PC11

KEY

Solving Quadratic Inequalities in One Variable

Practice

1. Solve $0 \leq (2x-1)(3+x)$

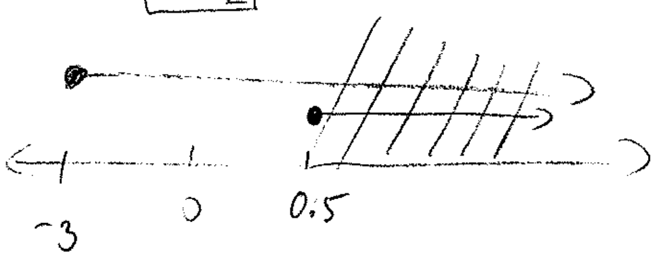
- The product of $(2x-1)(3+x)$ is positive. Thus two distinct cases exist:

Case I: (+)(+)

$2x-1 \geq 0$ $\qquad$ $3+x \geq 0$

$\frac{2x}{2} \geq \frac{1}{2}$ $\qquad$ $\boxed{x \geq -3}$

$\boxed{x \geq \frac{1}{2}}$



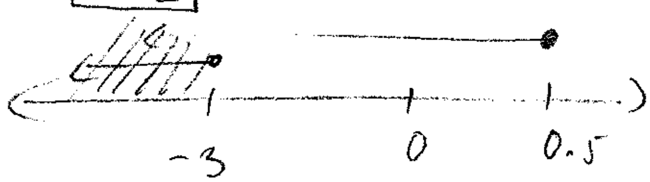
The solution is $x \geq 0.5$

Case II: (−)(−)

$2x-1 \leq 0$ $\qquad$ $3+x < 0$

$2x \leq 1$ $\qquad$ $\boxed{x < -3}$

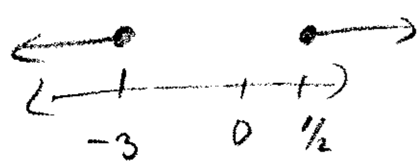
$\boxed{x \leq \frac{1}{2}}$



The solution is $x \leq -3$.

- Combining the solutions of the two cases, the overall solution is:

∴ the overall solution is



or $\qquad x \leq -3 \;\cup\; x \geq \frac{1}{2}$

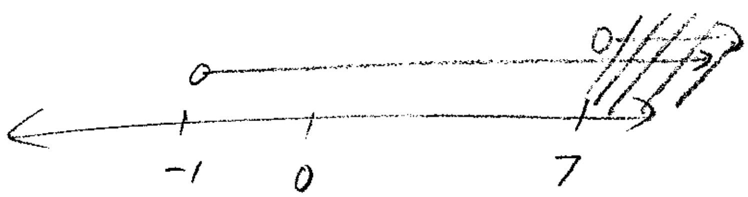
2. Solve $x^2 - 6x - 7 > 0$

- The product of $(x-7)(x+1)$ is <u>positive</u>. Thus two distinct cases exist:

Case I: (+)(+)

$x-7>0$ $\qquad$ $x+1>0$

$\boxed{x>7}$ $\qquad$ $\boxed{x>-1}$

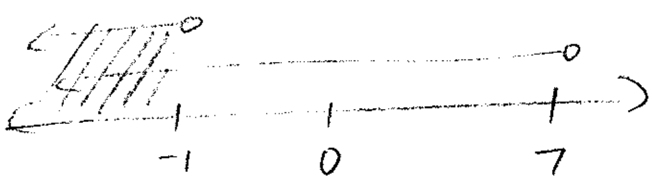


The solution is $x>7$.

Case II: (−)(−)

$x-7<0$ $\qquad$ $x+1<0$

$\boxed{x<7}$ $\qquad$ $\boxed{x<-1}$

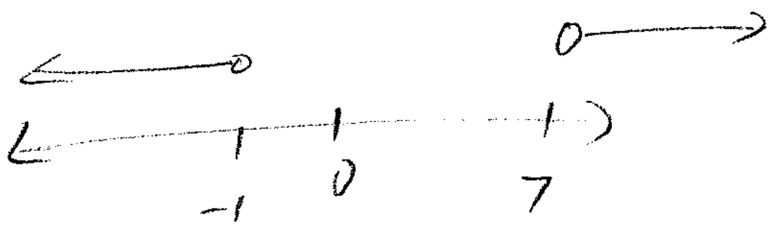


The solution is $x<-1$.

