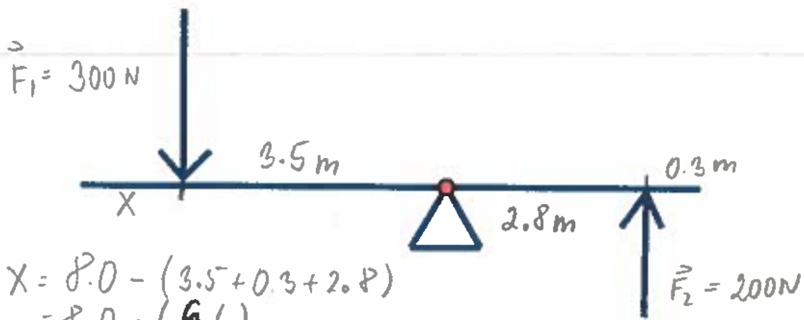
Answers

Rotational and Static Equilibrium

1. Find the force needed at the very left end of a lever in order to maintain rotational equilibrium of a 8 .0 m long lever.

$\Sigma\vec{\tau}=0$



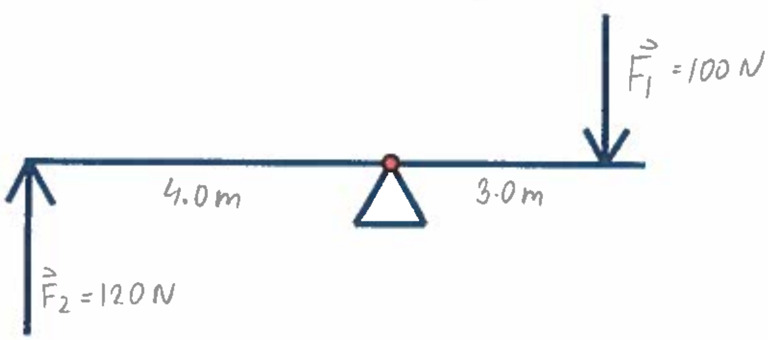
$X = 8.0 - (3.5+0.3+2.8)$
$= 8.0 - (6.6)$
$= 1.4$ m

$\vec{\tau}_1 = (300)(3.5) = 1050$ N·m [CCW]
$\vec{\tau}_2 = (200)(2.8) = 560$ N·m [CCW]
$\Sigma\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_F$
$0 = 1050 + 560 - \vec{\tau}_F$
$= 1610 - F(1.4+3.5)$
$F = \frac{1610}{4.9}$
$F \doteq 329$ N
$\vec{F} = 329$N [up]

2. a) How far left from the fulcrum would you have to place an object with mass 50.0 kg in order to ensure a rotational equilibrium?

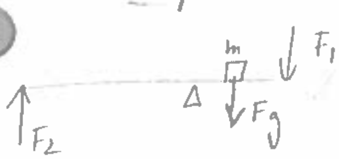
b) Predict how far right from the fulcrum would you have to place this 50.0 kg mass in order to ensure a rotational equilibrium? Why? Confirm your reasoning by calculating the $\sum\vec{\tau}$.

a)



a) $\vec{\tau}_1 = 100(3) = 300$ N m [CW]
$\vec{\tau}_2 = 120(4) = 480$ N·m [CW]
$\vec{\tau}_m = ?$
$\Sigma\vec{\tau} = 0 = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_m$
$0 = -300 - 480 + \vec{\tau}_m$
$\vec{\tau}_m = 780$ N m [CCW]
$\vec{\tau}_m = F_g d$
$780 = mgd$
$= (50.0)(9.8)d$
$d = \frac{780}{490}$
$d = 1.59$ m

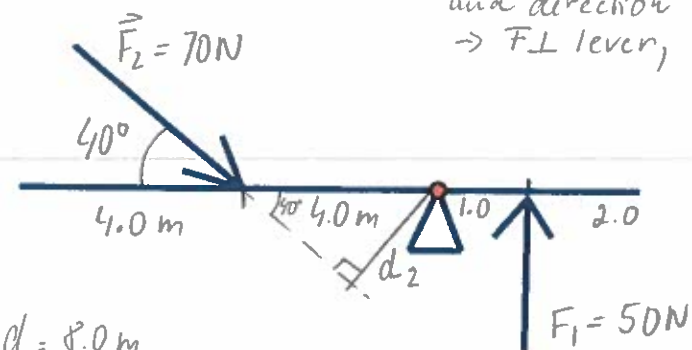
b) No matter how far to the right away from the fulcrum the 50.0kg mass will only increase the clock-wise torque.



