Answers

WORK, ENERGY, GPE, CENTRIPETAL ACCELERATION, ORBITS AND GRAVITATIONAL FORCE
Review Booklet

1.

A car completes a horizontal circle of radius r in time T. The same car then completes a larger horizontal circle of radius $2r$ in twice the time, $2T$. What is the ratio of the centripetal acceleration a_c for the car in the second circle to that in the first circle a_{c2}/a_{c1} ?

A. 1/4
B. 1/2
C. 2/1
D. 4/1

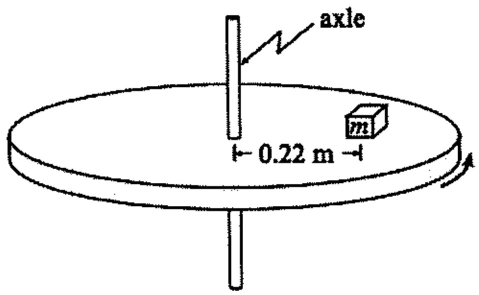
$$a_{c_1} = \frac{4\pi^2 r}{T^2}$$

$$a_{c_2} = \frac{4\pi^2 2r}{(2T)^2} = \frac{\cancel{2}^1 \cdot 4\pi^2 r}{\cancel{4}\,2 \cdot T^2} = \frac{1}{2} \cdot \frac{4\pi^2 r}{T^2}$$

$$\frac{a_{c_2}}{a_{c_1}} = \frac{\frac{1}{2}\left(\cancel{\frac{4\pi^2 r}{T^2}}\right)}{\left(\cancel{\frac{4\pi^2 r}{T^2}}\right)} = \frac{1}{2}$$

2.

An object of mass m is on a horizontal rotating platform. The mass is located 0.22 m from the axle and makes one revolution every 0.74 s.



horizontal circle

$\Rightarrow$ F_c is supplied by F_f

$\Rightarrow$ $F_c = F_f$

The friction force needed to keep the mass from sliding is 13 N. What is the object's mass?

A. 0.82 kg
B. 1.3 kg
C. 2.7 kg
D. 5.2 kg

- $F_f = 13\,N$
- $T = 0.74\,s$
- $r = 0.22\,m$

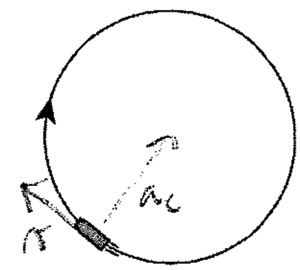
- $F_c = m a_c$
- $F_f = m \frac{4\pi^2 r}{T^2}$

$$m = \frac{F_f \cdot T^2}{4\pi^2 r}$$

$$m = \frac{(13)(0.74)^2}{4\pi^2(0.22)} = \underline{0.8196\ldots\,kg}$$

3.

An object is in uniform horizontal circular motion.



Top View

Which of the following shows the correct direction for the velocity, centripetal acceleration, and centripetal force on the object at the point shown?

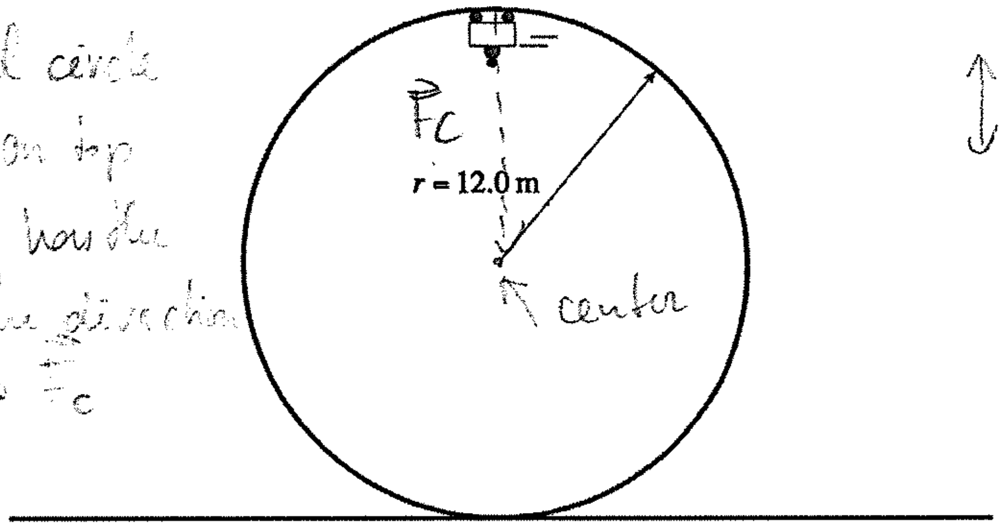
	Direction of the Velocity	**Direction of the Centripetal Acceleration**	**Direction of the Centripetal Force**
A.	↖	↙	↗
B.	↗	↙	↙
C.	↖	↗	↗
D.	↗	↗	↙

