PHYSICS 12 **VECTOR QUANTITIES AND OPERATIONS WITH VECTORS** KEY

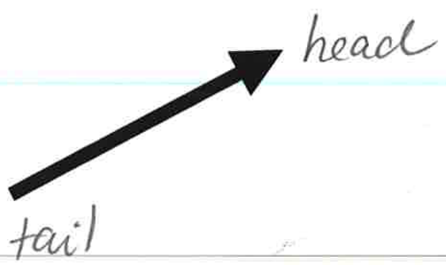
REVIEW

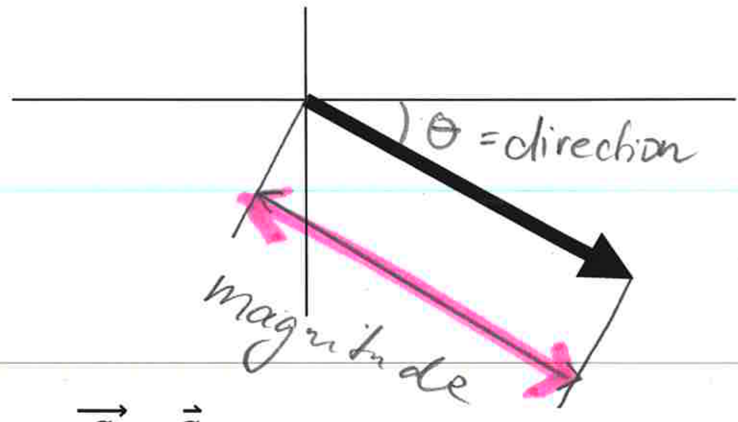
Physical quantities are either vectors or scalars.

While scalars have only magnitude, vectors have both magnitude and direction.

VECTOR		UNIT	
SYMBOL	**QUANTITY**	**NAME**	**SYMBOL**
$\vec{a}$	acceleration	meter per second squared	m/s^2
$\vec{v}$	velocity	meter per second	m/s
$\vec{d}$	displacement	meter	m
$\vec{F}$	force	Newton	N
$\vec{p}$	momentum	meter kilogram per second	$\frac{m \cdot kg}{s}$
$\vec{J}$	impulse	Newton second	$N \cdot s$
$\vec{E}$	electric field	Newton per coulomb	N/C
$\vec{B}$	magnetic field	Tesla	T
$\vec{\tau}$	torque	Newton meter	$N \cdot m$

VECTOR is an oriented ray with a head and a tail.





- A symbol of a vector quantity has either an arrow on top ($\overrightarrow{a}$ or $\vec{a}$) or in older texts is in bolded font (**a**).
- Vectors in two dimensions can be added, subtracted, multiplied by a scalar and multiplied using a **dot product(˙)**
- Vectors in three or more dimensions can be added, subtracted, multiplied by a scalar, multiplied using a dot product and multiplied using a **cross product** (×)
- graphical and numerical methods exists to carry on the above operations

VECTOR NOTATION

- Symbol
- Equal sign
- Square brackets
- Vector components separated by a comma
- Units

Examples:

- $\vec{v} = [v_x, v_y]$ m/s
- $\vec{d} = [d_x, d_y]$ m

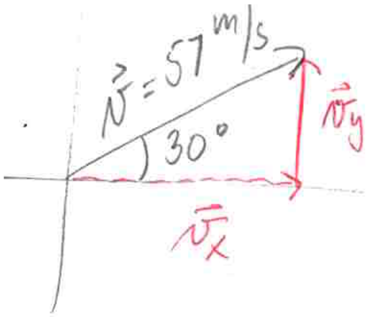
VECTOR COMPONENTS in 2 dimensions

- Vector components are vectors and as such they have direction and units
- Vector components are perpendicular to one another (= orthogonal)
- The horizontal vector component is listed first
- The vertical vector component is listed second

Resolving a vector into its components:

Example. Find vector components of initial velocity $\vec{v} = 57$ m/s [30° above horizontal]. Write the velocity vector in vector notation.

Sketch the velocity vector first. Label the angle and the vector components. Use trigonometry to find the vector components. Remember to have your calculator set to degrees.



