

Notes + Practice

PHYSICS 12

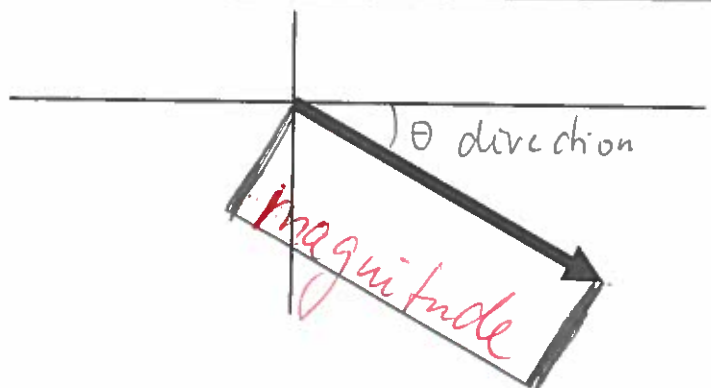
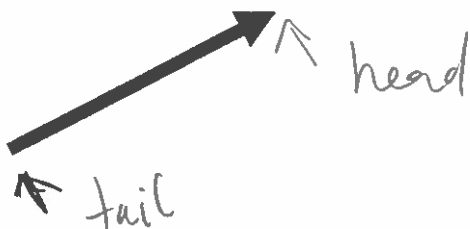
VECTOR QUANTITIES AND OPERATIONS WITH VECTORS

Physical quantities are either scalar or vector.

While scalars have only magnitude, vectors have both magnitude and direction.

VECTOR		SCALAR	
SYMBOL	QUANTITY	SYMBOL	QUANTITY
\vec{d}	displacement	m	mass
\vec{v}	velocity	t	time
\vec{F}	Force	W	work
a	acceleration	P	power
		KE	kinetic energy
		Q	heat
		PE	potential energy
		I	electric current
		V	voltage
		R	resistance

VECTOR is an oriented ray with a head and a tail.



- A symbol of a vector quantity has either an arrow on top (\vec{a} or \vec{a}) or in older texts is in bolded font (\mathbf{a}).
- Vectors in two dimensions can be added, subtracted, multiplied by a scalar and multiplied using a dot product (\cdot).
- Vectors in three or more dimensions can be added, subtracted, multiplied by a scalar, multiplied using a dot product and multiplied using a cross product (\times).
- algebraic = numerical and graphical methods exist to carry on the above operations

VECTOR NOTATION

- Symbol
- Equal sign
- Square brackets
- Vector components separated by a comma
- Units

Examples:

$$\vec{d} = [d_x, d_y] \text{ m}$$

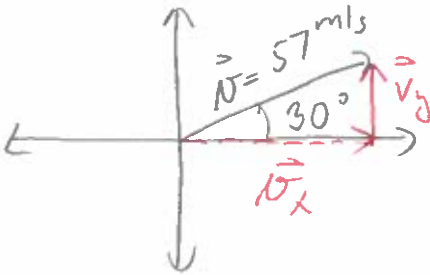
VECTOR COMPONENTS in 2 dimensions

- Vector components are vectors and as such they have direction and units
- Vector components are perpendicular to one another (= orthogonal)
- The horizontal vector component is listed first
- The vertical vector component is listed second

Resolving a vector into its components:

Example. Find vector components of initial velocity $\vec{v} = 57 \text{ m/s}$ [30° above horizontal]. Write the velocity vector in vector notation.

Sketch the velocity vector first. Label the angle and the vector components. Use trigonometry to find the vector components. Remember to have your calculator set to degrees.



$$\begin{aligned} \vec{v}_x &= 57 \cdot \cos 30^\circ \\ \vec{v}_x &= 49.4 \text{ m/s [R]} \\ \vec{v}_x &= [49.4, 0] \text{ m/s} \end{aligned}$$

$$\begin{aligned} \vec{v}_y &= 57 \cdot \sin 30^\circ \\ \vec{v}_y &= 28.5 \text{ m/s [U]} \\ \vec{v}_y &= [0, 28.5] \text{ m/s} \end{aligned}$$

$$\vec{v} = [49, 29] \text{ m/s}$$

General formulae for any two dimensional vector \vec{b} with direction θ :