$\Sigma\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_m$
$0 = -300 - 480 - F_g d$
$d = \frac{-780}{490}$
$d = -1.59$ m
? nonsense as magnitude cannot be negative

3. Consider an 11.0-m long lever. Where and at what direction would you apply a smallest possible force that would ensure a rotational equilibrium of the system? Find $\|\vec{F}\|$

Note: smallest possible Force must be applied at the point and direction where it have the maximum effect → F ⊥ lever, F as far from the fulcrum as possible



$\vec{\tau}_1 = (50)(1) = 50\ N\cdot m$ [CCW]

$\vec{\tau}_2 = (70)(2.571) = 180\ N\cdot m$ [CCW]

$d_2 = \sin(40°)(4.0) = 2.571\ m$

for $\Sigma\vec{\tau} = 0$ $\|\vec{\tau}_F\| = \|\vec{\tau}_1 + \vec{\tau}_2\|$

$\vec{\tau}_F = 50 + 180 = 230\ N\cdot m$ [CW]

$d_F = 8.0\ m$

$\vec{\tau}_F = \vec{F}\cdot\vec{d}_F$

$230 = F\ 8.0$

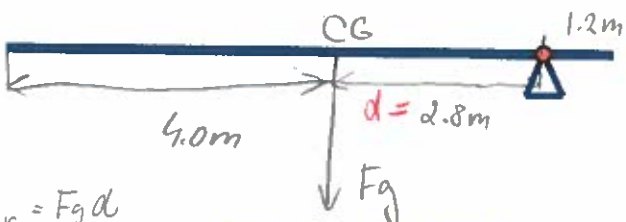
$F = \frac{230}{8} = 29\ N$ [up]

∴ The Force of 29N applied at the far left end of the lever perpendicularly upwards would ensure the rotational equilibrium

4. Provided that the bar below is 8.0 m long, its mass is 50.0 kg, and the pivot point is 1.2 m from the its right-most point, what is the torque of the bar?

$Fg_{bar} = mg = (50.0)(9.8) = 490\ N$

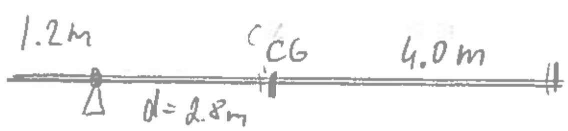
Center of gravity: 4m from left (or right) provided the mass of the bar is uniformly distributed



$\vec{\tau}_{bar} = Fg\,d$
$= (490)(2.8) = 1372\ N\cdot m$ [CCW]

5. How would the torque from question 4 change if the pivot point was 1.2 m from the left-most point of the bar? Sketch a labelled diagram. And show calculation supporting your prediction.

$Fg = 490\ N$



$\vec{\tau}_{bar} = Fg\ d$
$= (490)(2.8)$
$= 1372\ N\cdot m$ [CW]
$= -1372\ N\cdot m$

∴ Magnitude of the torque remains the same as Fg as well as $\|d\|$ do not change.
However, direction of the Torque is reversed.