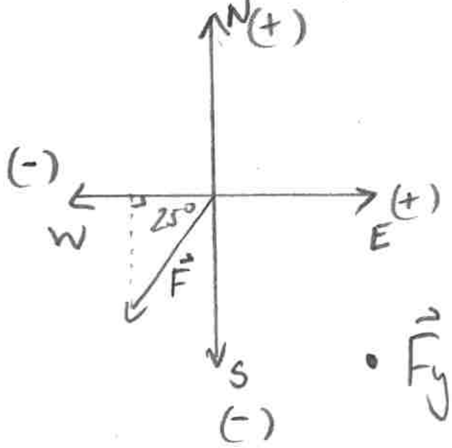
- $\vec{v}_x = \|\vec{v}\| \cdot \cos\theta$
  $= (57)(\cos 30°)$
  $\doteq 49$ m/s [Right]

- $\vec{v}_y = \|\vec{v}\| \cdot \sin\theta$
  $= (57)(\sin 30°)$
  $= 28.5 \doteq 29$ m/s [Up]

General formulae for any two dimensional vector $\vec{b}$ with direction θ:

Horizontal component:	Vertical components:
$\vec{b}_x = \pm \|\vec{b}\| \cdot \cos\theta$	$\vec{b}_y = \pm \|\vec{b}\| \cdot \sin\theta$

1. Find the vector components of net force $\vec{F}$ = 350 N [west 25° south]. **Include a diagram.** $W25°S = 25°S \text{ of } W$



- $\vec{F_x} = -350 \cdot \cos 25°$

$= -317.2077\ N$

$\vec{F_x} \doteq -3.2 \times 10^2\ N$ OR $\vec{F_x} = -317\ N$

OR $\vec{F_x} = 317\ N\ [W]$

- $\vec{F_y} = -350 \cdot \sin 25°$

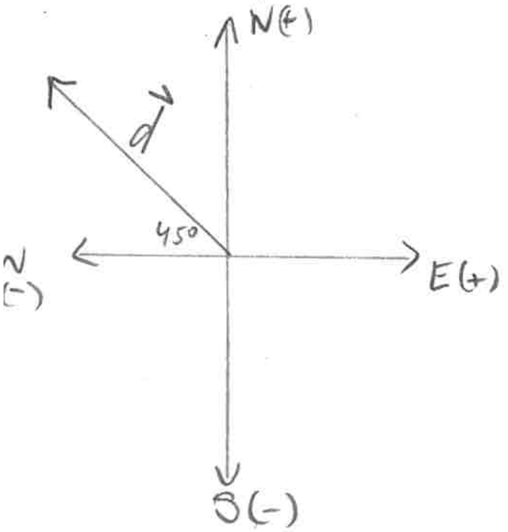
$= -147.9164\ N$

$\vec{F_y} = -1.5 \times 10^2\ N$ OR $\vec{F_y} = -148\ N$ OR $\vec{F_y} = 148\ N\ [$

Write the force vector using vector notation.

$\therefore \vec{F} = [-317, -148]\ N$

2. Resolve a displacement vector $\vec{d}$ = 65 km [NW] into two orthogonal components. **Include a diagram.**



- $\vec{d_x} = -65 \cdot \cos 45°$

$\vec{d_x} = -46\ m = 46\ m\ [W]$

- $\vec{d_y} = 65 \sin 45°$

$\vec{d_y} = 46\ m = 46\ m\ [N]$

Write the displacement vector using vector notation.

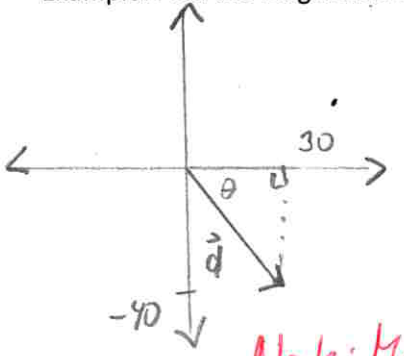
$\therefore \vec{d} = [-46, 46]\ m$

If you feel that you need more practice, please see me.

Magnitude of a vector

- Use the Pythagorean theorem to calculate the magnitude of a vector in vector notation
- Magnitude is the size of the vector (how fast, how far, how strong ...etc)

Example: Find the magnitude of a displacement vector $\vec{d}$ = [30,-40] km. **Include a diagram.**



$$\|\vec{d}\| = \sqrt{30^2 + (-40)^2}$$
$$= \sqrt{2500}$$
$$= 50 \text{ km}$$

$\therefore \|\vec{d}\| = 50 \text{ km}$

Note: Magnitude is always positive.