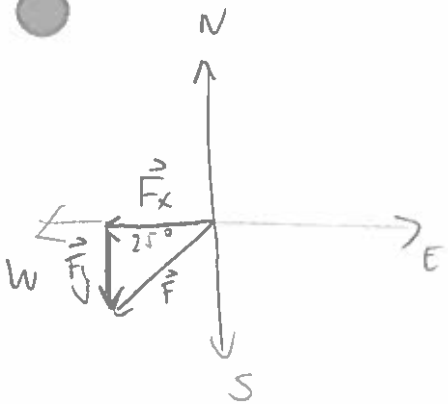
Horizontal component:

$$b_x = \|\vec{b}\| \cos \theta$$

Vertical components:

$$b_y = \|\vec{b}\| \sin \theta$$

1. Find the vector components of net force $\vec{F} = 350 \text{ N}$ [west 25° south]. Include a diagram.



$$\vec{F}_x = -350 \cdot \cos 25^\circ$$

$$\vec{F}_x = -317 \text{ N}$$

$$\vec{F}_x = [-317, 0] \text{ N}$$

$$\vec{F}_y = -350 \sin 25^\circ$$

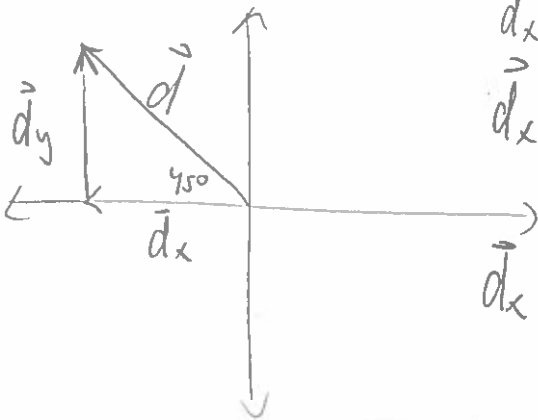
$$\vec{F}_y = -148 \text{ N}$$

$$\vec{F}_y = [0, -148] \text{ N}$$

Write the force vector using vector notation.

$$\vec{F} = [-317, -148] \text{ N}$$

2. Resolve a displacement vector $\vec{d} = 65 \text{ km}$ [NW] into two orthogonal components. Include a diagram.



$$\vec{d}_x = -65 \cdot \cos 45^\circ$$

$$\vec{d}_x = -46 \text{ km [E]}$$

$$= 46 \text{ km [W]}$$

$$\vec{d}_x = [-46, 0] \text{ km}$$

$$\vec{d}_y = 65 \sin 45^\circ$$

$$\vec{d}_y = 46 \text{ km [N]}$$

$$\vec{d}_y = [0, 46] \text{ km}$$

Write the displacement vector using vector notation.

$$\vec{d} = [-46, 46] \text{ km}$$

If you feel that you need more practice, please see me.

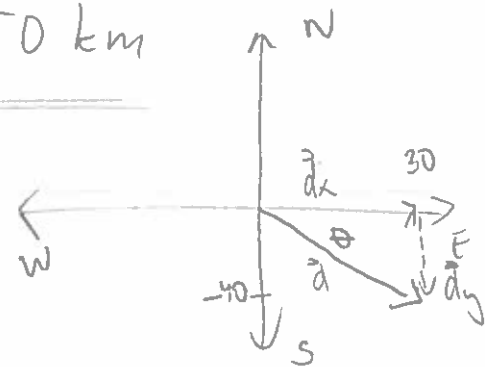
Magnitude of a vector

- Use the Pythagorean theorem to calculate the magnitude of a vector in vector notation
- Magnitude is the size of the vector (how fast, how far, how strong ...etc)

Example: Find the magnitude of a displacement vector $\vec{d} = [30, -40]$ km. Include a diagram.