- direction of $\vec{a}_c$ and $\vec{F}_c$ are always the same
- direction of a_c and F_c is <u>always</u> toward the center of the circle
- object is moving [NW] ↖ so $\vec{v}$ is [NW] ↖

4.

A roller coaster car carrying a 75.0 kg man has a speed of 11.0 m/s at the top of a circular loop.

- vertical circle
- object on top
  $\Rightarrow \vec{F}_g$ has the same direction as $\vec{F}_c$



What is the normal force acting on the man at the top of the loop?

A. 0.0 N
B. 21 N
C. 735 N
D. 756 N

- $F_{net} = F_c = m\frac{v^2}{r}$

$= (75)\left(\frac{11.0^2}{12.0}\right)$

$= 756.25\ N$

- $F_g = (75.0)(9.8)$

$= 735\ N$

- $\vec{F}_{net} = \vec{F}_g + \vec{F}_N$

$-756.25 = -735 + F_N$

$\vec{F}_N = -21.25\ N$

$\|\vec{F}_N\| = 21\ N$

5.

Objects dropped near the surface of the moon fall with one sixth the acceleration of objects dropped near the surface of the earth. Which of the following is the correct value for the gravitational field strength at the moon's surface?

A. 0.0027 N/kg
B. 0.27 N/kg
C. 1.6 N/kg
D. 9.8 N/kg

$g_E = 6 \times g_m$

OR $g_m = \frac{1}{6} g_E$

$g_E = 9.8$

$\Rightarrow g_m = \frac{1}{6} \cdot (9.8)$

$g_m = 1.6\overline{3}$ m/s^2

acceleration

$g_m = \underline{1.6 \text{ N/kg}}$

Strength of the Moon's gravitational field

- The strength of a gravitational field shows how much gravitational force is acting onto every one kg of matter

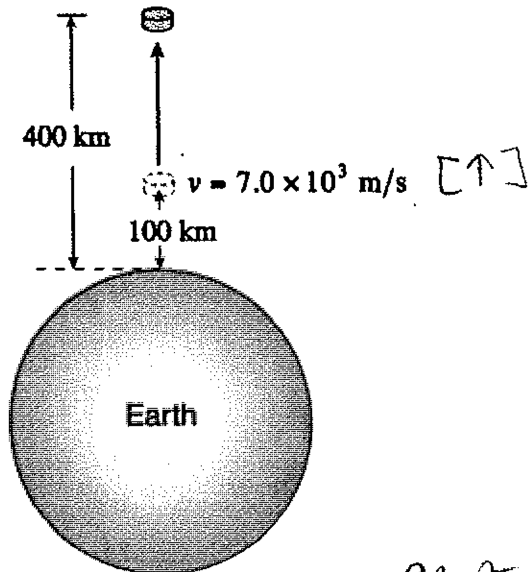
* The heat generated is $\underline{3.5 \times 10^{10}\ \text{J}}$.