General formula to find the magnitude of vector $\vec{b} = [b_x, b_y]$:

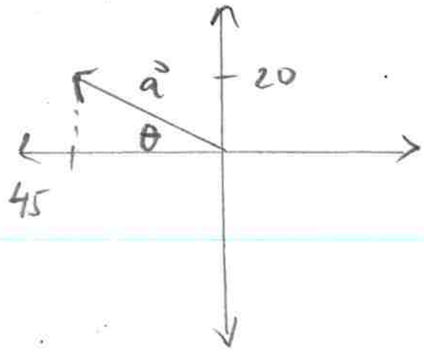
$$\boxed{\|\vec{b}\| = \sqrt{b_x^2 + b_y^2}}$$

Direction of a vector

- Use a tangent ratio to calculate the direction of a vector in vector notation

Example: Find the direction of the acceleration vector $\vec{a}$ = [- 45, 20] m/s^2

Always round to the nearest degree.



$$\theta = \tan^{-1}\left(\frac{20}{45}\right)$$
$$\theta = 23.9625\ldots^\circ$$
$$\therefore \theta = 24^\circ$$

$\therefore$ Direction of $\vec{a}$ is W 24° N (OR 24° N of W).

General formula to find the direction of vector $\vec{b} = [b_x, b_y]$:

$$\boxed{\theta = \tan^{-1}\left(\frac{|b_y|}{|b_x|}\right)}$$

GRAPHICAL METHOD

- A vector diagram must be drawn **to scale** using a straight edge and a protractor
- All angles have to be exact
- The order of vectors is irrelevant when adding vectors
- The order of vectors is **important** when subtracting vectors
- **Vector subtraction is an addition of a negative vector**
- Triangular and parallelogram method are the most frequent graphical methods of vector addition and subtraction

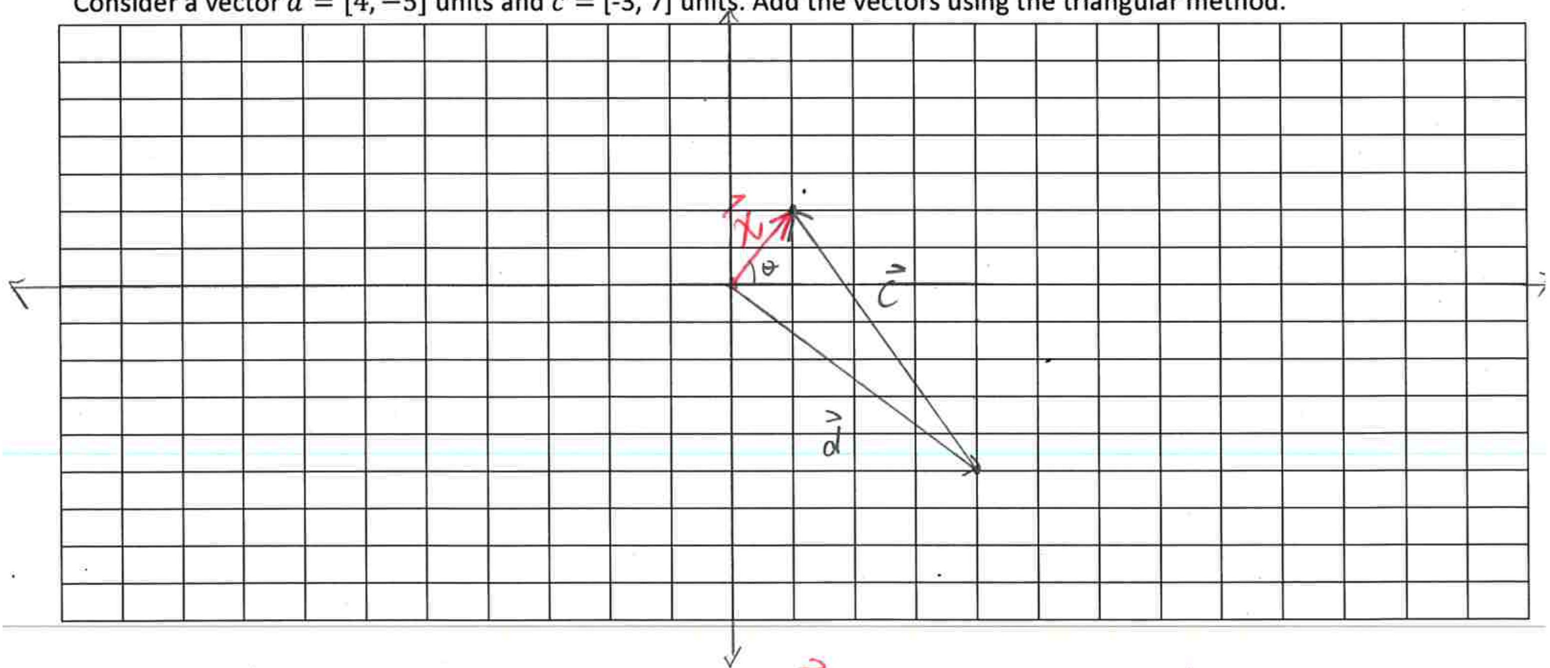
"Negative vector" = vector that has the same magnitude but opposite direction.