$$\begin{aligned}\|\vec{d}\| &= \sqrt{30^2 + (-40)^2} \\ &= \sqrt{900 + 1600} \\ &= \sqrt{2500}\end{aligned}$$

∴ $\|\vec{d}\| = 50 \text{ km}$



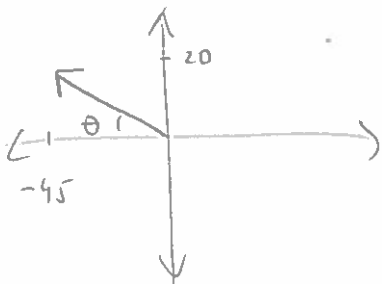
General formula to find the magnitude of vector $\vec{b} = [b_x, b_y]$:

$$\|\vec{b}\| = \sqrt{b_x^2 + b_y^2}$$

Direction of a vector

- Use a tangent ratio to calculate the direction of a vector in vector notation

Example: Find the direction of the acceleration vector $\vec{a} = [-45, 20]$ m/s²



$$\theta = \tan^{-1}\left(\frac{20}{45}\right) \therefore \text{The direction of } \vec{a} \text{ is } 24^\circ \text{U (or W } 24^\circ \text{N)}$$

(or 24°U of L ; 24°N of W)

General formula to find the direction of vector $\vec{b} = [b_x, b_y]$:

$$\theta = \tan^{-1}\left(\frac{|b_y|}{|b_x|}\right)$$

GRAPHICAL METHOD

- A vector diagram must be drawn to scale using a straight edge and a protractor
- All angles have to be exact
- The order of vectors is irrelevant when adding vectors
- The order of vectors is **important** when subtracting vectors
- **Vector subtraction is an addition of a negative vector**
- Triangular and parallelogram method are the most frequent graphical methods of vector addition and subtraction

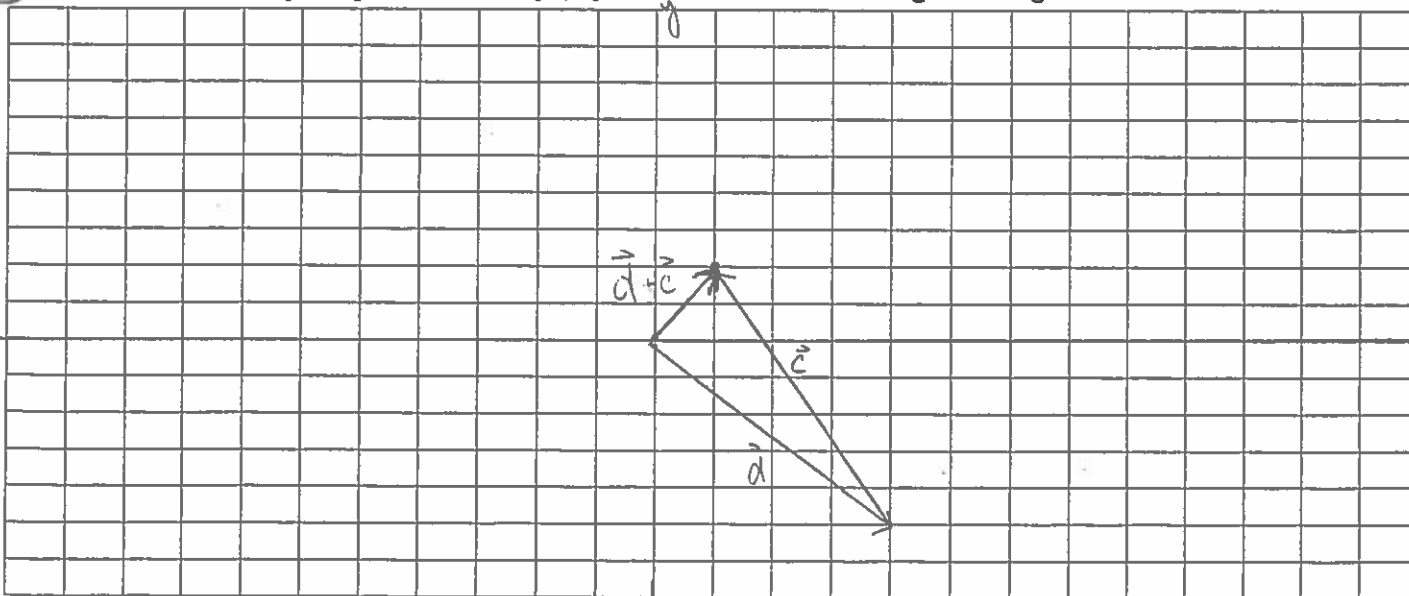
“Negative vector” = vector that has the same magnitude but opposite direction.

