Name:

UNIT 4 LEARNING GUIDE – QUADRATICS

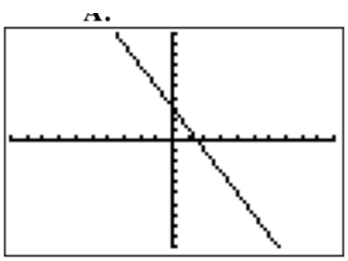
INSTRUCTIONS:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL of your work as is explained in the lessons. Do your best and always ask questions if there is anything that you don't understand.

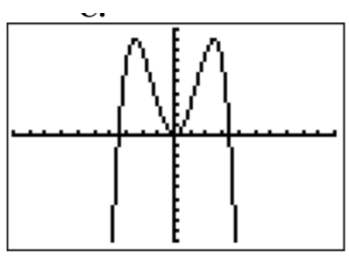
4.1 GRAPHS

1. Give the degree and name of the function shown in each graph.

Example 1 - Linear



a) ________________



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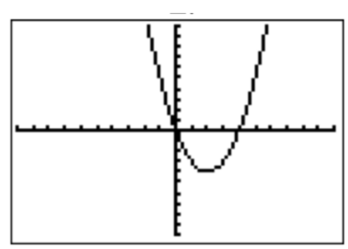

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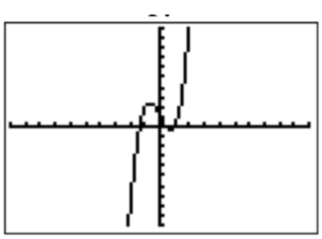
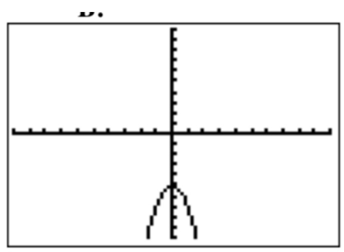
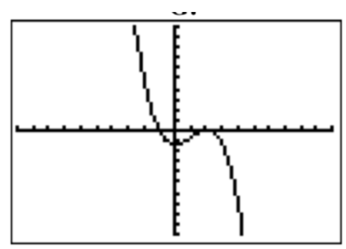


d) ________________



e) _________________



2. Which of the following relations are quadratic functions?

a) $y = 3x^2 + 7x - 2$ b) $f(x) = x^2 + \sqrt{x}$ c) $f(x) = 25 - 9x^2$

d) $y = 7 - 5x^2$ e) $y = 2x^2 + 11x - 4$ f) $f(x) = \frac{1}{4x^2 - 9x + 12}$

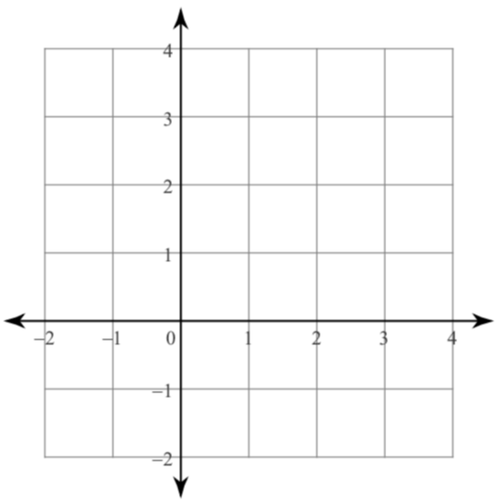
3. If you have a polynomial of degree n where n is even, how many x intercepts will there be?

