Notes:

PC 11

# SOLVING QUADRATIC EQUATIONS
# BY THE SQUARE ROOT PRINCIPLE

- This method is very fast when solving an equation that is either in vertex form or in standard form where b=0.
- This method makes it very obvious when there is no real solution to the equation:

$$\text{positive } \mathbb{R}(x \pm \ a\, number)^2 + \text{positive } \mathbb{R} \text{ number} = 0$$
$$\text{negative } \mathbb{R}(x \pm \ a\, number)^2 + \text{negative } \mathbb{R} \text{ number} = 0$$

OR

$$\text{positive}\mathbb{R}(x \pm \ a\, number)^2 = \text{negative } \mathbb{R} \text{ number}$$
$$\text{negative}\mathbb{R}(x \pm \ a\, number)^2 = \text{positive } \mathbb{R} \text{ number}$$

- This method makes is also very obvious when the quadratic equation has exactly one solution: $\mathbb{R}x^2 = 0$
- Any other form will result in two real solutions.

  - Remember that whenever you take a square root of a positive number you will get two answers: the positive root (=the principle root) and the negative root. You have to remember the negative root because your calculator is not as smart as you are and it does not give it to you.
  - In general, carry out all necessary steps to turn the quadratic equation into one of these forms:

$$(x \pm \ a\, number)^2 = \text{non-negative } \mathbb{R} \text{ number}$$

$$or$$

$$x^2 = \text{non-negative } \mathbb{R} \text{ number}$$

- **Then take a square root of both sides, remembering both roots.**
- **Simplify if necessary (= bring any numbers from the left side to the right side using the appropriate operation(s)).**

  - It is possible to solve an equation in standard form using the square root principle even when b ≠ 0, but you would have to complete the square first. It is much faster to use the quadratic formula.

Example 1: Solve $-5x^2 + 2 = 0$

$$\frac{-5x^2}{-5} = \frac{-2}{-5}$$

$$\sqrt{x^2} = \sqrt{\frac{2}{5}}$$

$$x = \pm\sqrt{\frac{2}{5}} = \pm\frac{\sqrt{2}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \pm\frac{\sqrt{10}}{5}$$

$\therefore$ The solutions are $x = \frac{\sqrt{10}}{5}$ and $x = \frac{-\sqrt{10}}{5}$.

Example 2: Solve $-(x + 3)^2 + 16 = 0$

$$\frac{-(x+3)^2}{-1} = \frac{-16}{-1}$$

$$\sqrt{(x+3)^2} = \sqrt{16}$$

$$x+3 = \pm 4$$

$$x = \pm 4 - 3$$

$$x = 4 - 3 = 1$$

$$x = -4 - 3 = -7$$

$\therefore$ The solutions are $x = 1$ and $x = -7$.

Practice:

(a) $x^2 - 81 = 0$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

$\therefore$ The solutions are $x = 9$ and $x = -9$.

(b) $4a^2 - 13 = 3$

$\frac{4a^2}{4} = \frac{16}{4}$

$\sqrt{a^2} = \sqrt{4}$

$a = \pm 2$

$\therefore$ The solutions are $a = 2$ and $a = -2$

(c) $(2y - 3)^2 - 25 = 0$

$\sqrt{(2y-3)^2} = \sqrt{25}$

$2y - 3 = \pm 5$

$\frac{2y}{2} = \frac{\pm 5 + 3}{2}$

$y = \frac{\pm 5 + 3}{2}$ *

(d) $(x + 1)^2 - 8 = 0$

$\sqrt{(x+1)^2} = \sqrt{8}$

$x + 1 = \pm \sqrt{8}$

$x = \pm \sqrt{8} - 1$ $\begin{cases} 2\sqrt{2} - 1 \\ -2\sqrt{2} - 1 \end{cases}$

$\therefore$ The solutions are $x = -1 + 2\sqrt{2}$ and $x = -1 - 2\sqrt{2}$

Answers: a) x=9 and x=-9, b) a=2 and a=-2, c) y=4 and y=-1, d) x=-1+2$\sqrt{2}$ and x=-1-2$\sqrt{2}$

(*) $y = \begin{cases} \frac{5+3}{2} = \frac{8}{2} = \underline{4} \\ \frac{-5+3}{2} = \frac{-2}{2} = \underline{-1} \end{cases}$

$\therefore$ The Solutions are $y = 4$ and $y = -1$.