Examples: Write negative vectors associated with the following vectors:

$\vec{d} = 35 \text{ km [W]}$	$\vec{v} = [20, -68] \text{ km/h}$	$\vec{a} = 9.8 \text{ m/s}^2 \text{ [left]}$	$\vec{F} = [-65, 30] \text{ N}$	$\vec{p} = 45 \text{ kg.m/s [NW]}$
$\vec{d} = 35 \text{ km [E]}$ $= -35 \text{ km [W]}$	$-\vec{v} = [-20, 68] \frac{\text{km}}{\text{h}}$	$-\vec{a} = 9.8 \text{ m/s}^2 \text{ [R]}$ $= -9.8 \text{ m/s}^2 \text{ [L]}$	$-\vec{F} = [65, -30] \text{ N}$	$-\vec{p} = 45 \text{ kg.m/s [SE]}$

Example.1. Vector addition – triangular method = “head-to-tail” method

Consider a vector $\vec{d} = [4, -5]$ units and $\vec{c} = [-3, 7]$ units. Add the vectors using the triangular method.



Write the resultant vector in vector notation. $\vec{d} + \vec{c} = [1, 2] \text{ units}$

What is the vector's magnitude?

$$\|\vec{d} + \vec{c}\| = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2.2 \text{ units}$$

What is the vector's direction?

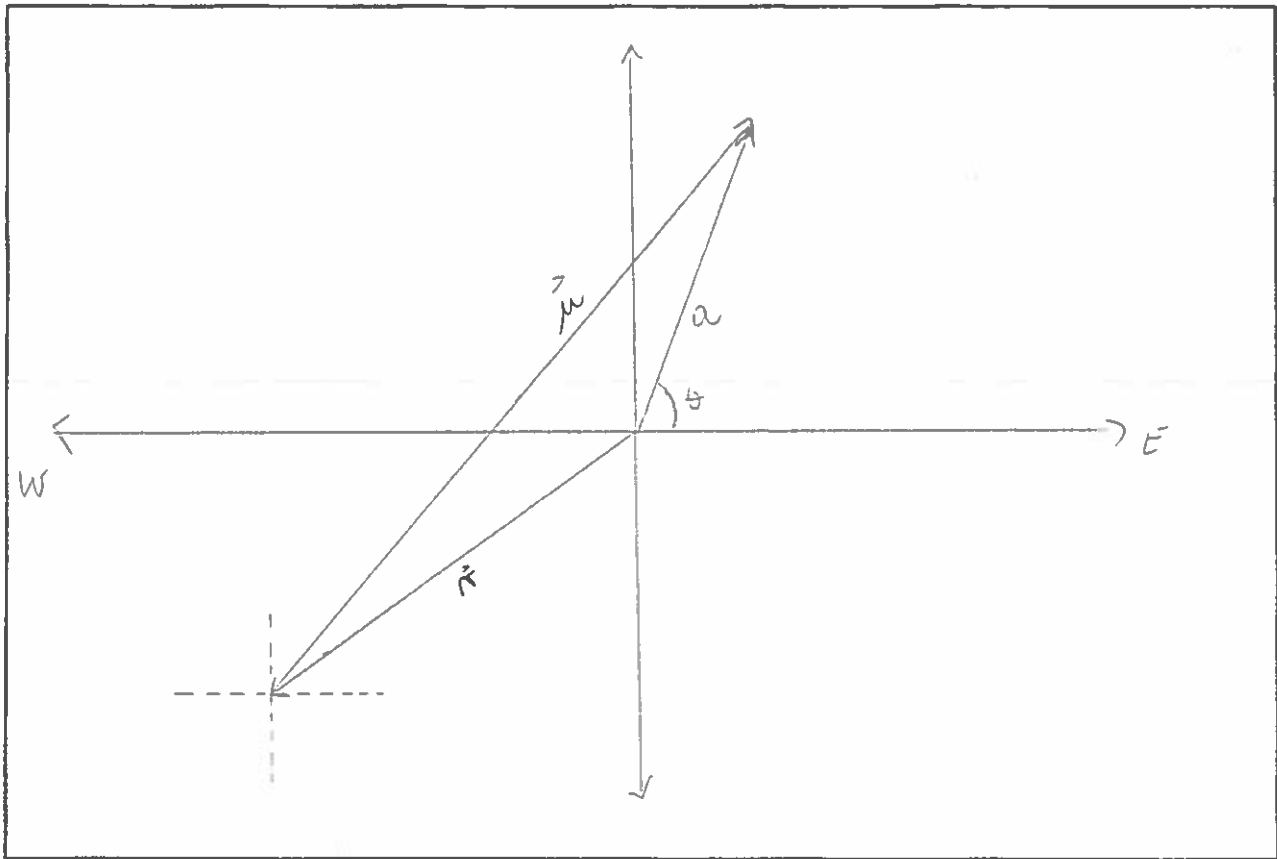
$$\theta = \tan^{-1}\left(\frac{2}{1}\right) = 63^\circ \therefore \text{direction is } R63^\circ U.$$