4. If you have a polynomial of degree n where n is odd, how many x intercepts will there be?

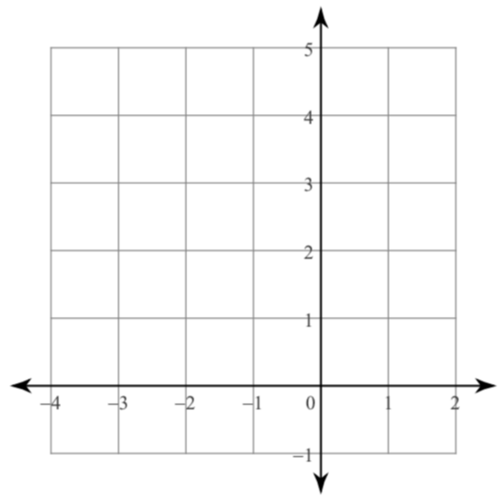
4.2 TRANSLATIONS

1. Sketch the graphs for each of the following parabolas:

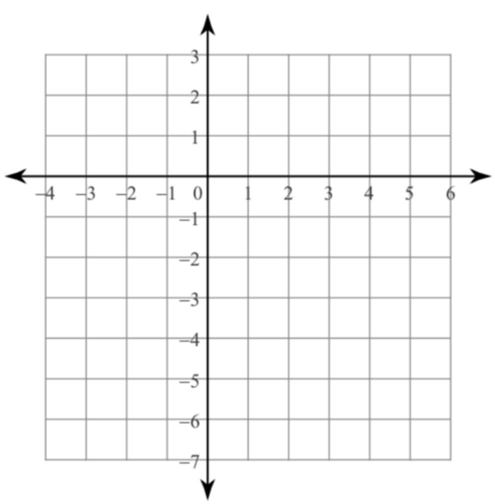
a) $f(x) = x^2 - 2x$



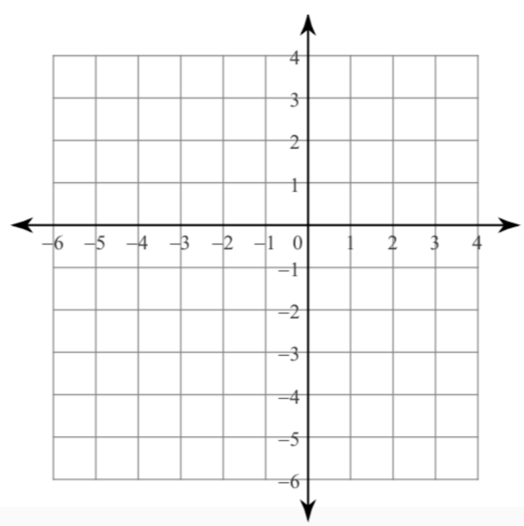
b) $f(x) = -x^2 - 2x + 3$



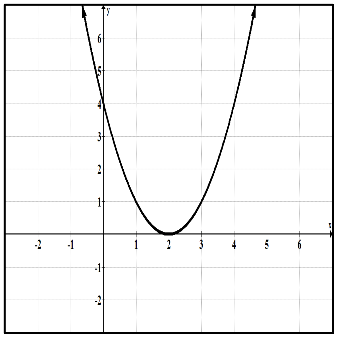
c) $f(x) = -2x^2 + 4x$

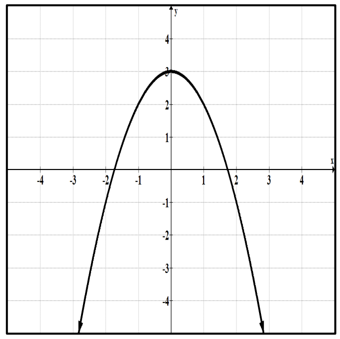


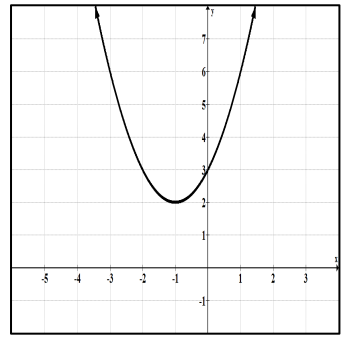
d) $f(x) = -2x^2 - 8x - 5$



2. Determine the equation of the following parabolas.

a) 

b) 

c) 

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3. Write the new equation for the parabola $y = x^2$ after the following:

a) a horizontal translation 2 units left and a vertical translation 7 units up.