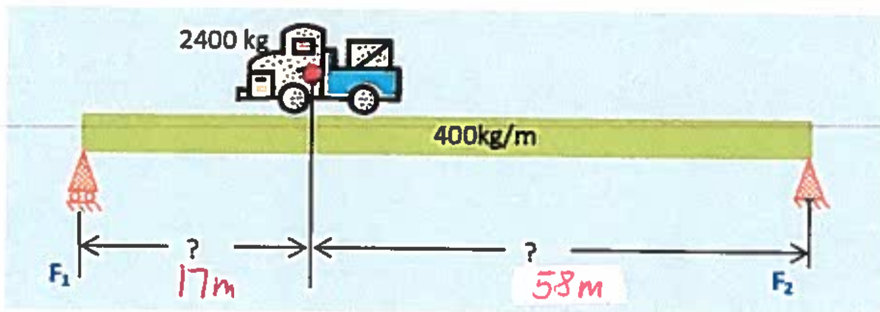
6. Consider a truck is parked on a 75.0-m bridge. The truck's mass is 2400 kg. Assume that the center of gravity of the track is 2m from its front bumper and the track is parked 15 m from the left-most support beam (measured from the front bumper). The bridge' mass is uniformly distributed at 400 kg per meter. Assume that the bridge is rigid.

$m_{bridge} = (75.0\,m)(400\,kg/m) = 3\times10^4\,kg$

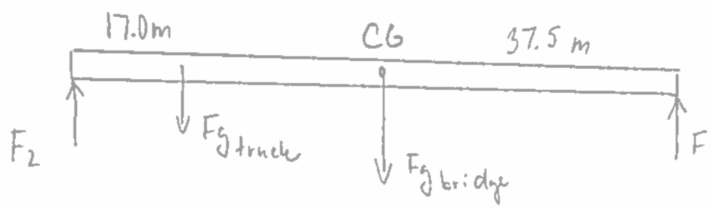
$Fg_{bridge} = (3.0\times10^4\,kg)(9.8\,\frac{m}{s^2})$
$= 2.94\times10^5\,N$

$CG = \frac{75}{2} = 37.5\,m$

$Fg_{truck} = (2400)(9.8) = 2.35\times10^4\,N$



a) Draw a FBD.



b) What will be the reaction forces at the supports of bridge at the two ends? (Hint: chose the left support beam as a pivot point and solve for F_2, then choose the right support beam as a pivot point and solve for F_1).

Pivot at the left:

$\vec{\tau}_1 = 0$

$\vec{\tau}_2 = ?$

$\vec{\tau}_t = (2.35\times10^4)(17) = 3.995\times10^5\,N\,m\,[CW]$

$\vec{\tau}_b = (2.94\times10^5)(37.5) = 1.1\times10^7\,N\cdot m\,[CW]$

for $\Sigma\vec{\tau}=0$ $\vec{\tau}_2$ must be in CCW direction and equal to $\vec{\tau}_b + \vec{\tau}_t$

$\Sigma\vec{\tau} = 0 = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_t + \vec{\tau}_b$

$0 = 0 + F_2(75) - 3.995\times10^5 - 1.1\times10^7$

$F_2 = \frac{1.142\times10^7}{75}$

$F_2 = 1.5\times10^5\,N$

Pivot at the right

$\vec{\tau}_2 = 0$

$\vec{\tau}_b = 1.1\times10^7\,N\cdot m\,[CCW]$

$\vec{\tau}_t = (2.35\times10^4)(58) = 1.363\times10^6\,N\cdot m\,[CCW]$

$\vec{\tau}_1 = ?$

for $\Sigma\vec{\tau}=0$ $\vec{\tau}_1$ has to be CW and equal $\vec{\tau}_b + \vec{\tau}_t$

$\Sigma\vec{\tau} = 0 = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_b + \vec{\tau}_t$

$0 = F_1(75) + 0 + 1.1\times10^7 + 1.363\times10^6$

$F_1 = \frac{1.236\times10^7}{75}$

$F_1 = 1.6\times10^5\,N$

p250 - 251 ex 4

7. Consider an 80-kg painter holding a 2.0-kg can of paint and standing 0.4 from the top of a 3.0-m long, 20 kg, ladder. The ladder is leaning against a frictionless (="smooth") wall and is placed at an angle of 55° against the ground. The ladder is in static equilibrium. Find:

a) The center of gravity of the ladder and label it in the diagram

b) $\vec{F_g}$ of the ladder, express it in vector notation and label it in the diagram.

c) $\vec{F_g}$ of the painter holding the can, express it in vector notation and label it in the diagram.

