P12

Special Theory of Relativity

- Proposed by Albert Einstein in 1905
- Describes events that are observed and measured from **<u>inertial reference frames.</u>**

Inertial Frame of Reference = a reference frame in which Newton's First Law is valid = if an object experiences a zero net force it will either remain at rest of it will continue moving at constant velocity.
= non-accelerating frame of reference

Relativity Principle = the basic laws of physics are the same in all inertial reference frames.

Galilean-Newtonian relativity works with unprovable assumptions that are confirmed by our everyday experiences. For example, it is assumed that the length, mass, forces, acceleration, and time are the same in one reference frame as in another.
Position and **velocity** are different when specified in different reference frames.

All inertial reference frames are equally valid.

Speed of light $3.00 \times 10^8 m/s$

Postulates of the Special Theory of Relativity

1. **The Relativity Principle = The laws of physics have the same form in all inertial reference frames.** In other words, there is not experiment you can carry out in an inertial reference frame to tell if you are at rest or moving in uniformly at constant velocity.

2. **Constancy of the Speed of Light = Light propagates through empty space with definite speed c independent of the speed of the source of observer.**

Simultaneity

- An important result of the special theory of relativity is that time is not an absolute quantity, that is the length of a time interval or the fact whether two event can be considered simultaneous is dependent on the observer's frame of reference.
- Two events are simultaneous if they occur exactly at the same time. If the two events happen at locations separated by great distance and one location is much closer to the observer, time it took for the light to travel from the location to the observer must be taken into an account.
- Furthermore, two events considered simultaneous by one observer are not necessarily simultaneous for another observer who moves with respect to the first observer.
- **Simultaneity** is not an absolute concept, it **is relative.** This suggests that time is also relative.

Result of the Special Theory of Relativity:

1. **Time dilation.**
2. **Length contraction.**
3. **Increase in mass = relativistic momentum and mass.**

Time Dilation

- Time passes differently in one reference frame than in another.
- **Clocks moving relative to an observer are measured by that observer to run more slowly as compared to clocks at rest.**

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

- c = speed of light
- v = speed of the object
- Δt_0= time interval measured in the fast moving reference frame (=proper time)
- Δt = time interval observed by an observer on Earth (at rest)
- Note: $\Delta t > \Delta t_0$ in most scenarios.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Gamma is a factor that occurs in relativity scenarios. In general, $\gamma \geq 1$ and $\gamma = 1$ for typical speeds.

Example 1A: Does time dilation exist for an object that moves at 100 km/h? Consider a car moving at 100 km/h. This car needs exactly 10.0s to cover certain distance as measured by a passenger's watch. What does an observer at rest on Earth measure for the time interval?

Example 1B: What if the car travelled at 95% speed of light? How would the passenger's 10.0 seconds compare to time measured by an observer on Earth?

Example2: If a spaceship travelled at $v = 0.999c$, how long would a 100-year trip take for the astronauts? (100 years measured on Earth).

Length Contraction

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

L_0 = proper length , measured by an observer at rest [

L = length measured by an observer when an object travels by them at speed v

v = speed of the object

<u>It is important to note that the length of contraction occurs only along the direction of motion.</u>

Example 1: A rectangular painting measuring 1.00 m tall and 1.5 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at speed of 0.90c. a) What are the dimensions of the picture according to the captain of the spaceship? b) What are the dimension as seen by an observer n the Earth?

Example 2: A very fast train with a proper length of 500 m is passing through a 200-m long tunnel. The train's speed is so great that the train fits completely within the tunnel as seen by an observer at rest on the Earth (on the mountain above the tunnel); that is, the engine is just about to emerge from one end of the tunnel as the last cart disappears onto the other end. What is the train's speed?

Note: Considerations like this led to the idea of four-dimensional space-time: space takes up three dimensions and time is a fourth dimension. Space and time are intimately connected. This refers to the idea that any object or event is specified by four quantities – three to describe where in space, and one to describe when in time. Space and time can inter-mix: a little bit of one can be exchanged for a little bit of the other when the reference frame is changed.

Relativistic Momentum and Mass

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_0 = proper mass = rest mass

v= speed of the object

c = speed of light in vacuum

p = relativistic momentum

Mass can increase with speed according to:

$$m_{rel} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Example 1: Compare the momentum of an electron when it has speed of a) $1.00\times 10^7 m/s$ and when it has a speed of b) 0.98 c.

- **A basic result of the special theory of relativity is that the speed of an object cannot equal or exceed the speed of light.** To accelerate an object to the speed of light would thus required infinite amount of energy, and so it is not possible.

Mass and Energy

$$\boxed{KE = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2}$$

$$\boxed{KE = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)}$$

For particles at rest = rest energy:

$$\boxed{\boldsymbol{E_0 = m_0 c^2}}$$

Example: A π^0 meson ($m_0 = 2.40 \times 10^{-28} kg$)travels at a speed of v = 0.80c. What is tis kinetic energy. Compare to a classical calculation.

Relativistic Addition of Velocities

$$\boxed{v_{AB} = \frac{v_{AC} + v_{CB}}{1 + \frac{v_{AC} v_{CB}}{c^2}}}$$

Example: A spacecraft approaching the Earth launches an exploration vehicle. After the launch, an observer on Earth sees the spacecraft approaching at speed of 0.50c and the exploration vehicle approaching at speed of 0.70c. What is the speed of the exploration vehicle relative to the spaceship?