Examples: Write negative vectors associated with the following vectors:

$\vec{d}$ = 35 km [W]	$\vec{v}$ = [20, - 68] km/h	$\vec{a}$ = 9.8 m/s^2 [left]	$\vec{F}$ = [-65, 30] N	$\vec{p}$ = 45 kg.m/s [NW]
$-\vec{d} = 35\,km$ [E]	$-\vec{v} = [-20, 68]\,\frac{km}{h}$	$-\vec{a} = 9.8\,\frac{m}{s^2}$ [right]	$-\vec{F} = [65, -30]\,N$	$-\vec{p} = 45\,\frac{kgm}{s}$ [SE]

Example.1. Vector addition – triangular method = "head-to-tail" method

Consider a vector $\vec{d} = [4, -5]$ units and $\vec{c}$ = [-3, 7] units. Add the vectors using the triangular method.



Write the resultant vector in vector notation. $\vec{x} = \vec{d} + \vec{c} = [1, 2]$ units

What is the vector's magnitude?

$$\|\vec{x}\| = \sqrt{1^2 + 2^2} = \sqrt{5} \doteq 2.2 \text{ units}$$

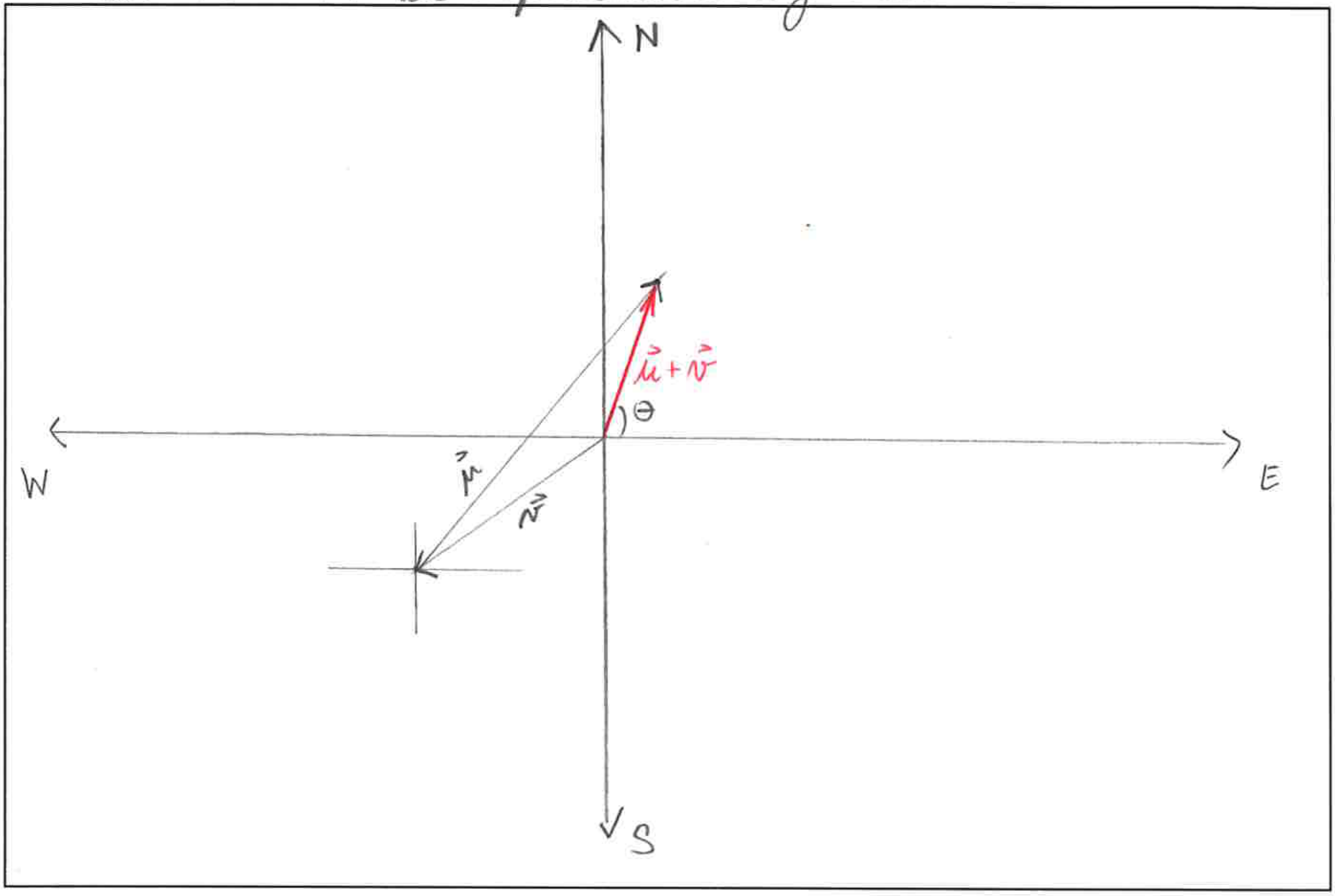
What is the vector's direction?