6. it slows down as

An unpowered 1600 kg object has an upward velocity of 7.0×10^3 m/s at an altitude of 100 km above the earth. The object reaches a maximum altitude of 400 km. → 400 000 m

100 000 m

↑ +
↓ −



as $v = 0$ m/s at maximum altitude

What is the heat energy generated during the object's increase in altitude from 100 km to 400 km?

$$KE_i + GPE_i + \underbrace{W_{in}}_{\text{none}} = \underbrace{KE_f}_{0} + GPE_f + W_{out}$$

→ heat

$$W_{out} = KE_i + GPE_i - GPE_f$$

$$= \frac{1}{2} m_o v^2 + \frac{-G m_E m_o}{r_E + 1.0 \times 10^5} - \frac{-G m_E m_o}{r_E + 4.0 \times 10^5}$$

$$= \frac{1}{2}(1.6 \times 10^3)(7.0 \times 10^3)^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.6 \times 10^3)}{(6.38 \times 10^6 + 1.0 \times 10^5)} + \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.6 \times 10^3)}{(6.38 \times 10^6 + 4.0 \times 10^5)}$$

$$= 3.482 \ldots \times 10^{10}$$

$$\doteq 3.5 \times 10^{10}\ \text{J} \quad *$$

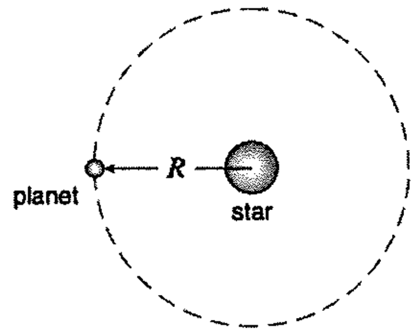
7.

A planet is in an orbit of radius R around a star. The star collapses to $\frac{1}{10}$ of its original volume while maintaining all of its mass.

- M is constan
- r is constant

↓

distance measued center-to-center



Before collapsed

What happens to the centripetal acceleration, a_c, of the planet due to the collapse of the star?

A. reduced to $\frac{1}{100}$ original a_c

B. reduced to $\frac{1}{10}$ original a_c

C. remains unchanged

D. increased to $10\times$ original a_c

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{v^2}{r}$$

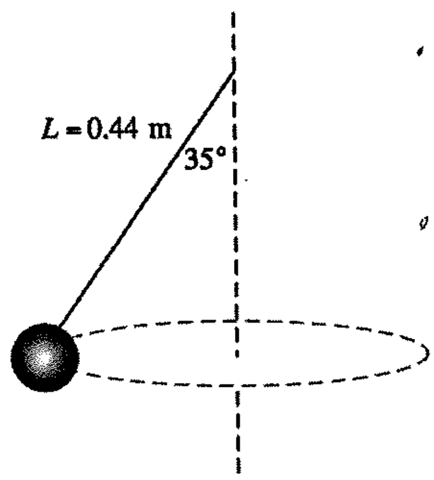
→ all variables are constant when volume changes

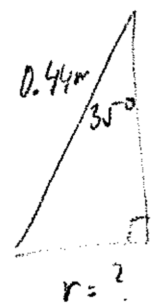
⇒ ac is also contant

8.
(5 marks)

A blue ball is swung in a horizontal circle and completes a single rotation in 1.2 s. The 0.44 m long cord makes an angle of 35° with the vertical during the ball's motion as shown.

$T = 1.2\,s$





$r = (0.44)(\sin 35°)$

$r = 0.25237363\,2\,m$

What is the centripetal acceleration of the ball?

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2(0.252373632)}{1.2^2}$$

$$a_c \doteq 6.9\ m/s^2$$

∴ The centripetal acceleration of the ball is $6.9\ m/s^2$.

