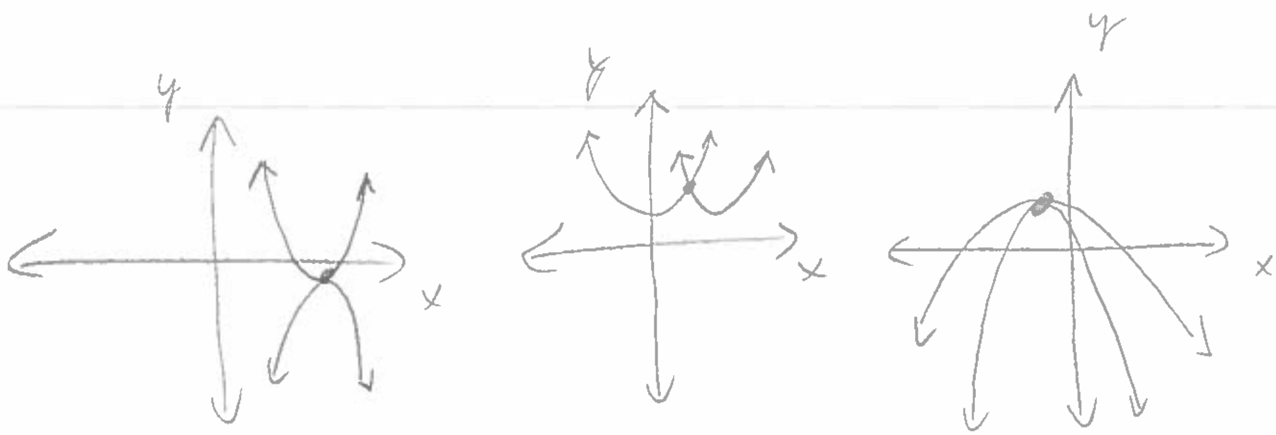
LS = RS ✓ ⇒ true statement

∴ The system has ifinitely many $\mathbb{R}$ solutions

3. **There is one solution to the system**

- When graphed, the parabolas either touch or intersect only once
- When solving algebraically, the variables raised to the power of two cancel out and the remaining equation is linear OR the resulting equation is a perfect square trinomial.

Examples:



Solve:

$$x^2 - 8x + 19 = y$$

$$x^2 - 16x + 59 = y$$

$\cancel{x^2} - 8x + 19 = \cancel{x^2} - 16x + 59$

$-8x + 16x = 59 - 19$

$\frac{8x}{8} = \frac{40}{8}$

$\boxed{x = 5}$

Find y:

$5^2 - 8(5) + 19 = y$

$25 - 40 + 19 = y$

$-15 + 19 = y$

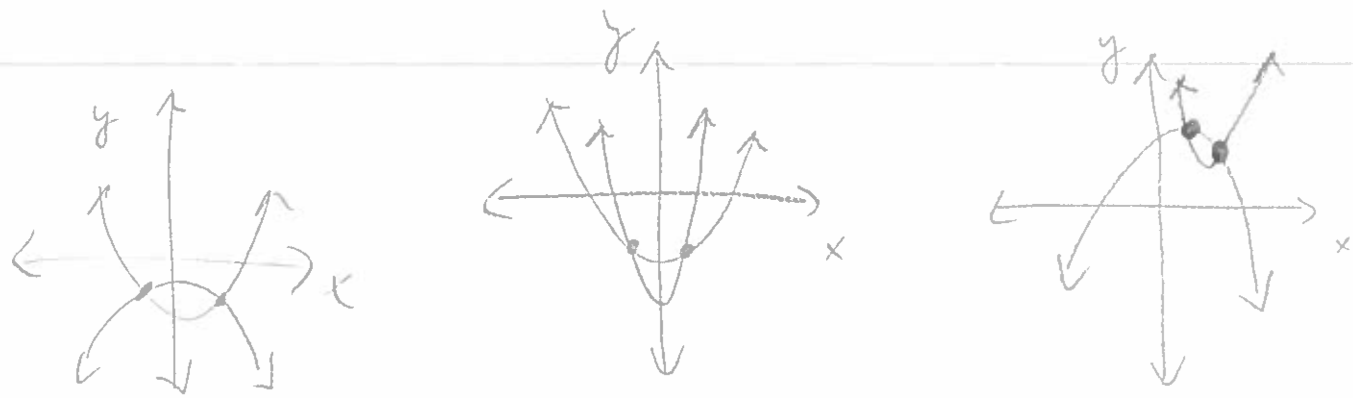
$y = 4$

∴ the solution to the system is $x = 5$ (OR (5,4)).

4. There are two solutions to the system

- When graphed, the parabolas intersect twice.
- When solving algebraically, the resulting equation has two solutions (discriminant is positive).

Examples:



Solve:

$$y = 3x^2 + 30x + 74$$

$$x^2 + 10x + 28 = y$$

$$3x^2 + 30x + 74 = x^2 + 10x + 28$$

$$2x^2 + 20x + 46 = 0$$

$$x^2 + 10x + 23 = 0$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(23)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{8}}{2} = \frac{-10 \pm 2\sqrt{2}}{2} = \frac{\overset{5}{\cancel{-10}} \pm \overset{1}{\cancel{2}}\sqrt{2}}{\cancel{2}\,1}$$

$$x = -5 + \sqrt{2}$$

$$x = -5 - \sqrt{2}$$

$\therefore$ the system has two solutions:

$x = -5 + \sqrt{2}$ and

$x = -5 - \sqrt{2}$