$$\theta = \tan^{-1}\left(\frac{2}{1}\right) = 63^\circ$$

∴ the direction is R 63° U.
E 63° N

Example 2. Draw a vector addition diagram of $\vec{v} = 30$ km/h [W 35° S] and $\vec{u} = 50$ km/h [N 40°E].

Let 10 km/h be represented by 1 cm.



Measure the magnitude of the resultant vector.

$\vec{v}+\vec{u} \doteq 22$ km/h

Measure the direction of the resultant vector.

$\theta \doteq 70°$ $\therefore$ direction of $\vec{u}+\vec{v}$ = E 70°N

How could you confirm the accuracy of the measured values?

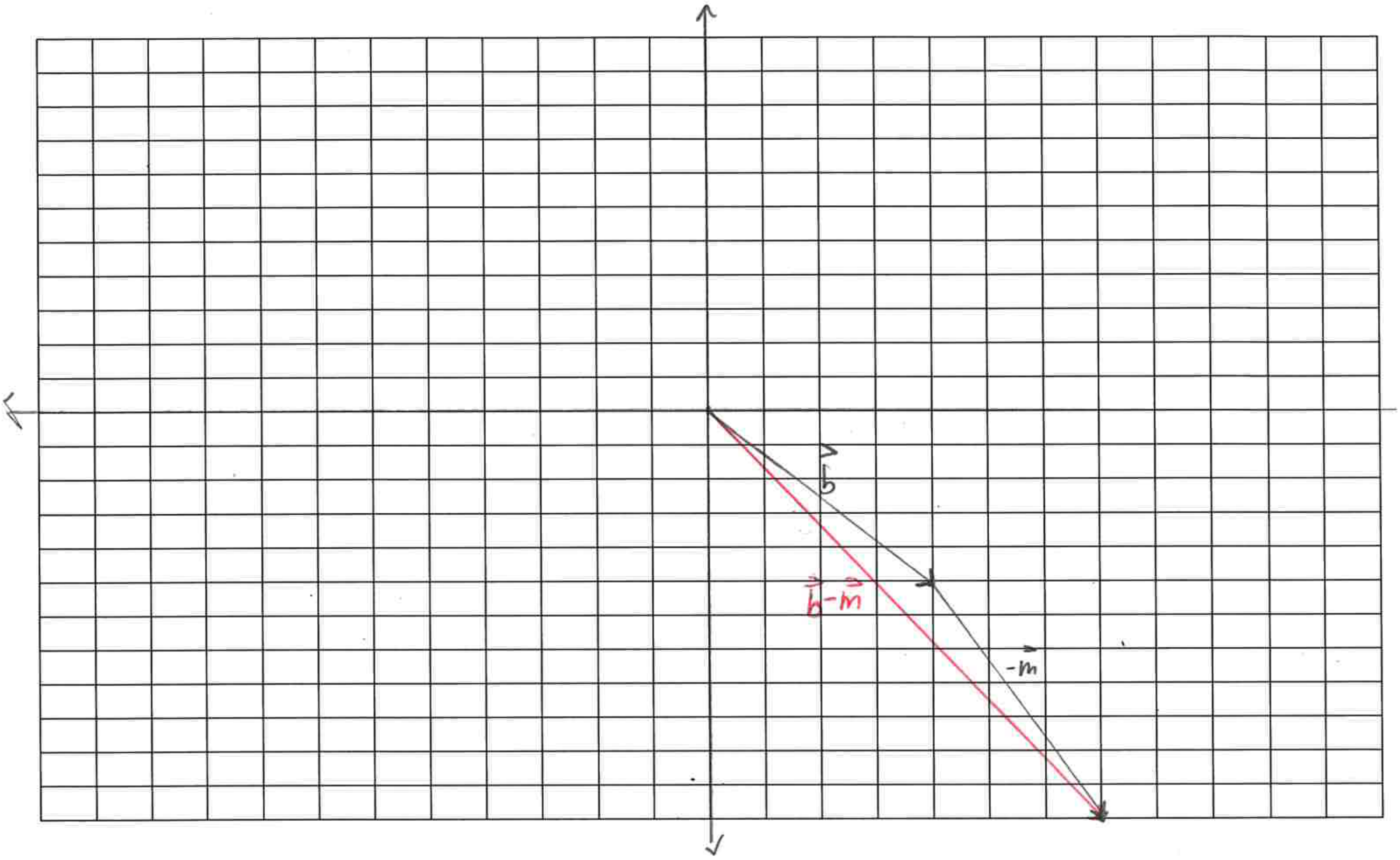
Using a numerical - algebraic method would verify the graphical method results.