$$\text{let } \vec{a} = \vec{r} + \vec{u}$$

Scale:

Example 2. Draw a vector addition diagram of $\vec{v} = 30 \text{ km/h [W } 35^\circ \text{ S]}$ and $\vec{u} = 50 \text{ km/h [N } 40^\circ \text{ E]}$.

1cm = 5 km/h



Measure the magnitude of the resultant vector. $\|\vec{a}\| = 22 \text{ km/h}$

Measure the direction of the resultant vector. $\theta = 69^\circ \text{ N of E}$

How could you confirm the accuracy of the measured values? Using algebra

$$\vec{a} = [-30 \cos 35^\circ, -30 \sin 35^\circ] + [50 \cos 50^\circ, 50 \sin 50^\circ]$$

$$\vec{a} = [-24.6, -17.21] + [32.14, 38.30]$$

$$\vec{a} = [7.54, 21.09] \text{ km/h}$$

$$\|\vec{a}\| = \sqrt{7.54^2 + 21.09^2} = 22.4 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{21.09}{7.54}\right) = \underline{\underline{70^\circ}}$$

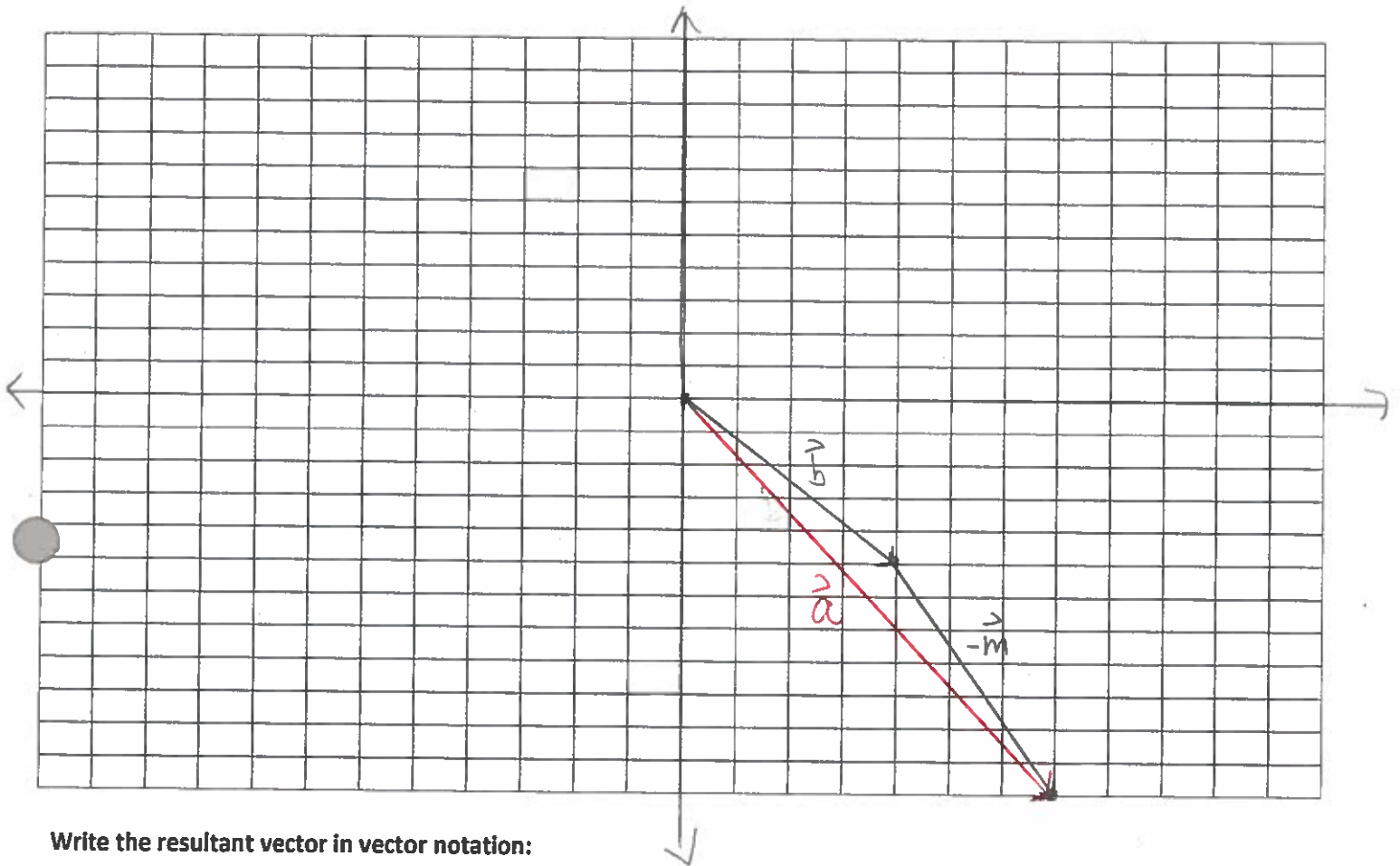
In your own words describe the "head-to-tail" method of vector addition

→ answers can vary

Example 3. Consider a vector $\vec{b} = [4, -5]$ units and $\vec{m} = [-3, 7]$ units. Subtract \vec{m} from \vec{b} using the triangular method.