d) $\vec{F_N}$ the ground exerts on the ladder, express it in vector notation and label it in the diagram

e) $\vec{F_f}$ between the ladder and the ground, express it in vector notation and label it in the diagram

f) $\vec{F_w}$ the force of the wall pushing against the ladder and express it in vector notation.

g) $\overrightarrow{F_{net}}$ and express in vector notation and as a vector sum of all forces above.

h) $\vec{\tau_p}$ = torque created by the weight of the painter holding the can. What do you think would vector notation look like for $\vec{\tau_p}$?

i) $\vec{\tau_l}$ = torque created by the weight of the ladder. What do you think would vector notation look like for $\vec{\tau_l}$?

b) $\vec{F}_{g_L} = (20)(9.8) = 196\text{ N} = [0, -196]\text{N}$

c) $\vec{F}_{g_p} = (82)(9.8) = 804\text{N} = [0, -804]\text{N}$

d) $\vec{F_N} = F_{g_L} + F_{g_P} = 196 + 804 = 1000\text{N} = [0, 1000]\text{N}$

e) $\|\vec{F_f}\| = \|\vec{F_w}\|$

-but not enough information is provided, so we have to look at $\Sigma\vec{\tau} = 0$ and find F_w first

$\Sigma\vec{\tau} = 0$

$0 = \vec{\tau}_L + \vec{\tau}_p + \vec{\tau}_w$

$0 = 169 + 1.2\times10^3 - 1.72F_w$

$F_w = \frac{1.3686\times10^3}{1.72}$

$= 796\text{ N}$

f) $\vec{F_w} = [796, 0]\text{N}$

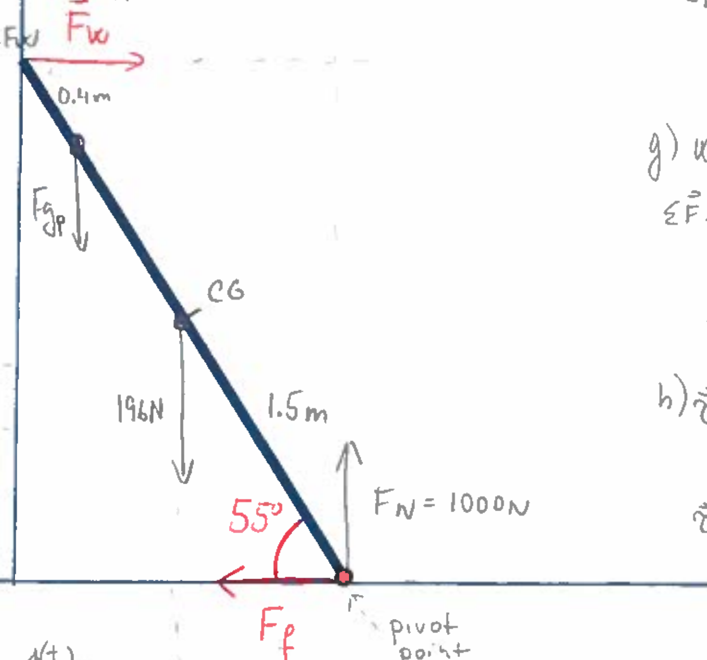
e) $\vec{F_f} = [-796, 0]\text{N}$

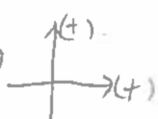
$\vec{\tau}_L = (196)(\cos55°)(1.5)$
$= 169\text{ N m [CCW]}$

$\vec{\tau}_p = (804)(\cos55°)(2.6)$
$= 1.2\times10^3\text{ N m [CCW]}$

$\vec{\tau}_w = ?$
$= -(F_w)(3.0)(\sin55°)$
$= -(F_w)(2.457)$

$F_w = 556.2822\text{ N}$





g) we need $\vec{F_w}$ first $\Sigma\vec{F} = [0,0]$

$\Sigma\vec{F} = \vec{F_w} + \vec{F_f} + \vec{F}_{g_p} + \vec{F}_{g_L} + \vec{F_N}$

