Answers

Physics 12

Projectile Motion Practice

1. An object spent 8.45 seconds in the air. It was launched at 35° above horizontal.
   a) Find the initial velocity of the object if it landed at the same level it was launched from.
   b) Find the initial velocity of the object if it landed 5.4 meters above its launching point.

G: $t_{air} = 8.45\ s$

a) $\theta = 35°$ above

$a_y = -9.8\ m/s^2$

R: $\vec{v}_i = ?\ [m/s]$

A:
- $t_{air} = 2t_{hmax}$
- $t_{hmax} = \frac{t_{air}}{2}$
- $v_i = \frac{v_{iy}}{\sin\theta}$
- $v_{iy} = v_{fy} - a_y \Delta t_{hmax}$

S: $= 0 - (-9.8)\left(\frac{8.45}{2}\right)$

$= 41.405\ m/s$

$v_i = \frac{4.405}{\sin 35°}$

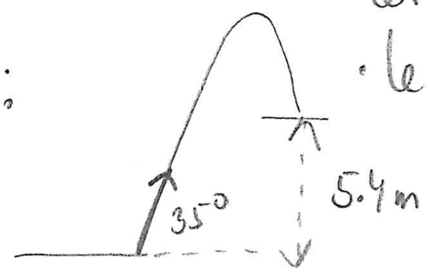
$v_i \doteq 72\ m/s$

∴ S: The initial velocity was 72 m/s [35° above horizontal].

#1 b)

· let $v_{iy} = y$
· let $t_{air} = t$
· let $a_y = a$

A:



$$t_{air} = t_{hmax} + t_{ff}$$

$$= \frac{0 - v_{iy}}{a_y} + \sqrt{\frac{2 d_y}{a_y}}$$

$$d_y = -(h_{max} - 5.4)$$

$$= -\left(\frac{0^2 - v_{iy}^2}{2a_y} - 5.4\right)$$

$$= -\left(\frac{-v_{iy}^2 - 10.8 a_y}{2a_y}\right)$$

$$= \frac{v_{iy}^2 + 10.8 a_y}{2a_y}$$

$$t = \frac{-y}{a} + \sqrt{\frac{2\left(\frac{y^2 + 10.8a}{2a}\right)}{a}}$$

$$t = \frac{-y}{a} + \sqrt{\frac{y^2 + 10.8a}{a^2}}$$

$$t + \frac{y}{a} = \frac{1}{\sqrt{a^2}} \cdot \sqrt{y^2 + 10.8a}$$

$$ta + y = \sqrt{y^2 + 10.8a}$$

$$(ta)^2 + 2tay + \cancel{y^2} = \cancel{y^2} + 10.8a$$

$$y = \frac{10.8a - t^2a^2}{2ta}$$

$$y = \frac{10.8 - t^2 a}{2t}$$

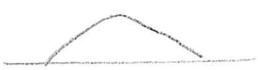
$$y = \frac{10.8 - (8.45)^2(-9.8)}{2(8.45)}$$

$$y = 42 \text{ m/s}$$

$$v_i = \frac{y}{\sin 35^\circ} = \underline{\underline{73 \text{ m/s}}}$$

2. Find the initial velocity of an object that was launched at 60° above the horizontal, given that its horizontal displacement was 78 m to the left of the launching point. Assume the same launching and landing level.

G: $\theta = 60°$ above h.
$\vec{d}_x = 78\text{ m [L]}$



R: $\vec{V}_i = ?$ [m/s]

A: $t_{air} = 2 \cdot h_{max}$

$= 2\left(\frac{0 - V_{iy}}{a_y}\right)$

$= 2\left(\frac{0 - V_i \sin 60°}{a_y}\right)$

S: $\vec{V}_i = 3.0 \times 10^1$ m/s [60° above horizontal].

A:

$d_x = V_{ix} \cdot t_{air}$

S: $78 = (V_i \cos 60°)\, 2\left(\frac{-V_i \sin 60°}{a_y}\right)$

$78 = +V_i^2 \cdot \cos 60° \cdot \sin 60° \cdot 2$

$+9.8$

$\frac{764.4}{0.8660} = \frac{V_i^2 \cdot 0.8660}{0.866}$

$\rightarrow V_i = \sqrt{882.6789\ldots}$

$V_i = 3.0 \times 10^1$ m/s

Extra:

3. Find the initial velocity of an object that covered 9.23 m of horizontal distance after it was thrown with speed of 15 m/s. It landed at the same level it was launched from.

This Can be solve by graphing.

G: $d_x = 9.23$ m
$V_i = 15$ m/s

R: $\theta = ?$ [°]

A: $t_{air} = 2t_{hmax}$
$d_x = t_{air} \cdot V_i \cos\theta$

$d_x = 2\left(\frac{0 - V_i \sin\theta}{a_y}\right) \cdot V_i \cos\theta$

$d_x \cdot a_y = -2(V_i \sin\theta \cdot V_i \cos\theta)$

$\frac{(9.23)(-9.8)}{(-2)(15)(15)} = \sin\theta \cdot \cos\theta$

$0.2010 = \sin\theta \cdot \cos\theta \leftarrow$ Desmos

$\therefore \theta \doteq 12°$