

Example 1: An object is thrown from the ground with initial velocity of 80m/s [40° above horizontal]. Find its maximum height and its range.

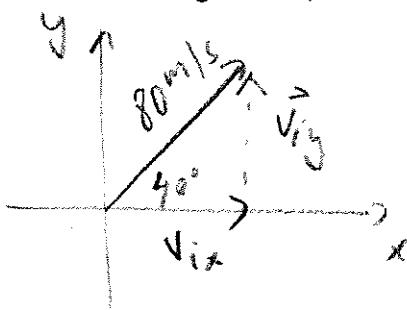
Given: $\vec{v}_i = 80 \text{ m/s} [40^\circ \text{ above horizontal}]$
 $a_y = g = -9.8 \text{ m/s}^2$



Diagram:



Vector diagram of v_i :



Vector components of v_i :

$$\begin{aligned}\vec{v}_{ix} &= 80 \cdot \cos 40^\circ \\ &= 61.2836 \text{ m/s} [\rightarrow] \\ \vec{v}_{iy} &= 80 \cdot \sin 40^\circ \\ &= 51.4230 \text{ m/s} [\uparrow]\end{aligned}$$

Time to reach maximum height:

$$t_{h_{\max}} = \frac{v_{iy} - v_{iy}}{g} = \frac{0 - 51.4230}{-9.8} = 5.24725 \approx 5.2 \text{ s}$$

Maximum height:

$$h_{\max} = \frac{v_{iy}^2 - v_{iy}^2}{2a_y} = \frac{0^2 - 51.4230^2}{2(-9.8)} = 134.914 \text{ m} \approx 1.3 \times 10^2 \text{ m}$$

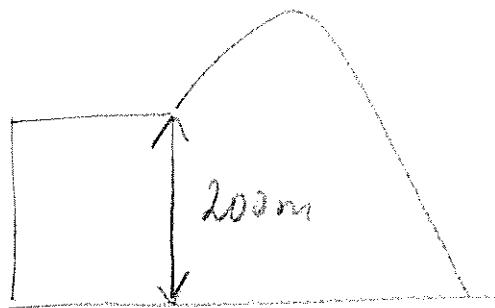
Time in the air:

$$t_{\text{air}} = 2t_{h_{\max}} = 2(5.2472) = 1.0 \times 10^1 \text{ s} \text{ OR } 10.5 \text{ s}$$

Range:

$$\begin{aligned}d_x &= v_{ix} \cdot t_{\text{air}} \\ &= (61.2836)(10.4945) \approx 6.4 \times 10^2 \text{ m}\end{aligned}$$

Example 2: In your notebook solve the above problem with a projectile launched 200 m above the ground level. Assume that the projectile lands on the ground. Compare your results with ex. 1.



→ lands below its launching level $\Rightarrow t_{ff} \uparrow \Rightarrow t_{air} \uparrow$

$$\begin{aligned}\vec{dy} &= (134.9145 + 200) \text{ m} & t_{air} &= t_{h_{max}} + t_{ff} \\ &= -334.9145 \text{ m} & &= 5.2472 + 8.2674 \\ & & &= 13.5146 \text{ s} \\ & & &= \underline{\underline{14 \text{ s}}}\end{aligned}$$

$$t_{ff} = \sqrt{\frac{2(-334.9145)}{-9.8}}$$

$$t_{ff} = \underline{\underline{8.2674 \text{ s}}}$$

$$\begin{aligned}\text{range: } d_x &= v_{ix} \cdot t_{air} \\ &= (61.2836)(13.5146) \\ &\doteq 828.2233 \text{ m} \\ &\doteq \underline{\underline{8.3 \times 10^2 \text{ m}}}\end{aligned}$$

∴ Comparison: the range increases from $6.4 \times 10^2 \text{ m}$ to $8.3 \times 10^2 \text{ m}$ because the time total increases due to longer free fall from 10.5 s to 13.5 s.