

PC 11

Quadratic Inequalities in One Variable

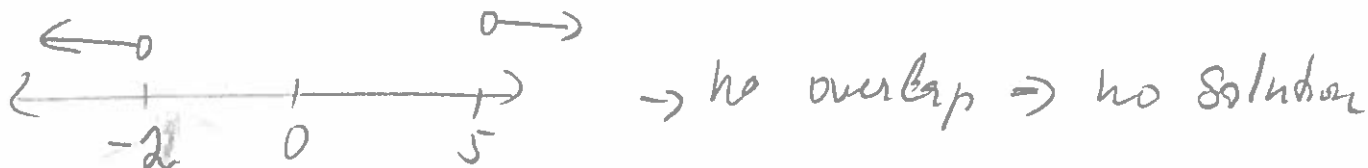
Solving Algebraically

Example 1: Solve. $x^2 - 3x - 10 < 0$

- Factor: $(x-5)(x+2) < 0$
- The product of $(x-5)$ and $(x+2)$ is negative, thus there are 2 cases to consider:

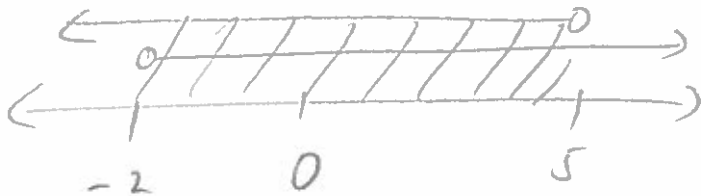
Case I: (+)(-)

$$\begin{array}{l} (x-5) > 0 \\ x-5 > 0 \\ x > 5 \end{array} \qquad \begin{array}{l} x+2 < 0 \\ x < -2 \end{array}$$



Case II: (-)(+)

$$\begin{array}{l} x-5 < 0 \\ x < 5 \end{array} \qquad \begin{array}{l} x+2 > 0 \\ x > -2 \end{array}$$



Combining the results of the two cases gives the overall solution:

\therefore The solution is or $-2 < x < 5$.

$$\begin{aligned} a &= 1 \\ b &= -7 \\ c &= 8 \end{aligned}$$

(*)

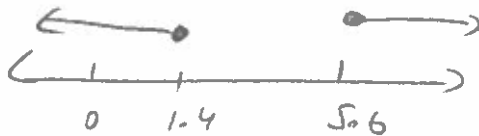
$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(8)}}{2(1)} \begin{cases} \frac{7 + \sqrt{17}}{2} \approx 5.6 \\ \frac{7 - \sqrt{17}}{2} \approx 1.4 \end{cases}$$

Example 2: Solve. $x^2 - 7x - 7 \leq -15$

- Set the left side less or equal to zero $x^2 - 7x + 8 \leq 0$
- Factor: Not factorable. Use the quadratic formula to find the x-intercepts. ⊕
- The product of $(x - 5.6)$ and $(x - 1.4)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

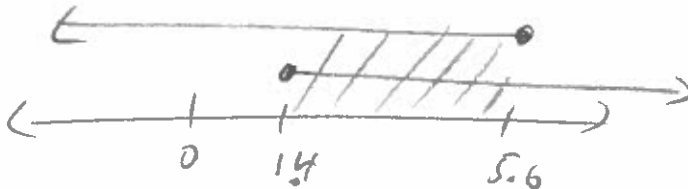
$$\begin{aligned} x - 5.6 &\geq 0 & x - 1.4 &\leq 0 \\ x &\geq 5.6 & x &\leq 1.4 \end{aligned}$$



- no overlap \Rightarrow no solution

Case II: (-)(+)

$$\begin{aligned} x - 5.6 &\leq 0 & x - 1.4 &\geq 0 \\ x &\leq 5.6 & x &\geq 1.4 \end{aligned}$$



Combining the results of the two cases gives the overall solution:

\therefore The solution is  OR

$$\frac{7 - \sqrt{17}}{2} \leq x \leq \frac{7 + \sqrt{17}}{2}$$

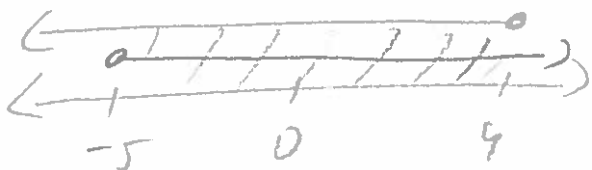
$$1.4 \leq x \leq 5.6$$

Example 3: Solve. $3x^2 + 3x \leq 60$

- Set the left side less or equal to zero $3x^2 + 3x - 60 \leq 0$
- Factor: $3(x^2 + x - 20) \leq 0 \rightarrow 3(x+5)(x-4) \leq 0$
- The product of $(x+5)$ and $(x-4)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$$\begin{array}{l} x+5 \geq 0 \\ x \geq -5 \end{array} \quad \begin{array}{l} x-4 \leq 0 \\ x \leq 4 \end{array}$$

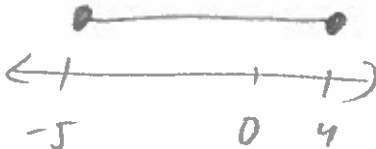


Case II: (-)(+)

$$\begin{array}{l} x+5 \leq 0 \\ x \leq -5 \end{array} \quad \begin{array}{l} x-4 \geq 0 \\ x \geq 4 \end{array}$$



Combining the results of the two cases gives the overall solution:

\therefore The solution is  OR $-5 \leq x \leq 4$

The number line shows a solid line segment between -5 and 4, with closed circles at both ends. Arrows point outwards from the ends of the number line.

Example 4: Solve. $x^2 + 12x \leq -35$

• Set the left side less or equal to zero $x^2 + 12x + 35 \leq 0$

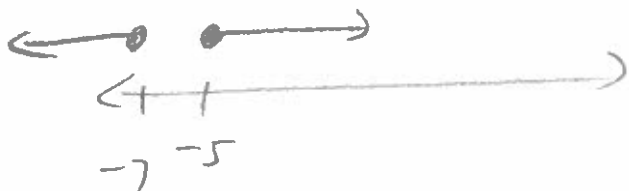
• Factor: $(x+5)(x+7) \leq 0$

• The product of $(x+5)$ and $(x+7)$ is negative, thus there are 2 cases to consider:

Case I: (+)(-)

$$x+5 \geq 5 \\ x \geq -5$$

$$x+7 \leq 0 \\ x \leq -7$$

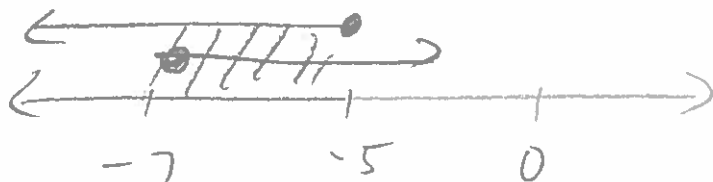


\rightarrow no solution

Case II: (-)(+)

$$x+5 \leq 0 \\ x \leq -5$$

$$x+7 \geq 0 \\ x \geq -7$$



Combining the results of the two cases gives the overall solution:

\therefore The solution is

A number line with tick marks at -7 and -5. Two solid dots are placed at -7 and -5. A horizontal line segment connects the two dots, representing the solution set $[-7, -5]$.