In your own words describe the "head-to-tail" method of vector addition

Example 3. Consider a vector $\vec{b} = [4, -5]$ units and $\vec{m} = [-3, 7]$ units. Subtract **m** from **b** using the triangular method.

Negative $\vec{m}$ = $[3, -7]$ units

$$\vec{b} - \vec{m} = \vec{b} + (-\vec{m}) = [4, -5] + [3, -7]$$



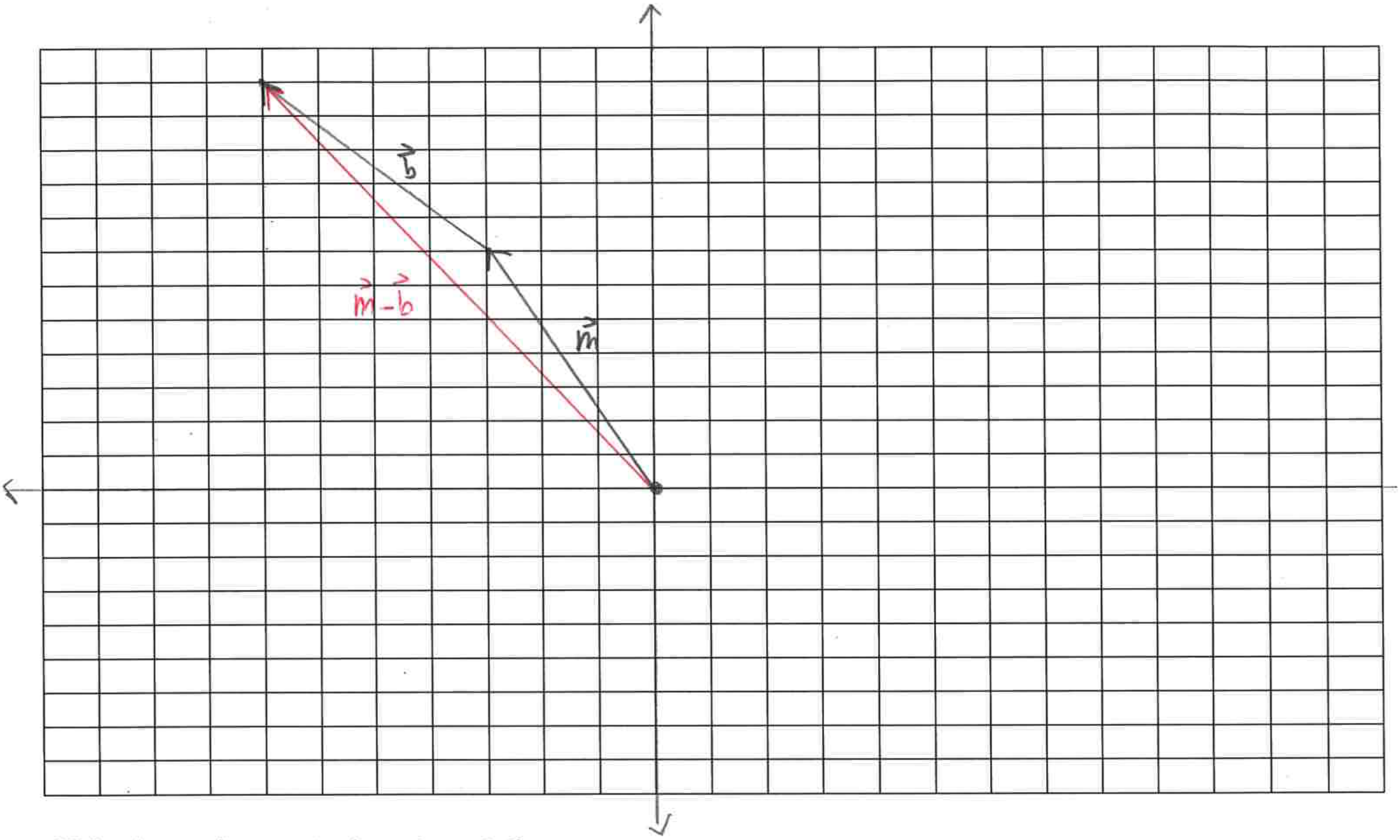
Write the resultant vector in vector notation:

$\vec{b} - \vec{m} = [7, -12]$ units

Example 4. Consider a vector $\vec{b} = [4, -5]$ units and $\vec{m} = [-3, 7]$ units. Subtract **b** from **m** using the triangular method.

Negative $\vec{b}$ = $-\vec{b} = [-4, 5]$ units

$\vec{m} - \vec{b} = \vec{m} + (-\vec{b}) = [-3, 7] + [-4, 5]$



Write the resultant vector in vector notation:

$\vec{m} - \vec{b} = [-7, 12]$ units

How the resultant vectors from example 3 and 4 compare? In what way are they same and in what way are they different?

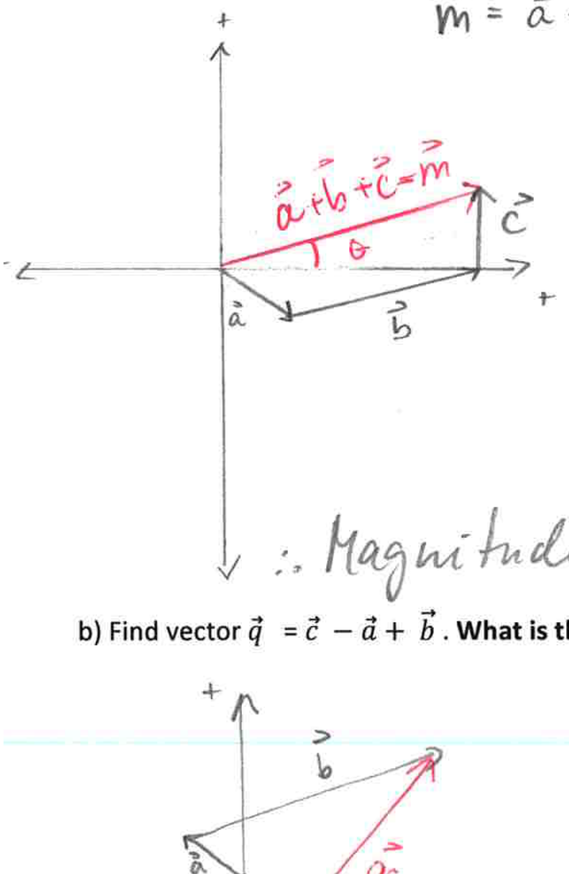
$(\vec{b} - \vec{m})$ and $(\vec{m} - \vec{b})$ have the same magnitude and opposite direction

NUMERICAL METHOD

1. Sketch a diagram of each given vector.
2. Label the vector components on the diagrams
3. Sketch a diagram of the vector addition or subtraction
4. Resolve all given vectors into their vector components (when necessary)
5. **Write a vector equation of the addition or subtraction**
6. Carry out all operations
7. Write the resultant vector in vector notation.
8. Sketch a diagram of the resultant vector or highlight it in the diagram from step 3.
9. Find the magnitude of the resultant vector using the Pythagorean theorem
10. Find the direction of the resultant vector using the tangent ratio

Example 1: Consider vectors $\vec{a}$ = [3, -2] m, $\vec{b}$ = [13, 2] m and $\vec{c}$ = [0, 5]m

a) Find vector $\vec{m} = \vec{a} + \vec{b} + \vec{c}$. **What is the magnitude and direction of the resultant vector?**