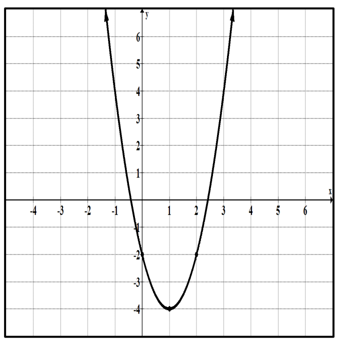
b) a horizontal translation 8 units right and a vertical translation 11 units down.

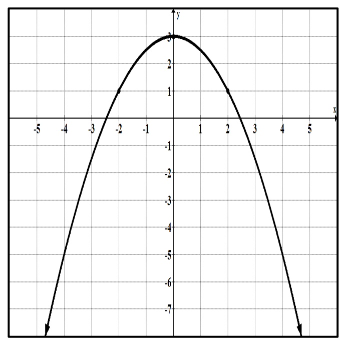
c) the parabola opens down and is translated 5 units down.

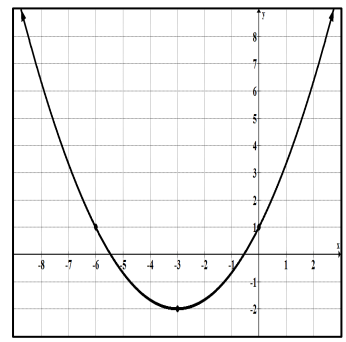
d) the parabola opens down and is translated 1 unit left and 3 units up.

4.3 EXPAND AND COMPRESS

1. Determine the equation of the following parabolas.

a) 

b) 

c) 

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2. Write the new equation for the parabola $y = x^2$ after the following:

a) a horizontal translation 3 units left and is congruent to $y = 7x^2$.

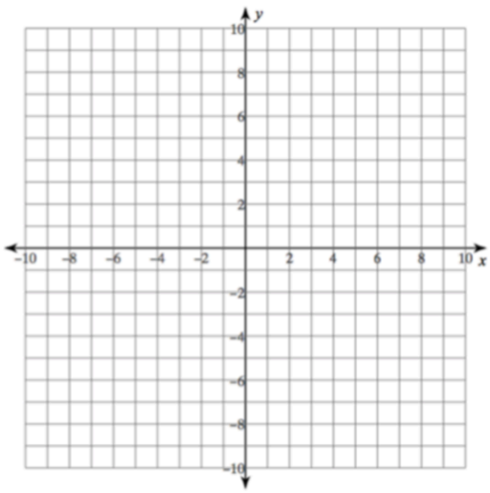
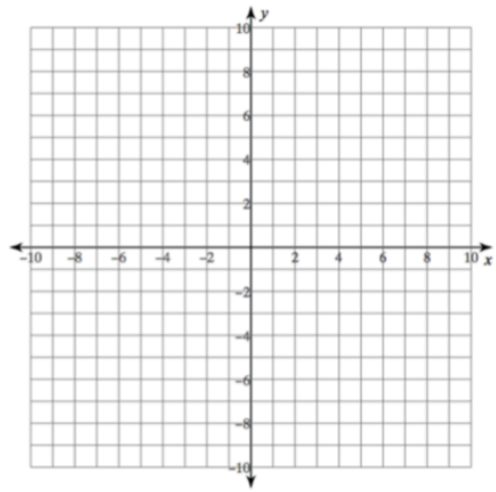
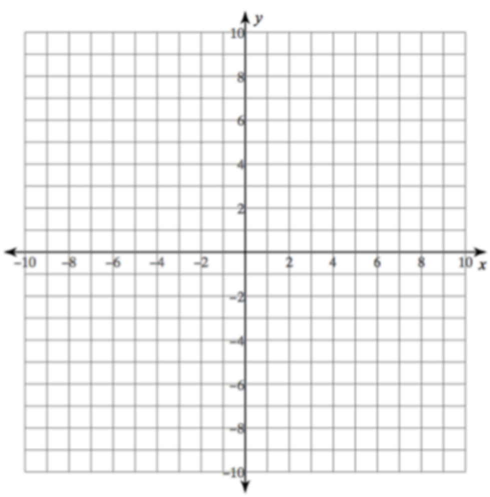
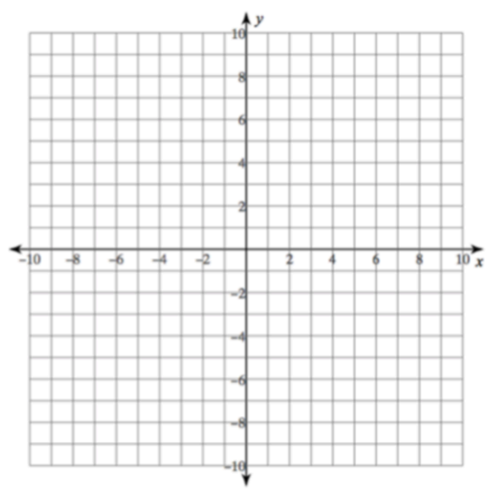
b) a horizontal translation 5 units right, a vertical translation 6 units down and congruent to $y = -\frac{1}{5}x^2$

