PRE-CALCULUS 11

RATIONAL EXPRESSIONS AND EQUATIONS

- A rational expression is an algebraic expression of the form $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x)$ is not zero.
- A rational equation is and equation that contains at least one rational expression

Recall: A polynomial is an algebraic expression that consists of one term or more terms that are added. A term is always of the form: ___________________________ That is a term has to have a real number coefficient, one or more variables that is (are) raised to a non-negative **integral** exponent. The term that has a variable raised to the power of zero is called the _____________________ term as the variable evaluates to one and does not have to be written down. The term that has the highest degree is called the leading term and its coefficient is called the leading coefficient.

Examples of Rational Expressions:

Non-Examples of Rational Expressions:

Equivalent Expressions:

- Rational expressions that are multiple of each other are equivalent.
- Examples of equivalent expressions:

- Any rational expression is defined only if its denominator is different from zero.
- Any rational expression is undefined when its denominator is equal to zero.

Restriction for Rational Expressions:

- Restriction is a rule or a set of rules a variable (or variables) has (have) to follow in order for the denominator to be different from zero, making the expression defined.
- To determine restrictions for a rational expression, solve an inequality: **denominator $\neq$0 Solution(s) to this inequality is (are) the restriction(s) for the expression.**

Example 1: State the restrictions for the following expressions:

a) $\frac{x^2+x-1}{3+x}$

b) $\frac{x+4}{x^2-4x-12}$

c) $\frac{6x^2+3x-2}{x^2+2}$

d) $\frac{3x+12}{9}$

e) $\frac{x^5+14x+y}{xy}$

<u>Non-Permissible Values (NPVs) for Rational Expressions:</u>

- Non-permissible values are the values of a variable that lead to a zero in the denominator of a rational expression, thus making this expression undefined.
- To determine NPVs for a rational expression, solve **an equation: denominator = 0**
  Solution(s) to this equation is (are) the NPV(s)

Example 2: State the NPVs for the following expressions:

a) $\frac{x^2+x-1}{-13+x}$

b) $\frac{x+4}{x^2+x-30}$

c) $\frac{6x^2+3x-2}{4x^2+2}$

d) $\frac{3x+12}{9}$

e) $\frac{x^5+14x+y}{xy}$

<u>Simplifying Rational Expressions</u>

- Factor the numerator and the denominator if possible.
- Cancel out any common factors if they exist.
- Rewrite the expression without the common factors and in its factored form.

Examples:

A Perfect Square Trinomials

- Any perfect square trinomial is a result of a binomial being raised to the power of two = square = being multiplied by itself.

- Every perfect square trinomial has one of the two patterns above.

To determine whether a trinomial is a (perfect) square trinomials, follow these steps:

- Determine if the first term is a square expression. Rewrite the first term as $(\ )^2$.
- Determine if the last term is a square number or expression. Rewrite the last term as $(\ )^2$.
- Check if the middle term is twice what is in the brackets for the first and last term.

Example: Determine whether the given trinomial is a perfect square. If it is, factor it.

a) $16x^2 + 24x + 9$

b) $x^4 - 2x^2y + y^2$

c) $25x^2 + 10x + 1$

d) $36x^2 + 14x + 1$

e) $121a^2 + 110a - 25$

f) $x^6 + 4x^3 + 4$

g) $289x^2 - 68x + 4$