$\vec{m} = \vec{a}+\vec{b}+\vec{c} = [3,-2]+[13,2]+[0,5]$

$= [3+13+0, -2+2+5]$

$= [16,5]$ m

$\|\vec{m}\| = \sqrt{16^2+5^2}$

$= \sqrt{281}$

$\doteq 16.7631$ m

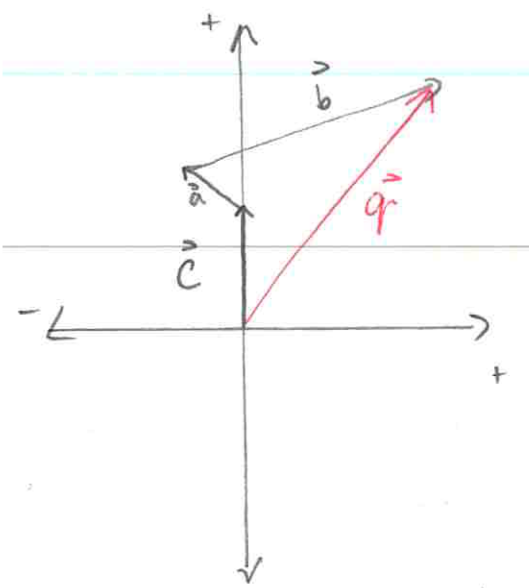
$\doteq 17$ m

$\theta = \tan^{-1}\left(\frac{5}{16}\right)$

$\theta \doteq 17°$

$\therefore$ Magnitude of $\vec{m}$ is 17m and its direction is E17°N.

b) Find vector $\vec{q} = \vec{c} - \vec{a} + \vec{b}$. **What is the magnitude and direction of the resultant vector?**



$-\vec{a} = [-3,2]$ m

$\vec{q} = \vec{c}-\vec{a}+\vec{b} = [0,5]+[-3,2]+[13,2]$

$= [0-3+13, 5+2+2]$

$= [10,9]$ m

$\|\vec{q}\| = \sqrt{10^2+9^2}$

$= \sqrt{181}$

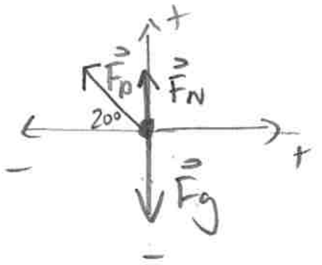
$\doteq 13$m

$\theta = \tan^{-1}\left(\frac{9}{10}\right)$

$\theta \doteq 42°$

$\therefore$ Magnitude of $\vec{q}$ is 13m and its direction is E42°N.

Example 2: Consider three forces $\overrightarrow{F_{pull}}$ = 25.0 N [left 20° up], $\overrightarrow{F_g}$ = 15.0 N [down], and $\overrightarrow{F_N}$ = 6.4 N [up], acting on an object at once. **Find the magnitude and direction of the net force.**



$$\bullet\ \vec{F}_{net} = \vec{F}_{pull} + \vec{F}_g + \vec{F}_N$$
$$= [-23.4923, 8.5505] + [0, -15] + [0, 6.4]$$
$$= [-23.4923, 0.0495]\ N$$
$$= \underline{\underline{[-23, 0]\ N}}$$

$\bullet\ \|\vec{F}_{net}\| = 23\ N$

$\vec{F}_{pull} = [-25 \cdot \cos 20°, 25 \cdot \sin 20°]\ N$

$\bullet\ \theta = N/A \rightarrow$ left

$\vec{F}_g = [0, -15]\ N$

$\vec{F}_N = [0, 6.4]\ N$

$\therefore \vec{F}_{net} = 23\ N$ [left]

Example 3: Consider an object that is pushed on a horizontal surface with force $\overrightarrow{F_{push}}$ = 75.0 N [left] and experiences force of friction F_f = 23 N. $\rightarrow \vec{F}_f = 23\ N$ [Right]

a) Write the force vectors using vector notation.

$\vec{F}_{push} = [-75.0, 0]\ N$ and $\vec{F}_f = [23, 0]\ N$

b) Write a vector equation that expresses the net force.

$\rightarrow$ assume on Earth

$$\vec{F}_{net} = \underbrace{\vec{F}_g + \vec{F}_N}_{0} + \vec{F}_{push} + \vec{F}_f \Rightarrow \vec{F}_{net} = [-75.0, 0] + [23, 0]$$
$$= [-52, 0]\ N$$

c) Calculate the magnitude and direction of the net force.

$\bullet\ \|\vec{F}_{net}\| = 52\ N$

$\bullet$ direction : Left

$\therefore \vec{F}_{net} = 52\ N$ [L]