c) the parabola opens downwards, then has been expanded vertically by a factor 5 and translated 7 units down.

d) the parabola opens upwards, then has been vertically compressed by a factor of $\frac{1}{3}$, translated 2 units left and 9 units up.

4.4 SOLVING FOR INTERCEPTS

1. Solve each quadratic equation by graphing.

a) $y=(x - 4)^2-1$	b) $y = 2(x - 1)^2 - 2$
	
c) $y = 2(x - 3)^2$	d) $y = -(x + 3)^2 + 4$
	

2. Solve by factoring.

a) $\mathrm{m}^2 - 12\mathrm{m} + 35 = 0$

b) $4\mathrm{c}^2 - 49 = 0$

c) $\mathrm{y}^2 - 11\mathrm{y} = 0$

d) $2\mathrm{x}^2 - 7\mathrm{x} - 15 = 0$

3. Solve by expanding, grouping like terms, and then factoring.

a) $m^2 + 8m = 3m + 24$

b) $x(x - 6) = 2(x - 8)$

c) $(2x - 1)(x - 3) = (x + 1)(x - 2)$

d) $(2p - 1)^2 - 3 = (p - 2)(p - 1)$

4. Solve using the quadratic formula. Give exact answers as integers, fractions, or simplest radical form.

a) $2x^2 - 5x + 2 = 0$

b) $3n^2 - 11n - 14 = 0$

c) $f^2 - 6f + 4 = 0$

d) $x^2 + 7x + 3 = 0$

e) $6t^2 - t - 1 = 0$

f) $5h^2 + 7h + 2 = 0$

5. Solve exactly using the quadratic formula.

a) $25q^2 + 70q + 49 = 0$

b) $12v^2 - 192 = 0$

c) $3r^2 - 4r = 0$

d) $5x^2 + 12x + 9 = 0$

6. Solve using the quadratic formula. Express answers to the nearest hundredth.

a) $0.2z^2 - z - 3.2 = 0$

b) $1.2x^2 = 1.4x - 1$

7. Solve using the quadratic formula. Answer in simplest radical form.

a) $2x^2 + 6x + 2 = 0$

b) $z^2 - 6z + 7 = 0$

8. Solve using the quadratic formula.

a) $(n - 4)(n - 2) = 12$

b) $(2x - 1)(3x + 5) = (x + 2)(2x - 1)$

c) $3m^2 - (5m + 1)(2m - 3) = 3$

d) $2(w - 2)(w + 1) - (w + 3) = 0$

9. Calculate the value of the discriminant and then state the nature of the roots.

a) $x^2 + 11x + 24 = 0$

b) $n^2 - 4n + 2 = 0$

c) $4m^2 - 20m + 25 = 0$

d) $2x^2 - 5x + 8 = 0$

e) $\sqrt{5}x^2 + 7x + 2\sqrt{5} = 0$

f) $\frac{x-1}{2} - x^2 = 3$

4.5 Identifying Characteristics

1. Complete the square for each of the following expressions

a) $x^2 + 6x + 3$

b) $x^2 - 10x - 6$

c) $x^2 + 20x + 160$

d) $x^2 - 5x + 2$

e) $x^2 + x + 1$

f) $x^2 - x + 1$

2. Complete the square for each of the following expressions

a) $2x^2 + 12x + 14$

b)) $3x^2 - 3x + 1$

c) $5x^2 + 4x + 3$ d) $4x^2 - 10x - 6$

3. Find the maximum or minimum value of each quadratic function. State the co-ordinate and whether it is a minimum or maximum.