Negative $\vec{m} = [3, -7]$ units

$$\text{let } \vec{a} = \vec{b} - \vec{m} = \vec{b} + (-\vec{m})$$

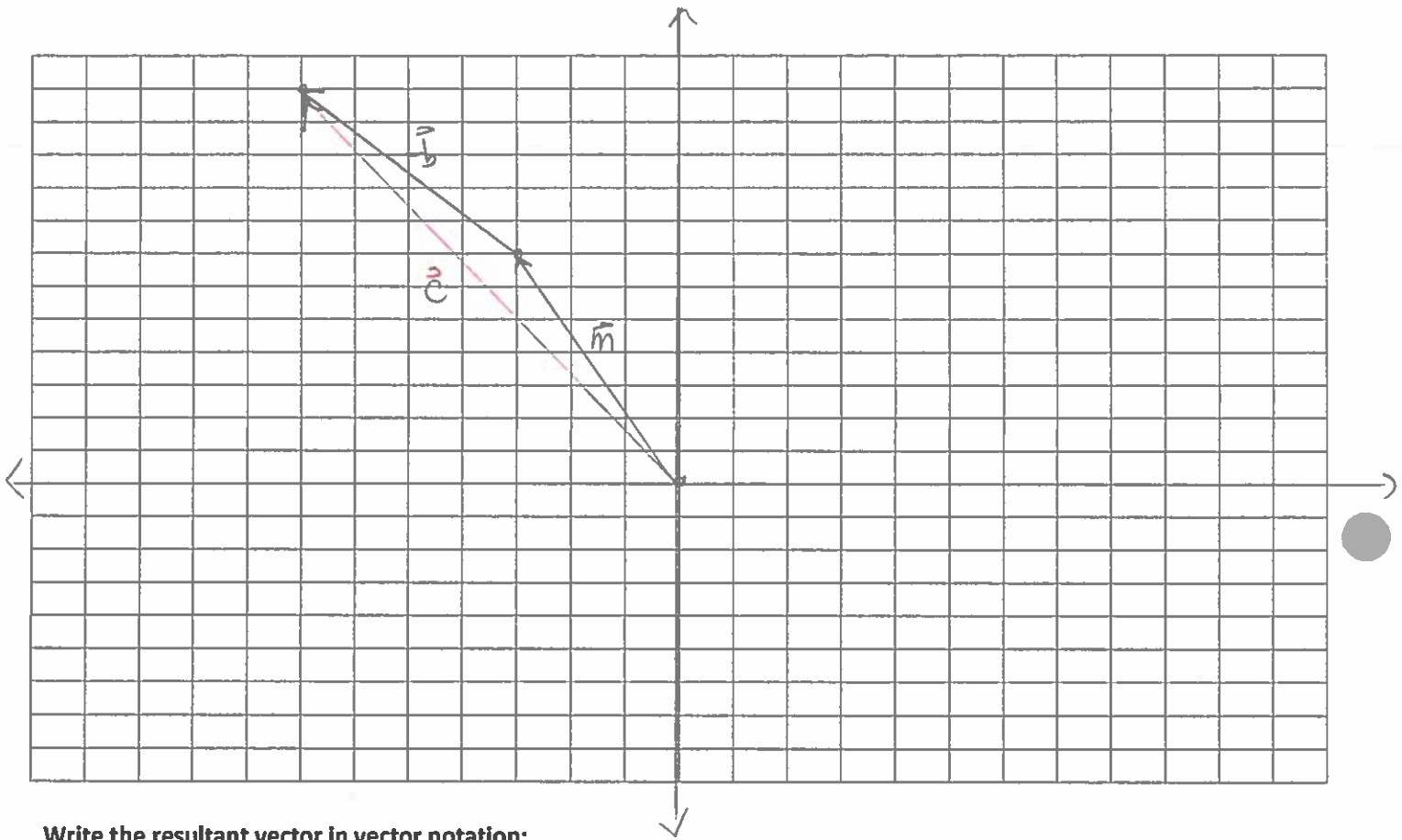


Write the resultant vector in vector notation:

$$\vec{a} = \vec{b} - \vec{m} = [7, -12] \text{ units}$$

Example 4. Consider a vector $\vec{b} = [4, -5]$ units and $\vec{m} = [-3, 7]$ units. Subtract \vec{b} from \vec{m} using the triangular method.

Negative $\vec{b} = [-4, 5]$ units let $\vec{c} = \vec{m} - \vec{b} = \vec{m} + (-\vec{b})$



Write the resultant vector in vector notation:

$$\vec{c} = [-7, 12] \text{ units}$$

How the resultant vectors from example 3 and 4 compare? In what way are they same and in what way are they different?

$$\vec{c} = -\vec{a} \quad \text{OR} \quad \vec{a} = -\vec{c}$$

- same magnitude
- opposite direction

NUMERICAL METHOD

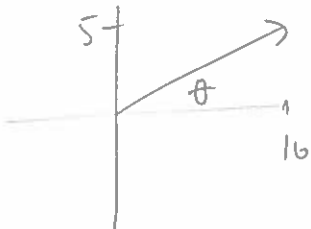
1. Sketch a diagram of each given vector.
2. Label the vector components on the diagrams
3. Sketch a diagram of the vector addition or subtraction
4. Resolve all given vectors into their vector components (when necessary)
5. Write a vector equation of the addition or subtraction
6. Carry out all operations
7. Write the resultant vector in vector notation.
8. Sketch a diagram of the resultant vector or highlight it in the diagram from step 3.
9. Find the magnitude of the resultant vector using the Pythagorean theorem
10. Find the direction of the resultant vector using the tangent ratio