$= [556, 0] + [-556, 0] + [0, -804] + [0, -196] + [0, 1000]$

$= [0, 0]$

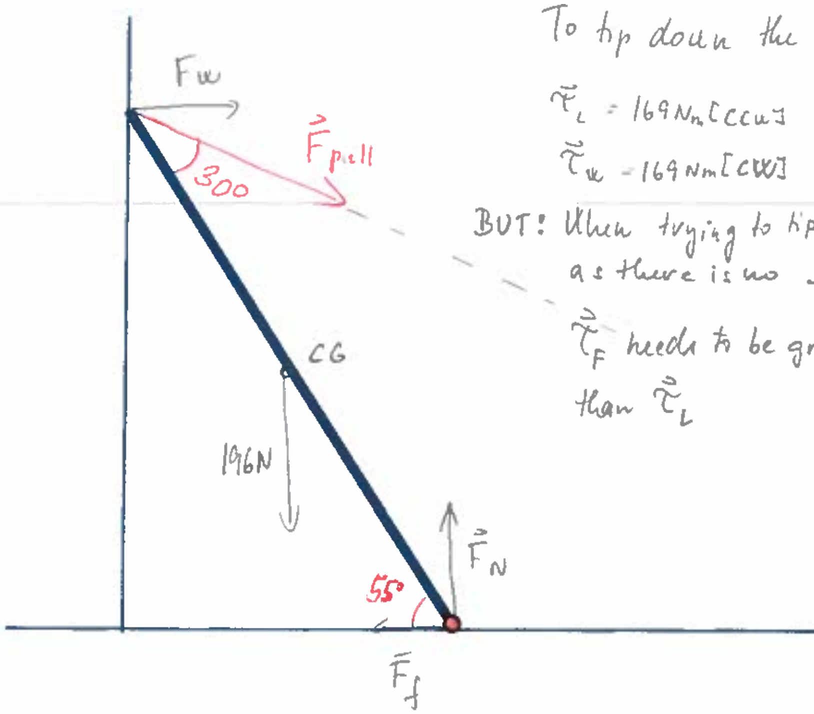
h) $\vec{\tau}_p = 1.2\times10^3\text{ N m [CCW]}$

$= [0, 0, 1.2\times10^3]\text{ N m}$

$\vec{\tau}_L = [0, 0, 169]\text{ N m}$

B

8. Consider the ladder from the previous question. If there were no objects on the ladder, with what force would you have to pull down on the ladder at angle 30° in order to tip the ladder over?



To tip down the ladder ⇒ $\Sigma\vec{\tau} \neq 0$

$\vec{\tau}_L = 169 Nm [CCW]$

$\vec{\tau}_w = 169 Nm [CW]$

BUT: When trying to tip the ladder over → no more $\vec{F}_w$ as there is no surface of contact

$\vec{\tau}_F$ needs to be greater and in opposite direction than $\vec{\tau}_L$

$\vec{\tau}_L = (196)(\cos 55°)(1.5)$
$\doteq 169 Nm [CCW]$

$\vec{\tau}_F = \vec{F}_p(\sin 30°)(3.0)$
$\doteq (1.5)\vec{F}_p$

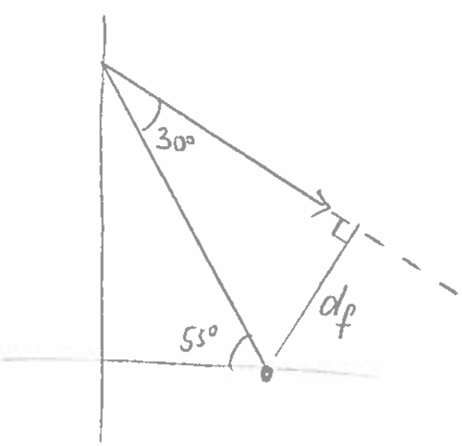
for rotational equilibrium

$\Sigma\vec{\tau} = 0 = \vec{\tau}_L + \vec{\tau}_F$

$0 = 169 - 1.5 F_p$

$F_p = \frac{169}{1.5}$

$= 113 N$



⇒ The pulling force applied at the very top of the ladder would have to be greater than 113 N in order for the ladder to tip over