- Combining the solutions of the two cases, the overall solution is:

$\therefore$ The overall solution is $x<-1 \cup x>7$

OR



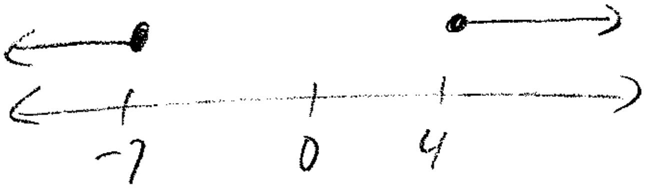
3. Solve $x^2 + 3x - 28 \leq 0$

- The product of $(x-4)(x+7)$ is negative. Thus two distinct cases exist:

Case I: (+)(−)

$x - 4 \geq 0$ $x + 7 \leq 0$

$\boxed{x \geq 4}$ $\boxed{x \leq -7}$

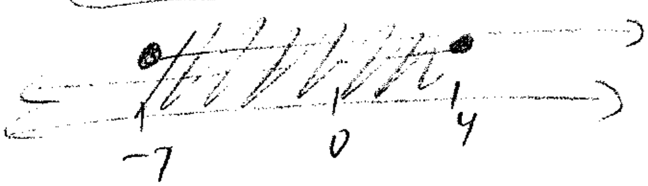


No solution

Case II: (−)(+)

$x - 4 \leq 0$ $x + 7 \geq 0$

$\boxed{x \leq 4}$ $\boxed{x \geq -7}$

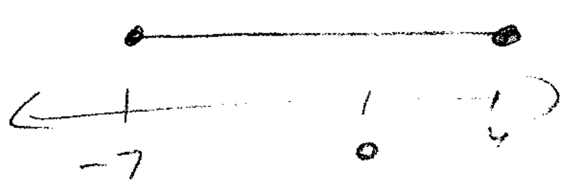


The solution is $-7 \leq x \leq 4$

- Combining the solutions of the two cases, the overall solution is:

$\therefore$ The overall solution is $-7 \leq x \leq 4$.

OR



4. Solve $3x^2 - 4x - 2 \leq 30$

$3x^2 - 4x - 32 \leq 0$

$a = 3$
$b = -4$
$c = -32$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-32)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{16 + 384}}{6}$$

$$x = \frac{4 \pm \sqrt{400}}{6}$$

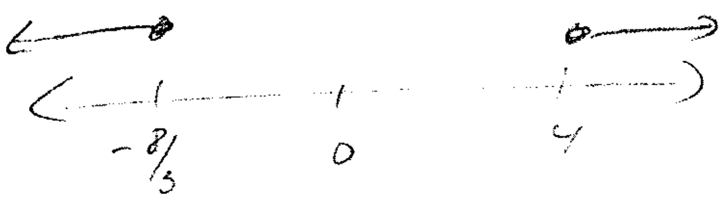
$$x = \frac{4 \pm 20}{6} \; \begin{cases} \underline{\underline{4}} \\ \frac{-16}{6} = \underline{\underline{\frac{-8}{3}}} \end{cases}$$

- The product of $(x-4)\left(x+\frac{8}{3}\right)$ is negative. Thus two distinct cases exist:

Case I: (+)(−)

$x - 4 \geq 0$ $\qquad$ $x + \frac{8}{3} \leq 0$

$\boxed{x \geq 4}$ $\qquad$ $\boxed{x \leq -\frac{8}{3}}$

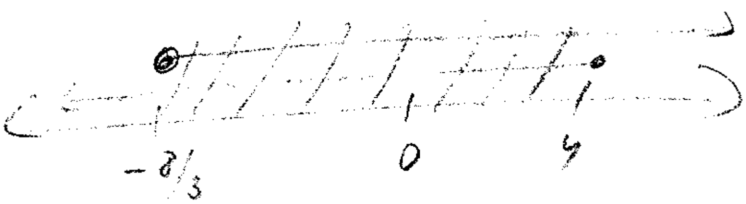


∴ No solution

Case II: (−)(+)

$x - 4 \leq 0$ $\qquad$ $x + \frac{8}{3} \geq 0$

$\boxed{x \leq 4}$ $\qquad$ $\boxed{x \geq -\frac{8}{3}}$

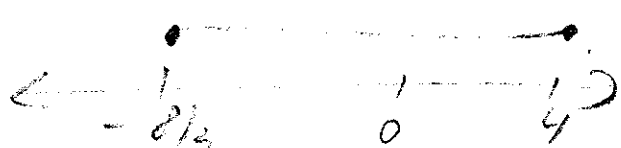


$\therefore -\frac{8}{3} \leq x \leq 4$

- Combining the solutions of the two cases, the overall solution is:

∴ The overall solution is $-\frac{8}{3} \leq x \leq 4$

OR



5. Solve $-x^2 \leq -36$

$$-x^2 + 36 \leq 0$$

$$\frac{-1(x^2 - 36)}{-1} \leq \frac{0}{-1}$$

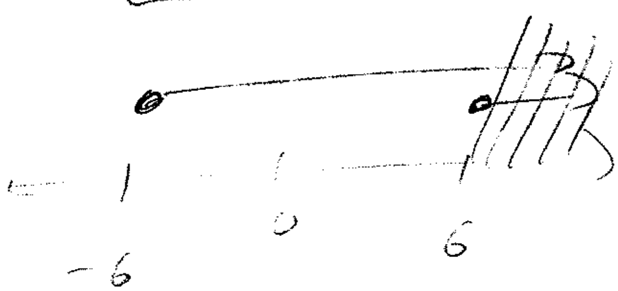
$$x^2 - 36 \geq 0 \longrightarrow (x-6)(x+6) \geq 0$$

- The product of $(x-6)(x+6)$ is positive. Thus two distinct cases exist:

Case I: (+)(+)

$x - 6 \geq 0$ $\quad\quad$ $x + 6 \geq 0$

$\boxed{x \geq 6}$ $\quad\quad$ $\boxed{x \geq -6}$

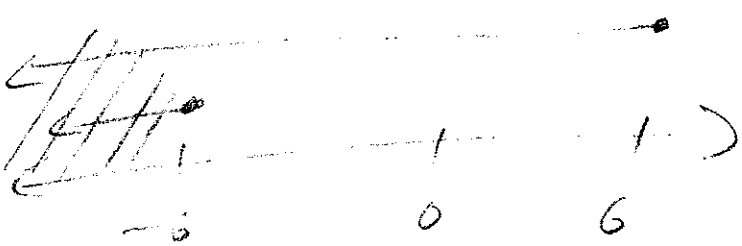


$\therefore x \geq 6$

Case II: (−)(−)

$x - 6 \leq 0$ $\quad\quad$ $x + 6 \leq 0$

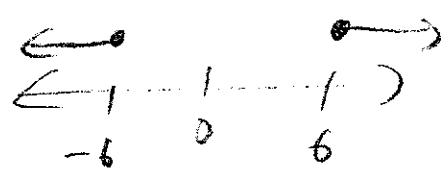
$\boxed{x \leq 6}$ $\quad\quad$ $\boxed{x \leq -6}$



$\therefore x \leq -6$

- Combining the solutions of the two cases, the overall solution is:

$\therefore$ The overall solution is



or $\quad x \geq 6 \cup x \leq -6$.

$5x(x+6)>0$

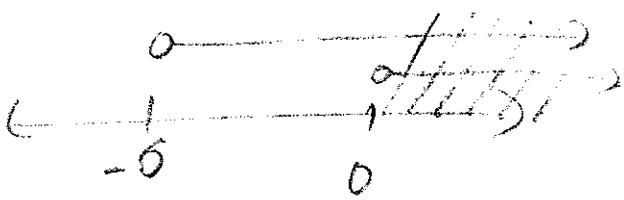
6. Solve $5x^2 + 30x > 0$

- The product of $(5x)(x+6)$ is positive. Thus two distinct cases exist:

Case I: $(+)(+)$

$5x > 0$ $\quad$ $x+6>0$

$\boxed{x>0}$ $\quad$ $\boxed{x>-6}$

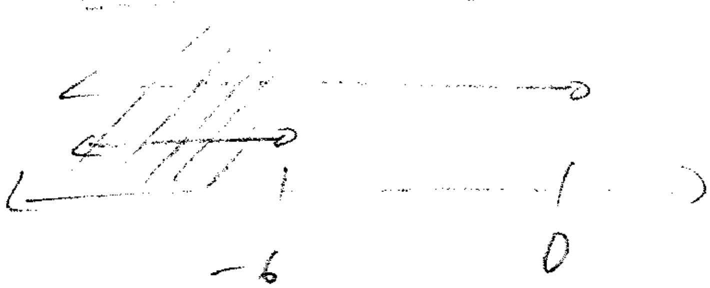


$\therefore\ x>0$

Case II: $(-)(-)$

$5x<0$ $\quad$ $x+6<0$

$\boxed{x<0}$ $\quad$ $\boxed{x<-6}$



$\therefore\ x<-6$

- Combining the solutions of the two cases, the overall solution is:

$\therefore$ the overall solution is $x<-6 \cup x>0$

or