

MULTIPLYING RATIONAL EXPRESSIONS

- Factor all the numerators and denominators if possible.
- Simplify all individual fractions (if possible) by cancelling common factors within each fraction (if these common factors exist).
- Simplify diagonally if possible. Remember that if you are multiplying more than two fractions, you can cancel common factors that are placed diagonally away from each other even if there is a fraction or are fractions in between:

Example:

$$\frac{\cancel{(x+1)}}{4} \times \frac{3}{(x-1)} \times \frac{\cancel{16}^4}{5\cancel{(x+1)}} = \frac{(3)(4)}{(x-1)(5)} = \boxed{\frac{12}{5x-5}}$$

- Only after all common factors are cancelled out, carry out the multiplication by following the rules for multiplication of fractions. (Multiply all the numerators. Multiply all denominators).
- Double check that no common factors are left over.
- Clearly state the final answer.

Example 1: Simplify.

$$\frac{x^2 - 25}{2x + 12} \times \frac{x^2 + 10x + 24}{x^2 + 3x - 10} \times \frac{3x + 6}{x^2 - 10x + 25}$$

$$= \frac{\cancel{(x-5)}\cancel{(x+5)}}{2\cancel{(x+6)}} \times \frac{\cancel{(x+5)}\cancel{(x-2)}(x+4)(x+6)}{\cancel{(x+5)}\cancel{(x-2)}} \times \frac{3\cancel{(x+2)}}{\cancel{(x-5)}\cancel{(x-5)}}$$

$$= \frac{(x+4)(3)}{(2)(x-5)}$$

$$= \boxed{\frac{3x+12}{2x-10}}$$

Example 2:

State the non-permissible values for

$$\frac{x^2 - 25}{2x + 12} \times \frac{x^2 + 8x + 24}{x^2 + 3x - 10} \times \frac{3x + 6}{x^2 - 10x + 25}$$

$$2x + 12 = 0$$

$$\frac{2(x+6)}{2} = \frac{0}{2}$$

$$x + 6 = 0$$

$$\boxed{x = -6}$$

$$(x+5)(x-2) = 0$$

↓

$$x+5=0$$

$$\boxed{x = -5}$$

↘

$$x-2=0$$

$$\boxed{x = 2}$$

$$x^2 - 10x + 25 = 0$$

$$(x-5)(x-5) = 0$$

↓

$$x-5=0$$

$$\boxed{x = 5}$$

∴ The NPVs are $x = -6$, $x = -5$, $x = 2$ and $x = 5$.

Example 3:

$$\frac{x^2 + 5xy + 6y^2}{x^2 + 4xy - 5y^2} \times \frac{x^2 + 3xy - 10y^2}{x^2 + xy - 6y^2}$$

$$= \frac{(x+2y)(x+3y)}{(x+5y)(x-y)} \times \frac{(x+5y)(x-2y)}{(x+3y)(x-2y)}$$

$$= \boxed{\frac{x+2y}{x-y}}$$

DIVIDING RATIONAL EXPRESSIONS

- Factor all the numerators and denominators if possible.
- Simplify all individual fractions (if possible) by cancelling common factors within each fraction (if these common factors exist).
- Keep the first fraction. Change division to multiplication. Reciprocate the second fraction. (If there are more than two fractions, reciprocate all fractions except the first).
- Simplify diagonally if possible. Remember that if you are multiplying more than two fractions, you can cancel common factors that are placed diagonally away from each other even if there is a fraction or are fractions in between.
- Only after all common factors are cancelled out, carry out the multiplication by following the rules for multiplication of fractions. (Multiply all the numerators. Multiply all denominators).
- Double check that no common factors are left over.
- Clearly state the final answer.

Example 1:

$$\frac{4mn^2}{39mn} \div \frac{(3mn)^2}{6mn}$$

$$= \frac{4n}{3} \div \frac{\cancel{3}n\cancel{n}(3mn)}{2\cancel{6}n\cancel{n}}$$

$$= \frac{4n}{3} \div \frac{3mn}{2}$$

$$= \frac{\cancel{4}n}{3} \times \frac{2}{3m\cancel{n}}$$

$$= \boxed{\frac{8}{9m}}$$

NPVs:

$$9mn = 0$$

$$\begin{matrix} \downarrow & \downarrow \\ m=0 & n=0 \end{matrix}$$

$$6mn = 0$$

$$\Rightarrow m=0, n=0$$

$$3mn = 0$$

$$\Rightarrow m=0, n=0$$

Example 2:

$$\frac{5x-10}{6x+6} \div \frac{2x-4}{x+1}$$

$$= \frac{5(x-2)}{6(x+1)} \div \frac{2(x-2)}{(x+1)}$$

$$= \frac{5(\cancel{x-2})}{6(\cancel{x+1})} \times \frac{(\cancel{x+1})^1}{2(\cancel{x-2})} \rightarrow \boxed{\frac{5}{12}}$$

When carrying a multiplication and division within the same question, remember to carry out multiplication and division in the order they are written left-to-right. Division and multiplication are operations that are equally important when the order of operations is concerned.

Example 3:

$$\frac{3x+6}{5-x} \div \frac{x^2-4}{x^2-8x+15} \times \frac{x^2-x-2}{x^2+x-12}$$

$$\frac{3(x+2)}{5-x} \div \frac{(x-2)(x+2)}{(x-5)(x-3)} \times \frac{(x-2)(x+1)}{(x+4)(x-3)}$$

$$\frac{3(\cancel{x+2})}{(5-x)} \times \frac{(\cancel{x-5})(\cancel{x-3})}{(\cancel{x-2})(\cancel{x+2})} \times \frac{(\cancel{x-2})(x+1)}{(x+4)(\cancel{x-3})}$$

$$\frac{3}{-(-5+x)} \times \frac{(x-5)}{1} \times \frac{(x+1)}{(x+4)}$$

$$\frac{3}{-(x-5)} \times \frac{(\cancel{x-5})}{1} \times \frac{(x+1)}{1} = -\frac{3(x+1)}{x+4} = -\frac{3x+3}{x+4}$$

Example 4: Simplify

$$\frac{\frac{3}{x} + 1}{1 - \frac{3}{x}} = \frac{\frac{3}{x} + \frac{1}{1}}{\frac{1}{1} - \frac{3}{x}} = \frac{\frac{3}{x} + \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}}$$

$$= \frac{\frac{(3+x)}{x}}{\frac{(x-3)}{x}} = \frac{(3+x)}{\cancel{x}} \times \frac{\cancel{x}}{(x-3)} = \boxed{\frac{3+x}{x-3}}$$