Notes:

**PHYSICS 12**

# Projectile Motion

## In-Class Examples

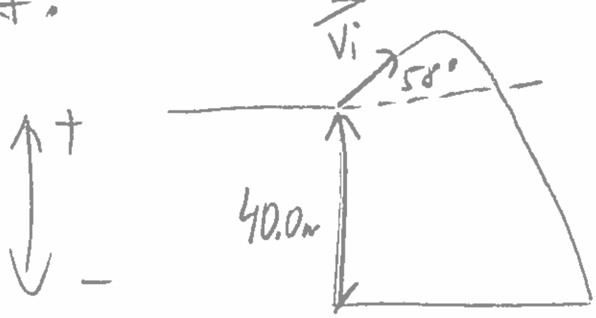
1. A 2.0-kg object is thrown at an angle of 58° above horizontal. This object land on the ground 10.0 seconds later. The ground level is 40.0 m below the launching level.
a) Find the object's initial velocity.
b) Find the horizontal displacement of the object.
c) What is the distance between the launching point and the point the object struck the ground?
d) What is the final velocity of the object just before it hits the ground?

G: $m = 2.0\,kg$
$a_y = g = 9.8\ m/s^2\,[D]$
$\theta = 58°$ above horizontal
$t_{air} = 10.0\,s$
it lands 40.0 m below launching level

- Let $\vec{v}_{iy} = \|\vec{v}_i\| \cdot \sin 58° = y$
- let $a_y = g = a$
- let $t_{air} = t$

R: $v_i = ?\,[m/s]$

A:




$t_{air} = t_{h_{max}} + t_{ff}$

$t_{air} = \frac{0-y}{a} + \sqrt{\frac{2\vec{d}_y}{a}}$

$\vec{d}_y = -(40 + h_{max})$

$= -\left(40 + \frac{0-y^2}{2a}\right)$

$= -\left(\frac{80a - y^2}{2a}\right)$

$= \frac{-80a + y^2}{2a}$

$$t = \frac{-y}{a} + \sqrt{\frac{\frac{2(-80a + y^2)}{2a}}{a}}$$

$$t = \frac{-y}{a} + \sqrt{\frac{-80a + y^2}{a} \cdot \frac{1}{a}}$$

$$t = \frac{-y}{a} + \frac{1}{a}\sqrt{\frac{-80a + y^2}{1}}$$

$$t + \frac{y}{a} = \frac{1}{a}\sqrt{-80a + y^2}$$

$$a\left(t + \frac{y}{a}\right) = \sqrt{-80a + y^2}$$

$$(at + y)^2 = \left(\sqrt{-80a + y^2}\right)^2$$

$$a^2t^2 + 2aty + y^2 = -80a + y^2$$

$$2aty = -80a - a^2t^2$$

$$y = \frac{-80a - a^2t^2}{2at}$$

$a = -9.8$
$t = 10.0$

$$y = \frac{-80(-9.8) - (-9.8)^2(10.0)^2}{2(-9.8)(10.0)}$$

$y = 45 \frac{m}{s} = V_{iy}$

$V_{iy} = \|V_i\| \cdot \sin\theta$

$$\|V_i\| = \frac{V_{iy}}{\sin\theta}$$

$$= \frac{45}{\sin 58^\circ}$$

$\doteq 53.1\ m/s$

S: the object's initial velocity is $53.1\ m/s$ $58^\circ$ above horizontal

b) $d_x = t_{air} \cdot v_{ix}$

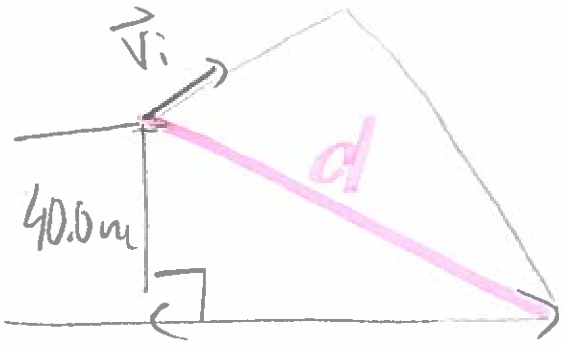
$d_x = (10.0)(v_i \cdot \cos 58°)$

$d_x = (10.0)(53.1)(\cos 58°)$

$d_x = 281\ m$

S: the object's horizontal displacement is 281 m [R] based on the assumption that it was launched to the right.

c)




$$d^2 = 40.0^2 + 281^2$$

$$d = \sqrt{80561}$$

$$d \doteq 284\,m$$

S: Distance between the launching and landing point is 284 m ($= 2.8 \times 10^2$ m).

d) $\vec{v}_f = [v_{ix}, v_{fy}]$ m/s

$v_{ix} = 28.1$ m/s

$$v_{fy} = -\sqrt{v_{iy}^2 + 2a_y d_y}$$

$$= -\sqrt{0^2 + 2(-9.8)\left(-\left(40 + \frac{-45^2}{2(-9.8)}\right)\right)}$$

$$= -\sqrt{2809}$$

$$= -53 \text{ m/s}$$

$$\|v_f\| = \sqrt{(28.1)^2 + (-53)^2}$$

$$= 59.988\ldots \text{ m/s}$$

$$= 6.0 \times 10^1 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{53}{28.1}\right)$$

$$\theta = 62°$$

∴ the object's final velocity is $6.0 \times 10^1$ m/s [R 62° D].