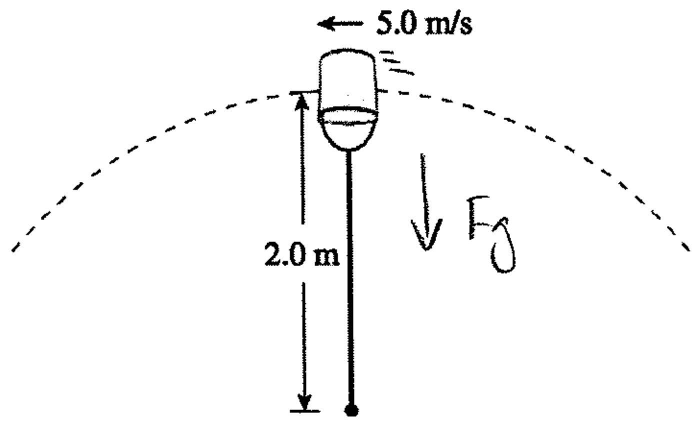
9.

m

A 4.0 kg bucket of paint tied to a rope is being swung in a vertical circle with a radius of 2.0 m. The speed of the bucket at the top of its swing is 5.0 m/s.

$\uparrow +$ $\downarrow -$

- $r = 2.0\,m$
- $v = 5.0\,m/s$
- $m = 4.0\,kg$



$\downarrow F_g$

$T \uparrow$

What is the tension in the rope at this point?

A. 11 N
B. 39 N
C. 50 N
D. 89 N

- $a_c = \frac{v^2}{r}$

$\vec{F_c} = \vec{F_g} + \vec{T}$

$-ma_c = -mg + T$

$T = -ma_c + mg$

$T = -(4.0)\left(\frac{5.0^2}{2.0}\right) + (4.0)(9.8)$

$T = -50 + 39.2$

$T = -10.8\ N$

$\underline{T \cong 11\,N}$

10.

$a = g = \text{constant}$

Two objects of unequal mass are dropped from the same height near the surface of the earth. Which of the following is the same for both objects just before they hit the surface? (Ignore friction.)

velocity is mass independent

A. velocity ✓
B. net force ×
C. momentum ×
D. kinetic energy ×

$F_{net} = ma$

$p_f = m v_f$

$KE = \frac{1}{2} m v^2$

$v_f = v_i t + \frac{1}{2} g t^2$

11.

What is the gravitational field strength on the surface of a moon with a mass of 3.7×10^{21} kg and a radius of 8.4×10^5 m?

A. 0.35 N/kg
B. 9.8 N/kg
C. 540 N/kg
D. 2.9×10^5 N/kg

- $M = m_m = 3.7 \times 10^{21}$ kg
- $r = 8.4 \times 10^5$ m

$$g = \frac{G m_m}{r^2}$$

$$g = \frac{(6.67 \times 10^{-11})(3.7 \times 10^{21})}{(8.4 \times 10^5)^2}$$

$$g \doteq 0.35 \text{ N/kg}$$

12.

$$F_g = F_c$$

What is the speed required to maintain a stable orbit around a planet of mass 2.5×10^{27} kg at a radius (from the centre of the planet) of 8.5×10^{7} m?

A. 23 m/s
B. 3.3×10^{4} m/s
C. 4.4×10^{4} m/s
D. 9.8×10^{8} m/s

- $m_p = 2.5\times10^{27}$ kg
- $r = 8.5\times10^{7}$ m
- $v = ?$ [m/s]

- m_s = mass of the orbiting body

$$F_c = F_g$$

$$\cancel{m_s}\frac{v^2}{r} = \frac{G m_p \cancel{m_s}}{r^2}$$

$$v = \sqrt{\frac{G m_p r}{r^2}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11})(2.5\times10^{27})}{8.5\times10^{7}}}$$

$$v = 44291.8 \ldots \text{ m/s}$$

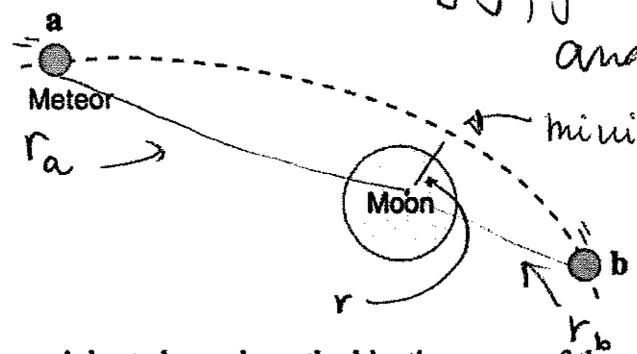
$$v \doteq 4.4\times10^{4}\text{ m/s}$$

- $E_{TOT_i} = E_{TOT_f}$ (the total energy of an object does not change!)
- $KE_i + GPE_i = KE_f + GPE_f$ ← assume no other sources of energy, just kinetic and gravitational

13.

