Notes:

PC11

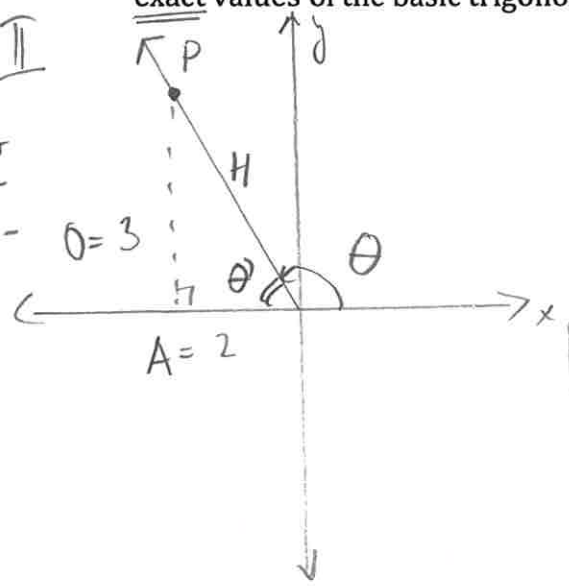
Exact Values of Basic Trigonometric Ratios

1. **Calculating exact values of basic trigonometric ratios given coordinates of a point on the terminal arm of an angle in standard position.**

Ex.1: Given that P (-2,3) is a point on the terminal arm of angle θ, determine the exact values of the basic trigonometric ratios.

II

Sin+
Cos-
tan-



$H^2 = 3^2 + 2^2$

$H^2 = 9+4$

$H = \sqrt{13}$

H is always (+)

S	A
T	C

$\Rightarrow$ Sin (+)

- $\sin\theta = \sin\theta'$

$= \frac{O}{H}$

$= \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$

- $\cos\theta = -\cos\theta'$

$= -\frac{A}{H}$

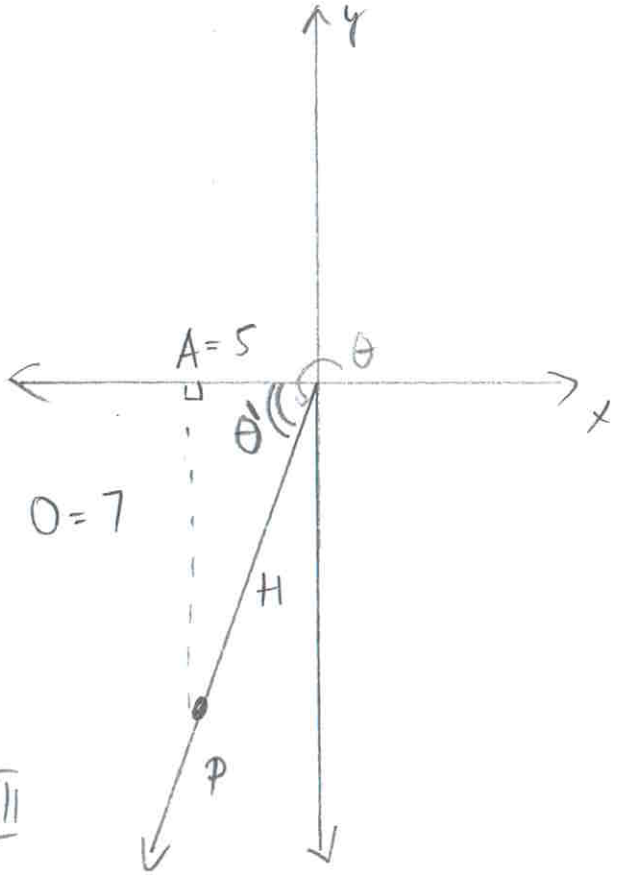
$= -\frac{2}{\sqrt{13}} = \boxed{\frac{-2\sqrt{13}}{13}}$

- $\tan\theta = -\tan\theta'$

$= -\frac{O}{A}$

$= \boxed{-\frac{3}{2}}$

Your Turn 1: Given that P (-5,-7) is a point on the terminal arm of angle θ, determine the exact values of the basic trigonometric ratios.