a) $f(x) = x^2 + 10x + 19$ b) $f(x) = -9x^2 + 42x - 53$

c) $f(x) = 2x^2 - 20x - 13$ d) $f(x) = -16x^2 + 24x - 5$

e) $f(x) = -4x^2 + 6x + 18$ f) $f(x) = -5x^2 + 16x - 7$

4. Determine the following for each quadratic. Show all work

a) $f(x) = -x^2 + 3x + 4$

Extrema

Line of Symmetry

X-intercepts

Direction of opening

Domain

Range

b) $f(x) = x^2 + 5x + 4$

Extrema

Line of Symmetry

X-intercepts

Direction of opening

Domain

Range

c) $f(x) = 2x^2 - 4x + 2$

Extrema

Line of Symmetry

X-intercepts

Direction of opening

Domain

Range

d) $f(x) = -x^2 + 3x + 10$

Extrema

Line of Symmetry

X-intercepts

Direction of opening

Domain

Range

4.6 APPLICATIONS

1. The height of a soccer ball, h (meters), as a function of the horizontal distance, d (meters), the ball travels until it first hits the ground is described by the function

$$h(d) = -0.025(d - 20)^2 + 10$$

Use Desmos.com to graph the function and answer the following questions:

a) What is the maximum height of the ball?

b) What is the horizontal distance of the ball from the kicker when it reaches its maximum height?

c) How far does the ball travel horizontally from when it is kicked until it hits the ground?

d) What is the height of the ball when it is 10*m* horizontally from the kicker?

e) Would an opposing player positioned under the path of the ball 34*m* from the kicker be able to head the ball? Explain.

2. The path a baseball takes after being hit can be modeled by the function $h(d) = -0.0095(d - 67)^2 + 43$, where h(d) is the height of the ball in meters and the horizontal distance the ball has travelled since it was struck.

Use Desmos.com to graph the function and answer the following questions:

a) What is the maximum height the baseball reaches?

b) How far has the ball travelled horizontally from where it was struck when it reaches its maximum height?

c) Calculate the horizontal distance the ball travelled?

d) The ball went over the fence 124 meters away, if the fence was 4 meters tall, by how much did the ball clear the fence?

e) How far had the ball travelled when it was 20 meters high for the first time?

f) State what represents the domain and the range in this example, then list both the domain and the range.

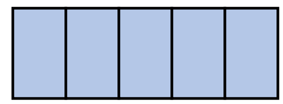
3. Find algebraically two numbers whose difference is 10 and whose product is a minimum.

4. Find algebraically two numbers whose sum is 26 and whose product is a maximum.

5. Two numbers have a sum of 34. Find algebraically the numbers if the sum of their squares is a minimum.

6. A rectangular field is to be enclosed by 400m of fence. What is the maximum area?

7. Helmut, the ostrich farmer, wants to build a rectangular fenced enclosure divided into five rectangular pens, as show in the diagram. A total length of 120m of fencing material is available. Find the overall dimensions of the enclosure that will make the total area a maximum. . (Note – there is a fence between each pen.)



8. An amusement park charges \$8 admission and averages 2000 visitors per day. A survey shows that, for each \$1 increase in the admission cost, 100 fewer people would visit the park. Write the equation to express the revenue, R(x) dollars, in terms of a price increase of x dollars and then determine the admission price that gives the maximum revenue.

9. The Soccer club sells hoodies as a fund-raiser. They sell 1200 shirts a year at \$20 each. They are planning to increase the price. A survey indicates that, for every \$2 increase in price, there will be a drop of 60 sales a year. What is the maximum revenue the club could make?