A meteor passes by a moon as shown below.

- $r_a > r_b$
- $r_a > r$
- $r > r_b$

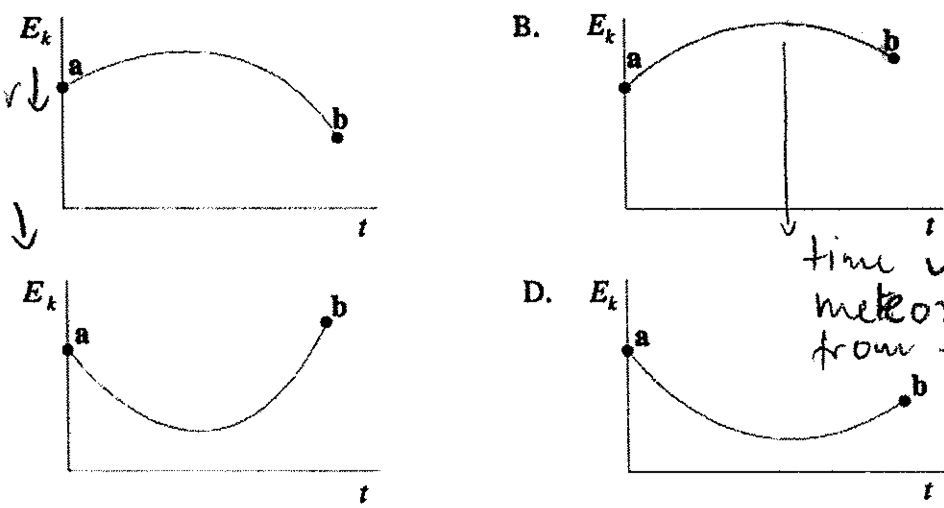


Which E_k versus time graph best shows how the kinetic energy of the meteor changes from position a to position b?

- as the meteor moves $r \downarrow$ and then $r \uparrow$

$\left(\frac{GMm}{r}\right) \uparrow$ when $r \downarrow$

but $\left(\frac{-GMm}{r}\right) \downarrow$ when $r \downarrow$



time when the meteor is r distance from the moon

- $KE_i + \frac{-GMm}{r_i} = KE_f + \frac{-GMm}{r_f}$
- $KE_i - \frac{GMm}{r_i} = KE_f - \frac{GMm}{r_f}$

As $\frac{-GMm}{r_a} > \frac{-GMm}{r}$ then $KE_a < KE_r \Rightarrow$

As $-\frac{GMm}{r} < \frac{-GMm}{r_b}$ then $KE_r > KE_b \Rightarrow$

14.

A 5.0×10^4 kg moonlet travels in a circular path around a planet. The moonlet's orbital radius is 2.5×10^7 m and the orbital period is 3.7×10^5 s. What is the mass of the planet?

A. 1.1×10^8 kg

B. 6.8×10^{22} kg

C. 3.4×10^{27} kg

D. 2.5×10^{28} kg

- $r = 2.5 \times 10^7 m$
- $T = 3.7 \times 10^5 s$
- $m = 5.0 \times 10^4 kg$
- $F_c = F_g$
- $m_m a_c = \frac{G m_p m_m}{r^2}$