$$H^2 = O^2 + A^2$$
$$H^2 = 7^2 + 5^2$$
$$H = \sqrt{74}$$

III quadrant $\Rightarrow$ S | A / T | C

$\sin\theta = (-)$
$\cos\theta = (-)$
$\tan\theta = (+)$

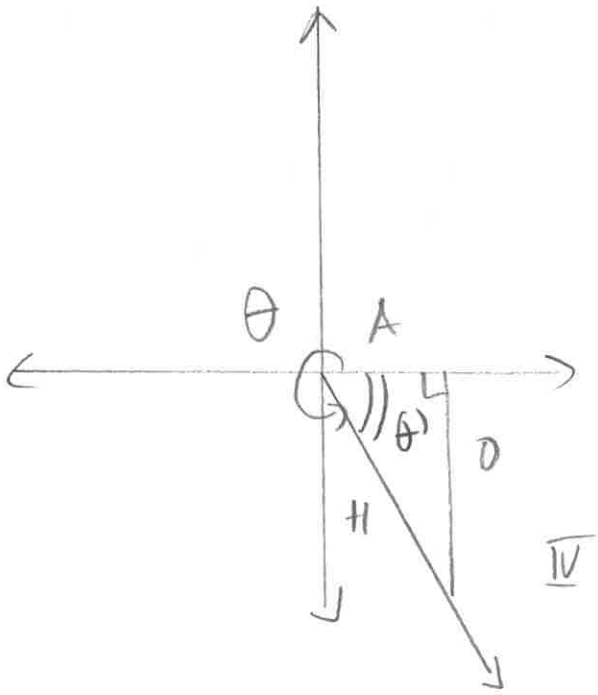
- $\sin\theta = -\sin\theta'$
  $= -\frac{O}{H}$
  $= -\frac{7}{\sqrt{74}} = \boxed{\frac{-7\sqrt{74}}{74}}$

- $\cos\theta = -\cos\theta'$
  $= -\frac{A}{H}$
  $= \frac{-5}{\sqrt{74}} = \boxed{\frac{-5\sqrt{74}}{74}}$

- $\tan\theta = \tan\theta$
  $= \frac{O}{A}$
  $= \boxed{\frac{7}{5}}$

2. Calculating the exact values of the basic trigonometric ratios given one of the 3 ratios and possibly information about the quadrant of the terminal arm.

Ex.2: Given that $cos\theta = \frac{3}{\sqrt{19}}$ and θ has its terminal arm in the fourth quadrant, find the values of the remaining trigonometric ratios. **Include a labeled diagram.**



$H = \sqrt{19}$

$A = 3$

$O^2 = H^2 - A^2$

$O^2 = (\sqrt{19})^2 - 3^2$

$O = \sqrt{19 - 9}$

$O = \sqrt{10}$

IV quadrant $\Rightarrow$ S | A / T | C

$\cos\theta = (+)$

$\sin\theta = (-)$

$\tan\theta = (-)$

- $\sin\theta = -\sin\theta'$

$= -\frac{O}{H}$

$= -\frac{\sqrt{10}}{\sqrt{19}} = \boxed{-\frac{\sqrt{190}}{19}}$

$\tan\theta = -\tan\theta'$

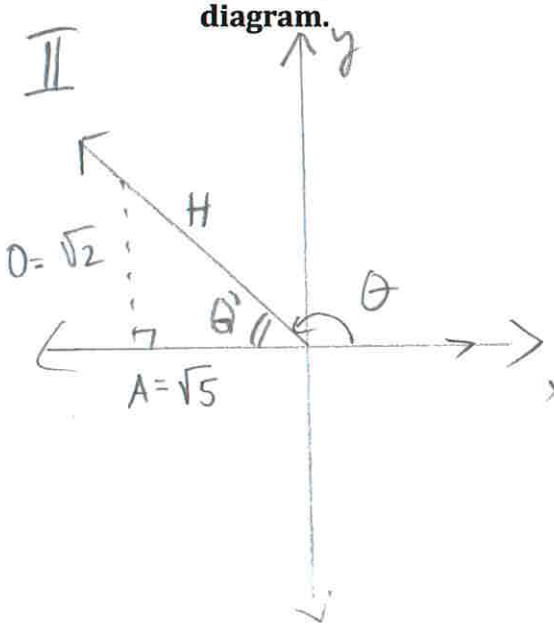
$= -\frac{O}{A}$

$= \boxed{-\frac{\sqrt{10}}{3}}$

S	A
T	C

$\Rightarrow$ sin(+)
cos(−)
tan(−)

Your Turn2: Given that $tan\theta = -\frac{\sqrt{2}}{\sqrt{5}}$ and θ has its terminal arm in the second quadrant, find the values of the remaining trigonometric ratios. **Include a labeled diagram.**



