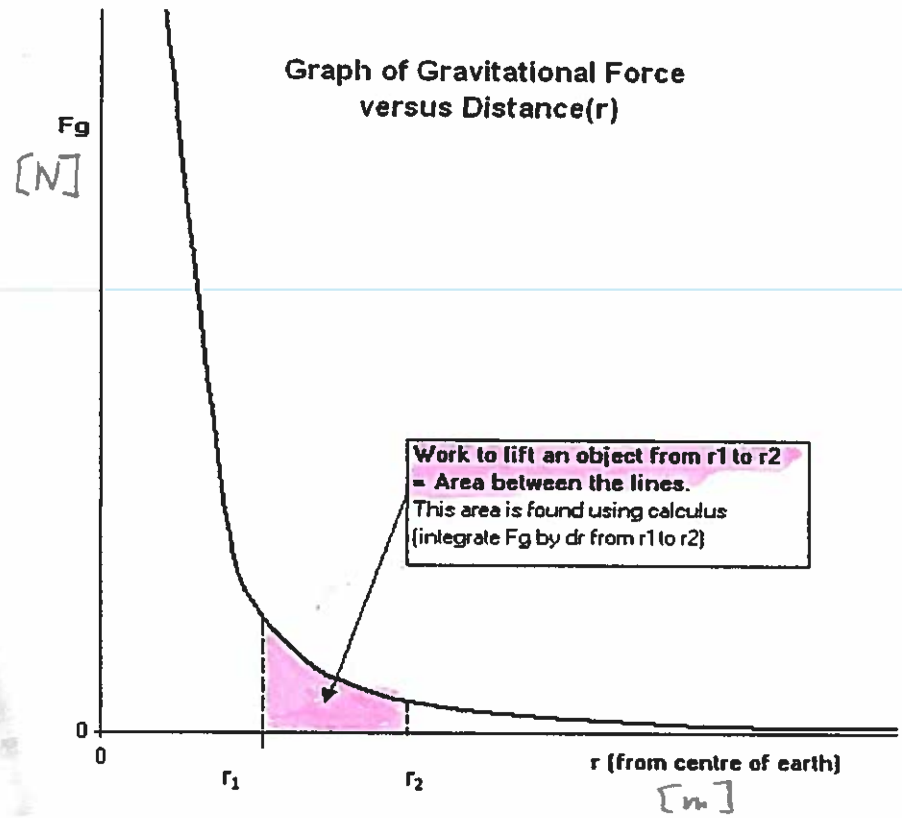
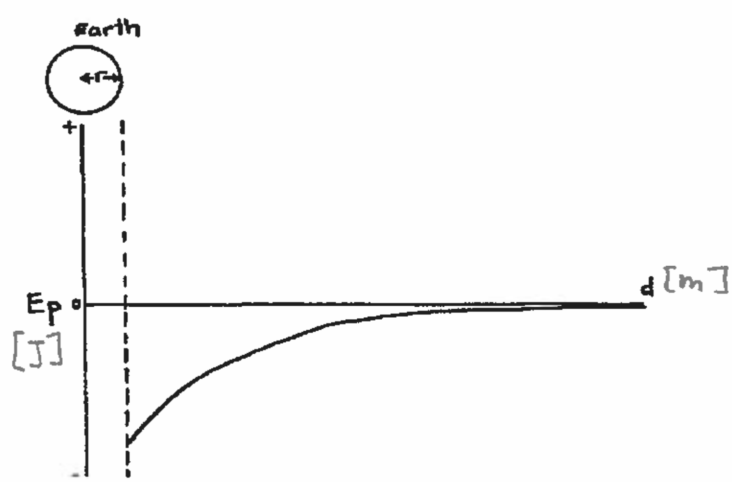
Notes

PHYSICS 12

Gravitational Potential Energy not on Earth



- A graph of the E_p =*GPE* surrounding a planet:



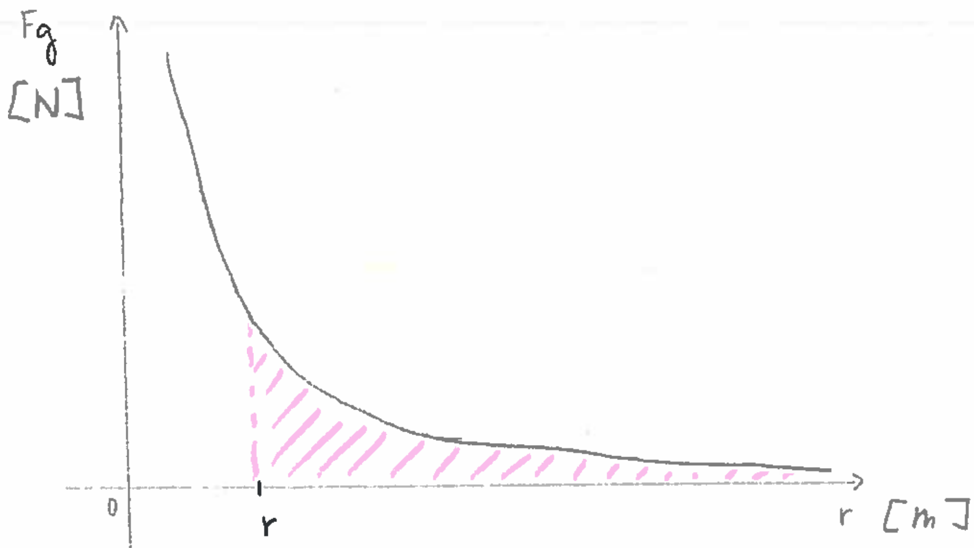
> ➢ When lifting an object against a gravitational field, e.g. launching a rocket, work is done on the object, that is, energy is transferred to the object. The object's gravitational potential energy, E_p =GPE, that is, the energy it has due to its position within the gravitational field, increases as a result.