$$\rightarrow \frac{\cancel{m_m} \cdot a_c \cdot r^2}{G \cancel{m_m}} = m_p$$

$$m_p = \frac{a_c \cdot r^2}{G}$$

$$= \frac{\frac{4\pi^2 r}{T^2} \cdot r^2}{G}$$

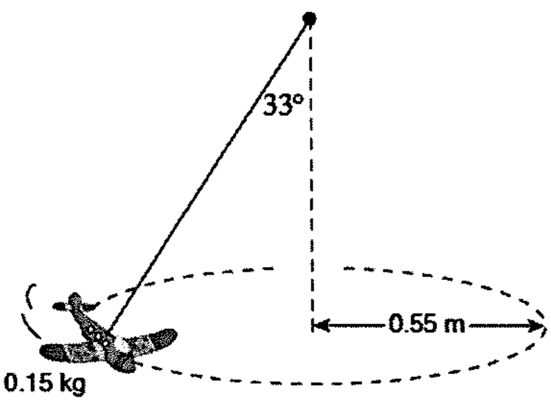
$$= \frac{4\pi^2 r^3}{T^2 G}$$

$$= \frac{4(\pi^2)(2.5 \times 10^7)^3}{(3.7 \times 10^5)^2 (6.67 \times 10^{-11})}$$

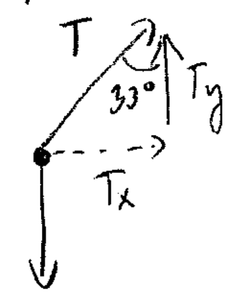
$$m_p \doteq 6.8 \times 10^{22} kg$$

15.

A 0.15 kg toy airplane is suspended as shown. It travels in a horizontal circle at a constant speed.



• FBD



• let F_T = tension

• $F_c = F_{T_x}$

• $F_g = F_{T_y}$

$mg = \cos 33° \cdot F_T$

$F_T = \frac{(0.15)(9.8)}{\cos 33°}$

$F_T = 1.75277404\ldots$

What is the period of the motion of this airplane?

A. 0.84 s
B. 1.6 s
C. 1.8 s
D. 2.0 s

• The centripetal force is supplied by the horizontal component of the force of tension.

$F_c = F_{T_x}$

$m a_c = \sin 33° \cdot F_T$

$m \frac{4\pi^2 r}{T^2} = \sin 33° \cdot F_T$

$T = \sqrt{\frac{m 4\pi^2 r}{(\sin 33° \cdot F_T)}}$

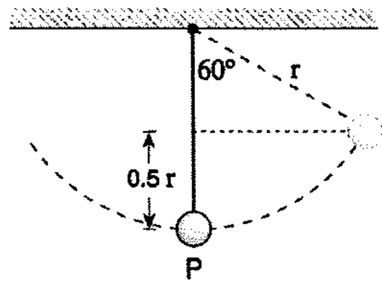
$T = \sqrt{\frac{(0.15)(4)(\pi^2)(0.55)}{[(\sin 33°)(1.75277\ldots)]}}$

$\underline{T \doteq 1.8 \text{ s}}$

vertical circle

16.

A small object of mass *m* is suspended from a fixed point by a light cord.



← at rest ⇒ $v_i = 0$ m/s

The object is raised to an angle of 60° and released from rest. The object moves in an arc of a circle as shown. When the object passes through its lowest position at point P, what is the tension in the cord in terms of the object's weight (*mg*)?

A. 0.5 *mg*
B. 1.0 *mg*
C. 1.5 *mg*
D. 2.0 *mg*

↑+ ↓−

· at point P:

$F_c = -mg + T$

$9.8m = -m(9.8) + T$

$2(9.8)m = T$

$\rightarrow T = 2\,mg$

and

vertical motion only

$v_f^2 = v_i^2 + 2ad$

$\vec{v}_f = \pm\sqrt{0 + 2(9.8)(-0.5r)}$

$\vec{v}_f = -\sqrt{9.8r}$ m/s

$F_c = m a_c$

$= m \frac{v^2}{r}$

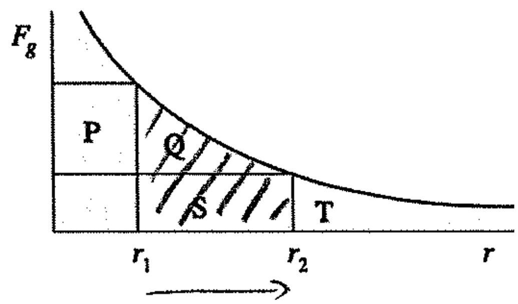
$= m \frac{(-\sqrt{9.8r})^2}{r}$

$= \frac{m(9.8)(\cancel{r})}{\cancel{r}}$

$= m \cdot (9.8)$

17.