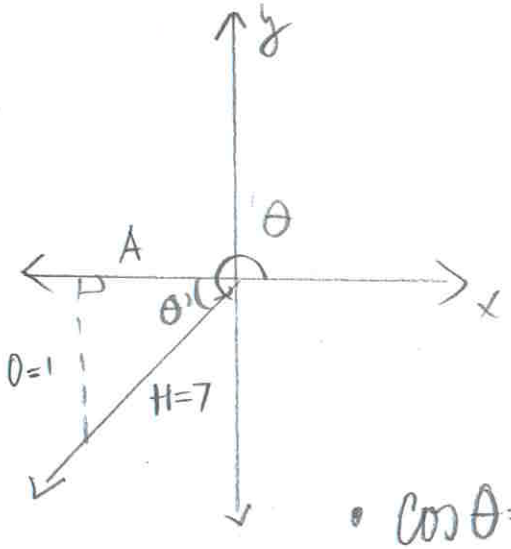
$H^2 = O^2 + A^2$

$H^2 = (\sqrt{2})^2 + (\sqrt{5})^2$

$H = \sqrt{7}$

- $\sin\theta = \sin\theta'$
  $= \frac{O}{H}$
  $= \frac{\sqrt{2}}{\sqrt{7}} = \boxed{\frac{\sqrt{14}}{7}}$
- $\cos\theta = -\cos\theta'$
  $= -\frac{A}{H}$
  $= -\frac{\sqrt{5}}{\sqrt{7}} = \boxed{\frac{-\sqrt{35}}{7}}$

Your Turn3: Given that $sin\theta = -\frac{1}{7}$ and θ has its terminal arm in the third quadrant, find the values of the remaining trigonometric ratios. **Include a labeled diagram.**



$A^2 = H^2 - O^2$

$A^2 = 7^2 - 1^2$

$A = \sqrt{48}$

$A = 4\sqrt{3}$

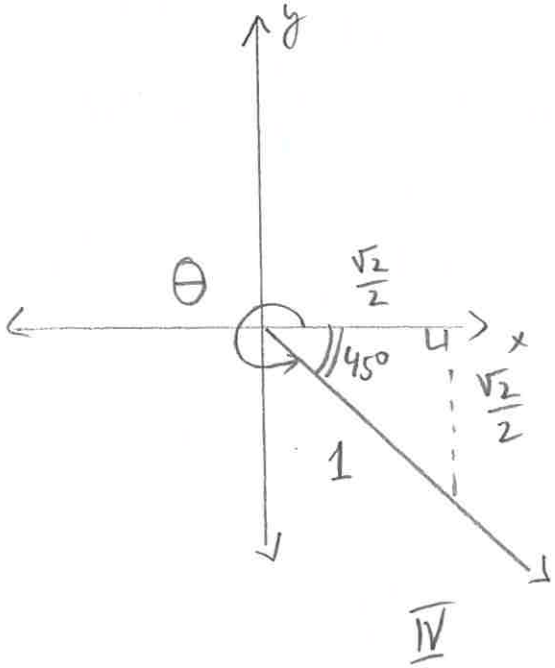
S	A
T	C

tan(+)
cos(−)
sin(−)

- $\cos\theta = -\cos\theta'$
  $= -\frac{A}{H}$
  $= \boxed{-\frac{4\sqrt{3}}{7}}$
- $\tan\theta = \tan\theta'$
  $= \frac{O}{A}$
  $= \frac{1}{4\sqrt{3}} = \boxed{\frac{\sqrt{3}}{12}}$

