Quadratic Functions Expansions, Compressions

1. Sketch the graph for each of the following without a calculator (label vertex and the coordinates of two other points) and then state:

   a) equation of axis of symmetry
   b) direction of opening and by what factor the graph has been stretched of compressed vertically
   c) the maximum or minimum value
   d) exact values of the x-intercept(s) (if any) and the y-intercept
   e) the domain and range

   i) $y = 2x^2 - 4$

   ii) $y = -\frac{1}{3}x^2 + 2$

   iii) $y = 4(x-2)^2 + 1$

   iv) $y = \frac{1}{2}(x+1)^2 - 2$

   v) $y = -5(x+3)^2$

   vi) $y = 2(x-1)^2$

   vii) $y = -3(x+3)^2 + 6$

   viii) $y = -(x-1)^2 - 3$

2. Write the new equation of the parabola $y = x^2$ if:

   a) it undergoes a horizontal translation 2 units to the left and a vertical translation 5 units down and is congruent to $y = 3x^2$.

   b) it undergoes a horizontal translation 2 units right and a vertical translation 3 units up and congruent to $y = -\frac{1}{2}x^2$.

   c) the parabola opens downwards and has been stretched vertically by a factor of 4.

   d) the parabola opens upwards and has been compressed vertically by a factor of one-third.

3. The path a tennis ball takes from a players forehand ground stroke can be modelled by the function: $h(d) = -0.015(d-9)^2 + 1.6$, where $h(d)$ is the height of the ball in metres and d is the horizontal distance the ball has travelled in metres since it was struck.

a) What is the maximum height the tennis ball reaches?

b) How far has the ball travelled horizontally from where it was struck when it reaches its maximum height?

c) What was the height of the ball when it was struck?

d) How far did the ball travel horizontally from where it was struck to where it landed inside the court?

e) If the opponent is standing 11 meters away, at what height would he make contact with the ball to volley it back?

f) State what represents the domain and the range in this example, then list both the domain and the range

4. The path a hit baseball takes after leaving a bat can be modelled by the function: $h(t) = -5.2(t-2.8)^2 + 41.5$, where $h(t)$ is the height of the ball in metres and t is the elapsed time in seconds since the ball was hit.

a) What is the maximum height the baseball reaches?

b) What was the height of the baseball when it was struck?

c) How long was the ball in the air before it landed on the ground?

d) What was the height of the ball after 4 seconds?

e) How long was the ball in the air if an outfielder caught the ball one meter off the ground?

f) State what represents the domain and the range in this example, then list both the domain and the range

5. The path a baseball takes after being hit can be modelled by the function $h(d) = -0.0095(d-60)^2 + 35$, where $h(d)$ is the height of the ball in metres and d is the horizontal distance the ball has travelled in metres since it was struck.

a) What is the maximum height the baseball reaches?

b) How far has the ball travelled horizontally from where it was struck when it reaches its maximum height?

c) Calculate the horizontal distance the ball travelled?

d) The ball went over the fence 112 meters away, if the fence was 3 meters tall, by how much did the ball clear the fence?

e) How far had the ball travelled when it was 20 meters high for the first time?

f) State what represents the domain and the range in this example, then list both the domain and the range.

Answers

1*ia*) $x = 0$

1*ib*) *up, stretched by a factor of* 2

1*ic*) min *of* -4 *when* $x = 0$

1*id*) $(\sqrt{2},0)\ (-\sqrt{2},0)\ (0,-4)$

1*ie*) *Domain*: $x \in R$ *Range*: $y \geq -4$

1*iia*) $x = 0$

1*iib*) *down, compressed by a factor of* $\frac{1}{3}$

1*iic*) max *of* 2 *when* $x = 0$

1*iid*) $(\sqrt{6},0)\ (-\sqrt{6},0)\ (0,2)$

1*iie*) *Domain*: $x \in R$ *Range*: $y \leq 2$

1*iiia*) $x = 2$

1*iiib*) *up, stretched by factor of* 4

1*iiic*) min *of* 1 *when* $x = 2$

1*iiid*) *none* $(0,17)$

1*iiie*) *Domain*: $x \in R$ *Range*: $y \geq 1$

1*iva*) $x = -1$

1*ivb*) *up, compressed by a factor of* $\frac{1}{2}$

1*ivc*) min *of* -2 *when* $x = -1$

1*ivd*) $(-3,0)\ (1,0)\ \left(0,-\frac{3}{2}\right)$

1*ive*) *Domain*: $x \in R$ *Range*: $y \geq -2$

1*va*) $x = -3$

1*vb*) *down, stretched by a factor of* 5

1*vc*) max *of* 0 *when x is* -3

1*vd*) $(-3,0)\ (0,-45)$

1*ve*) *Domain*: $x \in R$ *Range*: $y \leq 0$

1*via*) $x = 1$

1*vib*) *up, stretched by a factor of* 2

1*vic*) min *of* 0 *when x is* 1

1*vid*) $(1,0)\ (0,2)$

1*vie*) *Domain*: $x \in R$ *Range*: $y \geq 0$

1*viia*) $x = -3$

1*viib*) *down, stretched by a factor of* 3

1*viic*) max *of* 6 *when x is* -3

1*viid*) $(-3-\sqrt{2},0)\ (-3+\sqrt{2},0)$ y-intercept $(0,-21)$

1*viie*) *Domain*: $x \in R$ *Range*: $y \leq 6$

1*viiia*) $x = 1$

1*viiib*) *down, no stretched factor*

1*viiic*) max *of* -3 *when x is* 1

1*viid*) x-intercept: NONE y-intercept $(0,-4)$

1*viie*) *Domain*: $x \in R$ *Range*: $y \leq -3$

2a) $y = 3(x+2)^2 - 5$

2b) $y = -\frac{1}{2}(x-2)^2 + 3$

2c) $y = -4x^2$

2d) $y = \frac{1}{3}x^2$

3a) $1.6m$

3b) $9m$

3c) $0.385m$

3d) $19.3m$

3e) $1.54m$

3f) *Domain* : $0 \leq x \leq 19.3$

Range : $0 \leq y \leq 1.6$

4a) $41.5m$

4b) $0.732m$

4c) $5.63s$

4d) $34.01m$

4e) $5.59m$

4f) *Domain* : $0 \leq x \leq 5.63$

Range : $0 \leq y \leq 41.5$

5a) $35m$

5b) $60m$

5c) $120.7m$

5d) $6.312m$

5e) $20.26m$

5f) *Domain* : $0 \leq x \leq 120.7$

Range : $0 \leq y \leq 35$