The graph shows the gravitational force between the earth and an object as a function of the distance of separation, r, from the centre of the earth.



As the object is moved from r_1 to r_2, what is the work done?

A. Q
B. Q+S
C. P+Q
D. Q+S+T

Work is the area below the curve of Force versus displacement graph.

18.

A 200 kg object is released from rest at an altitude of 1.0×10^{7} m. What is its impact speed with the earth? Assume no air resistance.

A. 7.0×10^{3} m/s
B. 8.7×10^{3} m/s
C. 1.1×10^{4} m/s
D. 1.4×10^{4} m/s

• $m = 200$ kg

• $r_i = r_E + \text{altitude}$

$= 6.38\times10^{6} + 1.0\times10^{7}$

$= 1.638\times10^{7}$ m

• $KE_i + GPE_i = KE_f + GPE_f$

$$0 - \frac{G m_E \cancel{m_o}}{r_i} = \frac{1}{2}\cancel{m_o} v_f^2 - \frac{G m_E \cancel{m_o}}{r_E}$$

$$-\frac{G m_E}{r_i} + \frac{G m_E}{r_E} = \frac{1}{2} v_f^2$$

$$v_f = \sqrt{2\left(-\frac{G m_E}{r_i} + \frac{G m_E}{r_E}\right)}$$

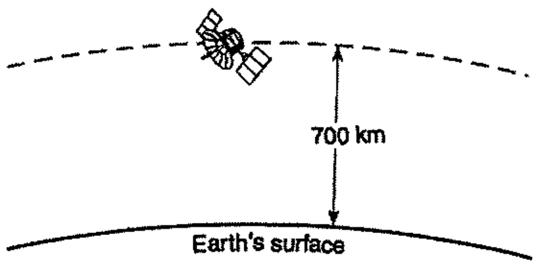
$$v_f = \sqrt{2\left(\frac{-6.67\times10^{-11}\times5.98\times10^{24}}{1.638\times10^{7}} + \frac{6.67\times10^{-11}\times5.98\times10^{24}}{6.38\times10^{6}}\right)}$$

$$v_f = 8.7\times10^{3}\ \text{m/s}$$

19.

A 4.20×10^4 kg satellite orbits the earth at an altitude of 700 km $(7.00 \times 10^5 \text{ m})$.

- $m_s = 4.20 \times 10^4$ kg
- altitude $= 7.00 \times 10^5$ m



a) What is the satellite's orbital speed at this altitude? **(4 marks)**

$$F_c = F_g$$

$$\cancel{m_s} a_c = \frac{G m_E \cancel{m_s}}{r^2}$$

$$\frac{v^2}{r} = \frac{G m_E}{r^2}$$

$$v_o = \sqrt{\frac{G m_E r}{r^2}}$$

$$v_o = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6 + 7.00 \times 10^5)}}$$

$$\underline{v_o = 7.51 \times 10^3 \text{ m/s}}$$

b) What is the satellite's total energy at this altitude? **(3 marks)**

$$E_{TOT} = KE + GPE$$

$$= \frac{1}{2} m_s v_o^2 + -\frac{G m_E m_s}{r_o}$$

$$= \frac{1}{2}(4.20 \times 10^4)(7.51 \times 10^3)^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(4.20 \times 10^4)}{(6.38 \times 10^6 + 7.00 \times 10^5)}$$

$$\underline{= -1.18 \times 10^{12} \text{ J}}$$