3. Determining exact values of the basic trigonometric ratios without a calculator.

Ex.3: Determine *sin* 315°. **Include a labeled diagram.**



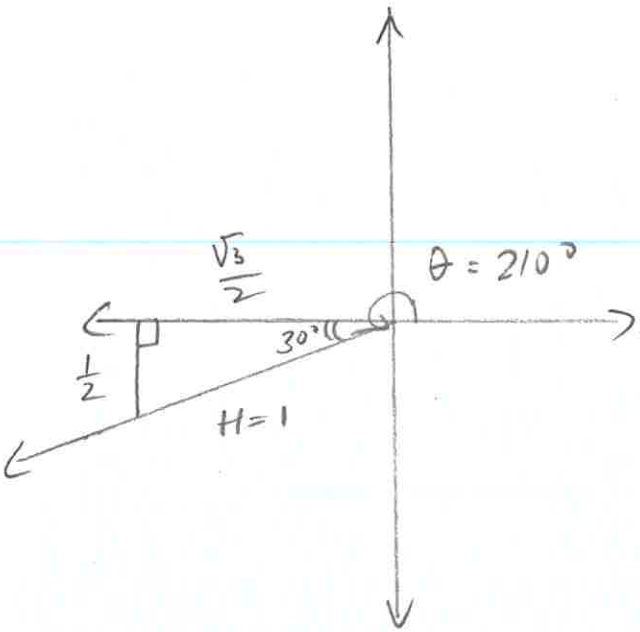
IV quadrant ⇒ $\sin\theta = (-)$

$\sin 315° = -\sin 45°$

$= -\dfrac{\frac{\sqrt{2}}{2}}{1}$

$= \boxed{-\dfrac{\sqrt{2}}{2}}$

Ex.4: Determine *tan* 210°. **Include a labeled diagram.**



III quadrant ⇒ $\tan\theta = (+)$

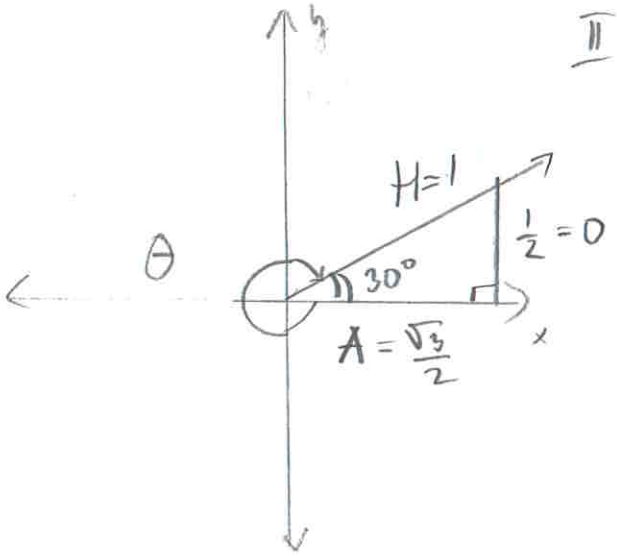
$\tan 210° = \tan 30°$

$= \dfrac{O}{A}$

$= \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$= \boxed{\dfrac{\sqrt{3}}{3}}$

Your Turn 4: Determine *cos* − 330°. **Include a labeled diagram.**



II quadrant ⇒ cos θ = (+)

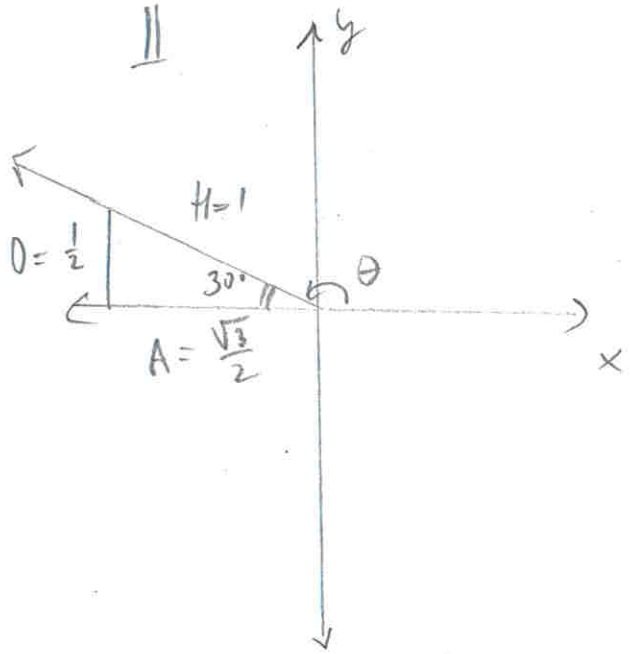
$\cos -330° = \cos 30°$

$= \frac{A}{H}$

$= \frac{\frac{\sqrt{3}}{2}}{1}$

$= \boxed{\frac{\sqrt{3}}{2}}$

Your Turn 5: Determine *tan* 150°. **Include a labeled diagram.**



II quadrant ⇒ tan θ = (−)

$\tan 150° = -\tan 30°$

$= -\frac{O}{A}$

$= \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2} \times \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$= \boxed{-\frac{\sqrt{3}}{3}}$