10. Find algebraically two consecutive integers with a product of 156.

11. Determine algebraically what number and its square differ by 30?

12. The hypotenuse of a right triangle is 29cm. If the other two sides differ by 1cm, what are their lengths?

13. When a football is kicked with a vertical speed of 20m/s, its height, h meters, after t seconds is approximated by the formula: $h = 20t - 5t^2$. How long after the kick is the football at a height of 15m?

14. The height, h meters, of a falling object is related to the time, t seconds, the object has been falling by the formula $h = -4.9t^2 + d$ where d meters is the initial height of the object above the ground. The Bankers Hall building in Calgary is 196m tall. Express the time an object takes to reach the ground from this height as an exact number of seconds.

15. The volume of a cone with height, h, and radius r, is given by the formula $v = \frac{1}{3}\pi r^2 h$. What is the radius of a cone with volume 168cm^3 and height 9cm.

16. Five is added to an integer that is being quartered and then squared. This sum equals nine. Determine the integer algebraically.

17. The surface area of a cube is 120cm^2. Determine the exact length of one of the edges.

18. Thirty less than the square of an integer is 139. Find the integer algebraically.

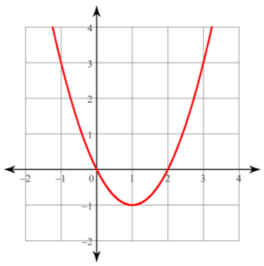
UNIT 4 – ANSWER KEY

SECTION 4.1

1. a) 4 - quartic b) 2 - quadratic c) 3 - cubic d) 2 - quadratic e) 3 - cubic 2. a) Yes b) No c) Yes d) Yes e) Yes f) No 3. 0 to n 4. 1 to n

SECTION 4.2

1. a)



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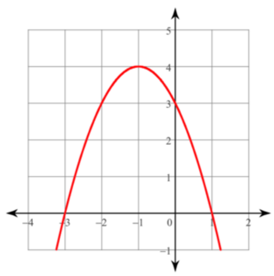
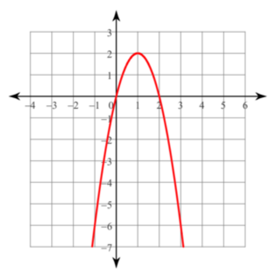
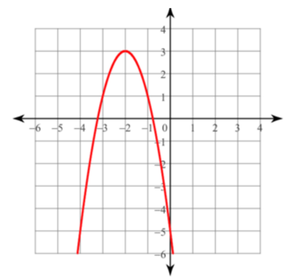


c)



d)



2. a) $y = (x-2)^2$ b) $y = -x^2 + 3$ c) $y = (x+1)^2 + 2$ 3. a) $y = (x+2)^2 + 7$ b) $y = (x-8)^2 - 11$ c) $y = -x^2 - 5$ d) $y = -(x+1)^2 + 3$

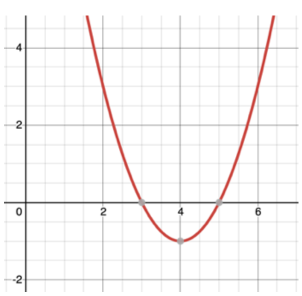
SECTION 4.3

1. a) $y = 2(x-1)^2 - 4$ b) $y = -\frac{1}{2}x^2 + 3$ c) $y = \frac{1}{3}(x+3)^2 - 2$ 2. a) $y = 7(x+3)^2$b) $y = -\frac{1}{5}(x-5)^2 - 6$ c) $y = -5x^2 - 7$ d) $y = \frac{1}{3}(x+2)^2 + 9$

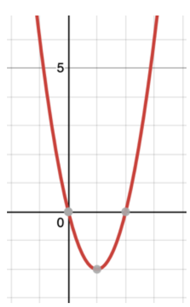
SECTION 4.4

1. a)