> ➢ When an object moves toward the source of the gravitational field, such as when dropping a stone, energy due to position in a field is transformed into kinetic energy (the stone speeds up).

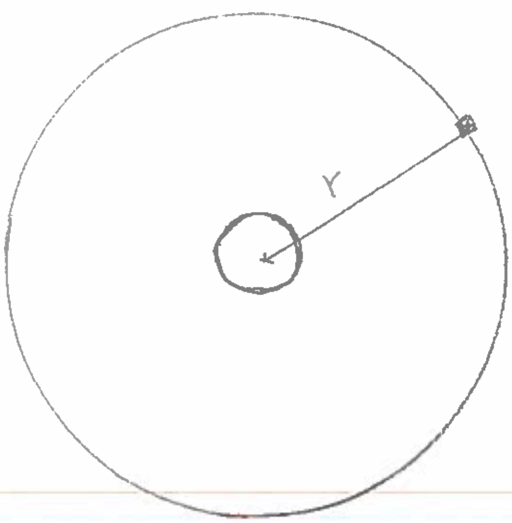
- Hence the position of lowest E_p=GPE in the gravitational field surrounding a planet is at the surface of the planet.

- An object only has zero E_p= GPE when it is no longer within the gravitational field, that is, a very large distance away. (Mathematically, distance must be infinite.) We say: **gravitational potential energy relative to zero at infinity.**

Example 1. Which of the indicated areas of the graph represent the work needed to send an object from separation distance r to infinity?



Example2: State the expression for the total energy of the orbiting satellite shown below?



$$E_{TOT} = KE + GPE$$

$$= \frac{1}{2} m_s v^2 + \frac{-G m_E \cdot m_s}{r}$$

$$\therefore E_{TOT} = \frac{1}{2} m_s v^2 - \frac{G m_E m_s}{r}$$

! orbital radius = r_E + altitude = center-to-center

Example 3. A 2.0×10^3 kg satellite is in a circular orbit around the earth. The satellite has a speed of 3.6×10^3 m/s at an orbital radius of 3.1×10^7 m. What is the total energy of this orbiting satellite?

G: $v = 3.6 \times 10^3 \, m/s$
$r = 3.1 \times 10^7 \, m$
$m_E = 5.98 \times 10^{24} \, kg$
$m_S = 2.0 \times 10^3 \, kg$

R: $E_{TOT} = ? \, [J]$

A: $E_{TOT} = KE + GPE$

$$= \frac{1}{2} m v_c^2 - \frac{G m_E m_S}{r}$$

S: $E_{TOT} = \frac{1}{2}(2000)(3.6\times10^3)^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(2000)}{3.1\times10^7}$

$= 1.296\times10^{10} - 2.5733\times10^{10}$

$\doteq -1.2773\times10^{3}$ J

∴ Satellite's total energy is 1.3×10^{10} J.

Example4. A 5.2×10^4 kg rocket is initially at rest on the surface of the earth. If 3.0×10^{11} J of work is done on this rocket, what maximum altitude h will the rocket reach? (Assume the rocket's mass does not change.)

$KE = 0$ J

G: $m_R = 5.2\times10^4 \, kg$
$v_i = 0 \, m/s \Rightarrow KE_i = 0 J$
$v_f = 0 \, m/s$ (at max altitude)
$\Rightarrow KE_f = 0 J$
$r_E = 6.38\times10^6 \, m$
$W_{in} = 3.0\times10^{11} J$
$m_E = 5.98\times10^{24} \, kg$

R: $h = ? \, [m]$

A: By the law of Conservation of energy:

$KE_i + GPE_i + W_{in} = KE_f + GPE_f + W_{out}$ ← assume 0 J as no work is mentioned.

$$-\frac{G m_E m_R}{r_E} + W_{in} = \frac{-G m_E m_R}{(r_E + h)} \rightarrow \frac{-G m_E m_R + W_{in} r_E}{r_E} = \frac{-G m_E m_R}{(r_E + h)}$$

S: $r_E + h = \frac{(-6.67\times10^{-11})(5.98\times10^{24})(5.2\times10^4)(6.38\times10^6)}{[(-6.67\times10^{-11})(5.98\times10^{24})(5.2\times10^4) + (3.0\times10^{11})(6.38\times10^6)]}$

$h = 7.028605\times10^6 - 6.38\times10^6$

$h \doteq 6.5\times10^5 \, m$

S: Maximum altitude is 6.5×10^5 m

Example 5. How much work is required to move a 560-kg satellite from altitude $3.0\text{x}10^{6}$ m above Earth to $4.0\text{x}10^{7}$m above Earth?

G: $m_s = 560\ kg$

$m_E = 5.98 \times 10^{24}\ kg$

$r_i = 6.38 \times 10^{6} + 3.0 \times 10^{6}\ m = 9.38 \times 10^{6}\ m$

$r_f = 6.38 \times 10^{6} + 4.0 \times 10^{7}\ m = 4.64 \times 10^{7}\ m$

R: $W = ?\ [J]$

A: $W = \Delta\ GPE$ (work-energy theorem)

$$W = GPE_f - GPE_i$$

$$W = \frac{-Gm_Em_s}{r_f} - \frac{-Gm_Em_s}{r_i}$$

$$W = -Gm_Em_s\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

S: $$W = -(6.67\times10^{-11})(5.98\times10^{24})(560)\left[\frac{1}{4.64\times10^{7}} - \frac{1}{9.38\times10^{6}}\right]$$

$$W = 1.89969\ldots \times 10^{10}\ J$$

S: The work done is 1.9×10^{9} J.