Example 1: Consider vectors $\vec{a} = [3, -2] \text{ m}$, $\vec{b} = [13, 2] \text{ m}$ and $\vec{c} = [0, 5] \text{ m}$

a) Find vector $\vec{m} = \vec{a} + \vec{b} + \vec{c}$. What is the magnitude and direction of the resultant vector?

$$\begin{aligned}\vec{m} &= [3, -2] + [13, 2] + [0, 5] \\ &= [3+13+0, -2+2+5]\end{aligned}$$

$$= [16, 5] \text{ m}$$



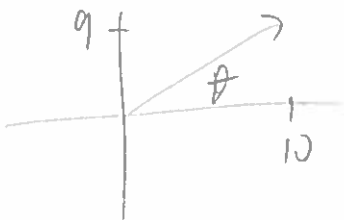
$$\begin{aligned}\|\vec{m}\| &= \sqrt{16^2 + 5^2} \\ &= \sqrt{281} \\ &\approx 16.76 \text{ m} \\ &\approx 17 \text{ m}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{5}{16}\right) \\ &\approx 17^\circ\end{aligned}$$

b) Find vector $\vec{q} = \vec{c} - \vec{a} + \vec{b}$. What is the magnitude and direction of the resultant vector?

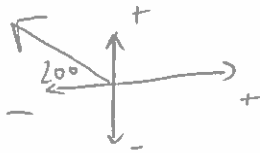
$$\begin{aligned}\vec{q} &= [0, 5] + [-3, 2] + [13, 2] \\ &= [0-3+13, 5+2+2]\end{aligned}$$

$$= [10, 9] \text{ m}$$



$$\begin{aligned}\|\vec{q}\| &= \sqrt{10^2 + 9^2} \\ &= \sqrt{181} \\ &\approx 13.5 \text{ m} \\ &\approx 14 \text{ m}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{9}{10}\right) \\ &\approx 42^\circ\end{aligned}$$



Example 2: Consider three forces $\vec{F}_{pull} = 25.0 \text{ N}$ [left 20° up], $\vec{F}_g = 15.0 \text{ N}$ [down], and $\vec{F}_N = 6.4 \text{ N}$ [up], acting on an object at once. Find the magnitude and direction of the net force.

$$\vec{F}_{net} = \vec{F}_{pull} + \vec{F}_g + \vec{F}_N$$

$$= [-25 \cos 20^\circ, 25 \cdot \sin 20^\circ] + [0, -15] + [0, 6.4]$$

$$= [-23.49 + 0 + 0, 8.55 - 15 + 6.4]$$

$$= [-23, 0] \text{ N}$$

$$\vec{F}_{net} = 23 \text{ N [L]}$$

Example 3: Consider an object that is pushed on a horizontal surface with force $\vec{F}_{push} = 75.0 \text{ N}$ [left] and experiences force of friction $F_f = 23 \text{ N}$.

a) Write the force vectors using vector notation.

$$\vec{F}_{push} = [-75, 0] \text{ N}$$

$$\vec{F}_g = [0, -mg] \text{ N}$$

$$\vec{F}_f = [23, 0] \text{ N}$$

$$\vec{F}_N = [0, mg] \text{ N}$$

b) Write a vector equation that expresses the net force.

$$\vec{F}_{net} = \vec{F}_{push} + \vec{F}_f + \vec{F}_g + \vec{F}_N$$

$$= [-75, 0] + [23, 0] + [0, -mg] + [0, mg]$$

$$= [-52, 0] \text{ N}$$

c) Calculate the magnitude and direction of the net force.

$$\bullet \|\vec{F}_{net}\| = 52 \text{ N}$$

$\theta = 0^\circ \rightarrow$ negative x-component and zero y-component

\bullet direction is to the left.