Notes :

PHYSICS 12

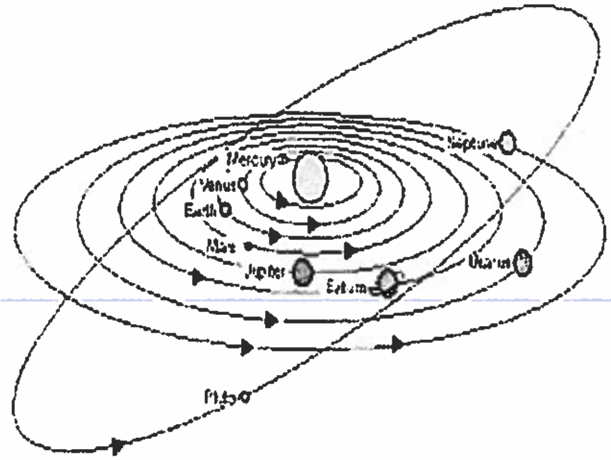
ORBITS

The object is specifically stated to be in orbit or it is located in space.

To find force, Newton's Law of Universal Gravitation must be used:

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$

$G = 6.67\times10^{-11}\ N{\cdot}m^2{\cdot}kg^{-2}$



$$\boxed{F_g = F_c}$$

- Be careful not to substitute object's altitude = surface-to-surface distance but always center-to-center

Example: Find the speed of Apollo 13 when it is in orbit about the Moon at an altitude of 3000km.

$m_M = 7.35\times10^{22}$ kg

$r_M = 1.74\times10^{6}$ m

! altitude = 3000 km → 3.0×10^{6} m

orbiting radius = r_M + altitude (⇒ neglecting the size of Apollo 13)

$$F_c = F_g$$

$$\cancel{m_A}\frac{v^2}{\cancel{r}} = \frac{G m_M \cancel{m_A}}{r^{\cancel{2}}}$$

$$v^2 = \frac{G m_M}{r}$$

$$v = \sqrt{\frac{G m_M}{r}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11})(7.35\times10^{22})}{[1.74\times10^{6} + 3.0\times10^{6}]}}$$

$v = 1016.99\ldots$ m/s

∴ Apollo's speed is 1.02×10^{3} m/s.

1. a) Find the altitude of a satellite orbiting the Earth at 5000 m/s. Mass of the Earth is 5.98x10^24 kg.

$v = 5000\ m/s$

$m_E = 5.98 \times 10^{24}\ kg$

$r_E = 6.38 \times 10^{6}\ m$

$\text{altitude} = r - r_E$

$$v = \sqrt{\frac{G m_E}{r}}$$

$$v^2 = \frac{G m_E}{r}$$

$$r = \frac{G m_E}{v^2}$$

$$r = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(5.0\times10^{3})^2}$$

$r = 15\,954\,640\ m$

$\text{altitude} = 15\,954\,640 - 6.38\times10^{6}$

$\doteq 9.6\times10^{6}\ m$

$\therefore$ Satellite's altitude is $9.6\times10^{6}\ m$.

b) Find the period of the probe.

Period $= T =$ time to complete 1 revolution

$$t = \frac{d}{v}$$

$$= \frac{2\pi r}{v}$$

$$= \frac{2\pi(15\,954\,640)}{5000}$$

$\doteq 20\,049\ s$

$\hookrightarrow = 5.6\ hrs$

$\therefore$ The probe completes one revolution around the Earth in 5.6 hours (or $2.0\times10^{4}\ s$).

POTENTIAL ENERGY ASSOCIATED WITH GRAVITY

(for objects high above the Earth's surface)

GPE = gravitational potential energy

- This energy is found by evaluating work required to move an object from Earth's surface to infinity. This work is done by applied force that counters gravity.

$$\boxed{GPE = \frac{-GmM}{r}}$$

- GPE = 0 in infinity as objects separated by infinitely large radius do not interact.

Recall: Total energy of an object is given by the sum of the potential and kinetic energy of the object.

- For an object to escape Earth, the total energy has to be zero.
- Total energy of an orbiting satellite is always negative as the satellite is not free to move away.

$$E_{TOT} = KE + GPE$$

Derive the formula for escape velocity: $\Rightarrow E_{TOT} = 0\,J$

$$KE + GPE = 0$$

$$KE = GPE$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{Gm_E\cancel{m_2}}{r}$$

$$\Rightarrow v^2 = \frac{2Gm_E}{r}$$

$$v = \sqrt{\frac{2Gm_E}{r}}$$

Escape Velocity

$$\boxed{v = \sqrt{\frac{2Gm_E}{r}}}$$

2. Read textbook p. 144-145 Chapter 5.5

3. a) What is the speed a satellite needs to maintain an orbit with a fixed radius?

$$F_g = F_c$$

$$\frac{G m_E \cancel{m_2}}{r^{\cancel{2}}} = \cancel{m_2}\frac{v^2}{\cancel{r}}$$

$$\frac{G m_E}{r} = v^2$$

$$\rightarrow v = \sqrt{\frac{G m_E}{r}}$$

b) How does the magnitude of this speed depend on the mass of the satellite?

- v_{orb} is mass independent.
- v_{orb} is dependent on the mass of the object that creates the gravitational field and the orbiting radius.

4. What is a geosynchronous satellite?

A satellite whose period is the same time needed for Earth to complete one revolution about its axis of rotation. This type of a satellite is always above the same spot on Earth.

5. How is time needed for one revolution about the orbit related to the speed of a satellite?

$$v = \frac{d}{t} = \frac{2\pi r}{T} \Rightarrow \text{as } T\uparrow \quad v\downarrow$$

$\therefore$ Speed and period are inversely proportional.

6. Calculate the magnitude of escape velocity of the Earth.

$m = 5.98 \times 10^{24}$ kg

$$v_{esc} = \sqrt{\frac{2Gm_E}{r}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^{6}}} = 1.1 \times 10^{4} \text{ m/s}$$

7. If you wanted to calculate escape speed from another planet, what variables would change in the formula?

- the mass of the planet and its radius.