Notes

PC 11

Quadratic Inequalities in One Variable

Solving Algebraically

Example 1: Solve. $x^2 - 3x - 10 < 0$

- Factor: $(x-5)(x+2) < 0$
- The product of $(x-5)$ and $(x+2)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$(x-5) > 0$ $\qquad$ $x+2 < 0$

$x-5 > 0$ $\qquad$ $x < -2$

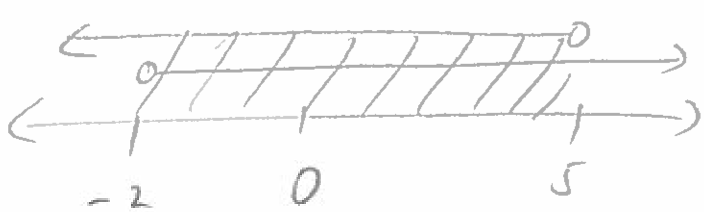
$x > 5$

→ no overlap ⇒ no solution

Case II: (-)(+)

$x-5 < 0$ $\qquad$ $x+2 > 0$

$x < 5$ $\qquad$ $x > -2$



Combining the results of the two cases gives the overall solution:

∴ the solution is 

or $-2 < x < 5$.

$a = 1$
$b = -7$
$c = 8$

$(*)$ $\dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(8)}}{2(1)}$ $\quad \dfrac{7+\sqrt{17}}{2} \doteq 5.6$ $\quad \dfrac{7-\sqrt{17}}{2} \doteq 1.4$

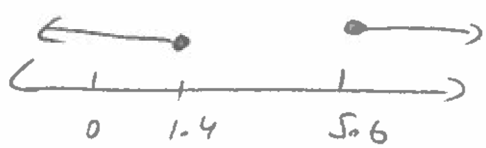
Example 2: Solve. $x^2 - 7x - 7 \leq -15$

- Set the left side less or equal to zero $x^2 - 7x + 8 \leq 0$
- Factor: Not factorable. Use the quadratic formula to find the x-intercepts. $(*)$
- The product of $(x - 5.6)$ and $(x - 1.4)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$x - 5.6 \geq 0$ $\qquad$ $x - 1.4 \leq 0$

$x \geq 5.6$ $\qquad$ $x \leq 1.4$

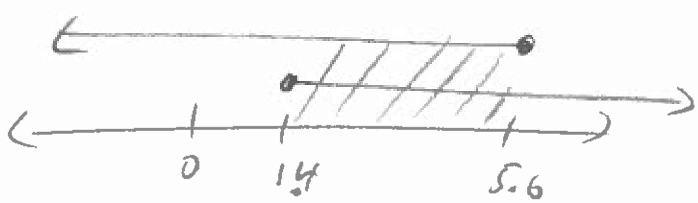


– no overlap ⇒ no solution

Case II: (-)(+)

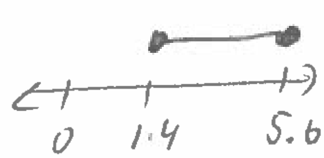
$x - 5.6 \leq 0$ $\qquad$ $x - 1.4 \geq 0$

$x \leq 5.6$ $\qquad$ $x \geq 1.4$



Combining the results of the two cases gives the overall solution:

∴ the Solution is



OR

$\dfrac{7-\sqrt{17}}{2} \leq x \leq \dfrac{7+\sqrt{17}}{2}$ $\qquad$ $1.4 \leq x \leq 5.6$

Example 3: Solve. $3x^2 + 3x \leq 60$

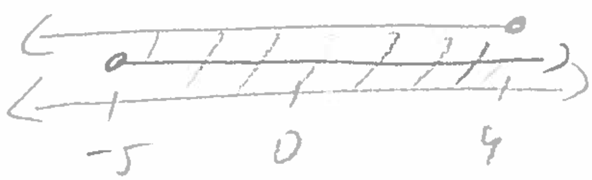
- Set the left side less or equal to zero $3x^2 + 3x - 60 \leq 0$
- Factor: $3(x^2 + x - 20) \leq 0 \rightarrow 3(x+5)(x-4) \leq 0$

- The product of $(x+5)$ and $(x-4)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$x+5 \geq 0$ $\quad\quad$ $x-4 \leq 0$

$x \geq -5$ $\quad\quad$ $x \leq 4$



Case II: (-)(+)

$x+5 \leq 0$ $\quad\quad$ $x-4 \geq 0$

$x \leq -5$ $\quad\quad$ $x \geq 4$

no solution

Combining the results of the two cases gives the overall solution:

∴ The solution is

OR $-5 \leq x \leq 4$

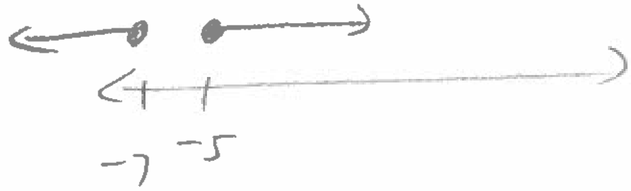
Example 4: *Solve.* $x^2 + 12x \leq -35$

- Set the left side less or equal to zero $x^2+12x+35 \leq 0$
- Factor: $(x+5)(x+7) \leq 0$
- The product of $(x+5)$ and $(x+7)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$x+5 \geq 5$ $\qquad$ $x+7 \leq 0$

$x \geq -5$ $\qquad$ $x \leq -7$

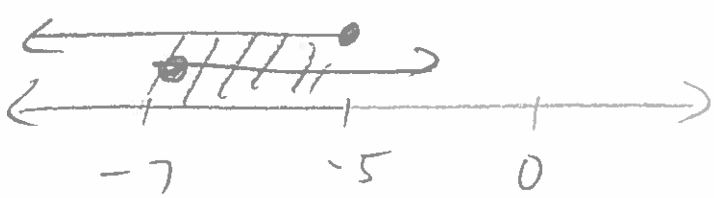


→ no solution

Case II: (-)(+)

$x+5 \leq 0$ $\qquad$ $x+7 \geq 0$

$x \leq -5$ $\qquad$ $x \geq -7$



Combining the results of the two cases gives the overall solution:

∴ the solution is