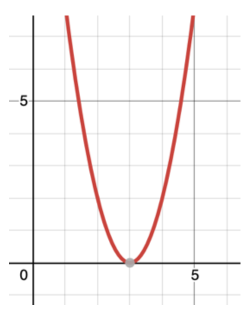
a) x = 3, x = 5



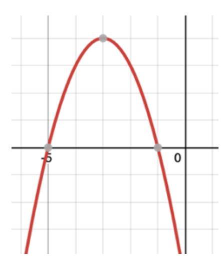
b) x = 0 x = 2



c) x = 3



d)x = -5 x = -1



2. a) m=5,7 b) $c = \pm\frac{7}{2}$ c) y= 0,11 d) $x = -\frac{3}{2}, 5$ 3. a) m=-8,3 b) x = 4 c) x = 1,5

d) $p = -1, \frac{4}{3}$ 4. a) $x = \frac{1}{2}, 2$ b) $n = -1, \frac{14}{3}$ c) $f = 3 \pm \sqrt{5}$ d) $x = \frac{-7 \pm \sqrt{37}}{2}$ e) $t = -\frac{1}{3}, \frac{1}{2}$
f) $h = -1, -\frac{2}{5}$ 5. a) $q = -\frac{7}{5}$ b) $v = \pm 4$ c) $r = 0, \frac{4}{3}$ d) no real roots
6. a) $z = -2.22, 7.22$ b) no real roots 7. a) $x = \frac{-3 \pm \sqrt{5}}{2}$ b) $z = 3 \pm \sqrt{2}$
8. a) $n = \frac{6 \pm \sqrt{52}}{2}$ b) $x = -\frac{3}{2}, \frac{1}{2}$ c) $m = 0, \frac{13}{7}$ d) $w = \frac{3 \pm \sqrt{65}}{4}$
9.a) Two Real Roots b) Two Real Roots c) One Real Root d) No Real Roots
e) Two Real Roots f) No Real Roots

SECTION 4.5

1. a) $(x + 3)^2 - 6$ b) $(x - 5)^2 - 31$ c) $(x + 10)^2 + 60$ d) $(x - 5/2)^2 - 17/4$ e) $(x + 1/2)^2 + 3/4$
f) $(x - 1/2)^2 + 3/4$ 2. a) $2(x + 3)^2 - 4$ b) $3(x - 1/2)^2 + 1/4$ c) $5(x + 2/5)^2 + 11/5$ d) $4(x - 5/4)^2 - 49/4$
3. a) minimum at (-5, -6) b) maximum at (7/3, -4) c) minimum at (5,-63) d) maximum at (3/4, 4) e) maximum at (3/4, 81/4) f) maximum at (8/5. 29/5) 4. a) (1.5, 6.25) x = 1.5 -1, 4 downward $-\infty < x < \infty,\ x \in R\ \ y \le 6.25, y \in R$ b) (-2.5, -2.25) x = -2.5 -4, -1 upwards $-\infty < x < \infty,\ x \in R\ \ y \ge -2.25, y \in R$ c) (1,0) x = 1 1 upward $-\infty < x < \infty,\ x \in R\ \ y \ge 0, y \in R$ d) (1.5, 12.25) x = 1.5 -2 5 downwards $-\infty < x < \infty,\ x \in R\ \ y \le 12.25, y \in R$

SECTION 4.6

1. a) 10m b) 20m c) 40m d) 7.5m e) No, the player would need to be able to reach 5.1m
2. a) 43m b) 67 m c) 134.3m d) 8.13m e) 17.8m f) Domain (Horizontal distance) : 0 < d < 134.3 Range(Height of the ball): 0 < h < 43 3. 5 and -5 4. 13 and 13 5. 17 and 17 6. 10 000 m^2
7. 10m by 30m 8.R(x) = (2000 – 100x)(8 + x) $14 9. $27 000 10. 12 and 13 or -13 and -12 11. 6 or -5 12. 20 cm and 21 cm 13. 1 sec and 3 sec 14. $2\sqrt{10}$ 15. 4.22 cm 16. 8 17. $2\sqrt{5}$ 18. 13