Answers:

FORCES AND OBJECTS ON INCLINED PLANES

1. Consider a 10-kg object on 3 different inclined planes (3°, 38° and 70°).
   a) Find $\overrightarrow{F_g}$, $\overrightarrow{F_{g//}}$, $\overrightarrow{F_N}$, $\overrightarrow{F_f}$ *and* $\overrightarrow{F_{net}}$ for each plane provided that the coefficient of friction between the surfaces of contact is 0.20
   b) Decide whether the object is moving or at rest. If it is at rest find the magnitude and direction of an applied force that is needed to accelerate the object. If you find that the object is not at rest, find its acceleration.

2. Consider an object of 50 kg that is placed on a 15°inclined plane.
   a) Is the object stationary if the surfaces of contact are frictionless? If not, find the object's acceleration.
   b) If μ is 0.3 find the net force and decide whether the object accelerates. If it does not accelerate state what is required for the object to accelerate.

3. Consider a 20-kg object that is pushed with 100 N force at 20° angle down an inclined plane that is elevated 30° above horizontal. The coefficient of friction is 0.15. Find the magnitude of $\overrightarrow{F_g}$, $\overrightarrow{F_{g//}}$, $\overrightarrow{F_N}$, $\overrightarrow{F_f}$, $\overrightarrow{F_{net}}$ and $\vec{a}$.

Forces and Objects on Inclined Planes

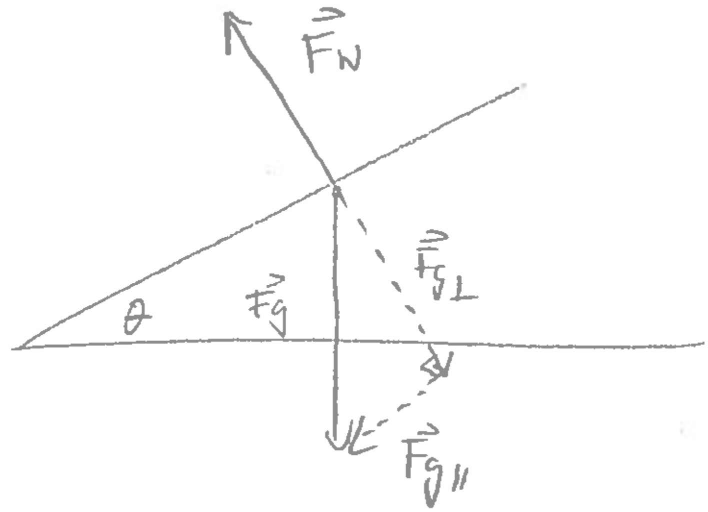
1. a) $m = 10\,kg$

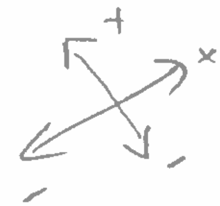
$\theta_1 = 30°$

$\theta_2 = 38°$

$\theta_2 = 70°$

$\mu = 0.20$





θ_1

$Fg = mg$

$= 98N$

$Fg_\perp = Fg\cos\theta$

$= -97.8657N$

$Fg_\parallel = Fg\sin\theta$

$= -5.1289N$

$\vec{F_N} = 97.8657N$

$\vec{F_f} \leq F_N \cdot \mu$

$\leq 19.5731N$

$\vec{F}_{net} = \underbrace{\vec{Fg_\perp} + \vec{F_N}}_{0} + \vec{Fg_\parallel} + \vec{F_f}$

$= -5.1289 + 5.1289$

$= 0N$

θ_2

$\vec{Fg_\perp} = (98.0)(\cos 38°)$

$= -77.2251\,N$

$Fg_\parallel = -60.3348N$

$\vec{F_N} = 77.2251N$

$\vec{F_f} \leq 15.4450N$

$\vec{F}_{net} = \vec{F_f} + \vec{Fg_\parallel} + \vec{F_N} + \vec{Fg_\perp}$

$= -60.3348 + 15.4450$

$= -44.8898N$

θ_3

$\vec{Fg_\perp} = Fg\cos 70°$

$= -33.5180N$

$\vec{Fg_\parallel} = -92.0899N$

$\vec{F_N} = 33.5180N$

$\vec{F_f} \leq 6.7031\,N$

$\vec{F}_{net} = -85.3868N$

#1 b)

θ_1 = Object is at rest $\qquad \|\vec{F}_f\| \geq \|\vec{F}_{g\parallel}\|$

→ to accelerate the object up the plane:

$$\vec{F}_a \geq 5.1289 + 19.5731$$
$$\geq 24.702$$
$$\geq 25\,N$$

∴ $\vec{F}_a$ is 25N [up the plane].

→ to accelerate down the plane:

$$\vec{F}_a \geq 19.5731 - 5.1289$$
$$\geq 14.4442\,N$$

∴ $\vec{F}_a$ is 14N [down the plane].

θ_2 :

$\| \vec{F}_f \| < \| \vec{F}_{g_{\parallel}} \|$ → object is in motion

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

$$= \frac{-44.8898}{10}$$

$$= \underline{4.5 \ m/s^2} \ [\text{down the plane}]$$

θ_3 :

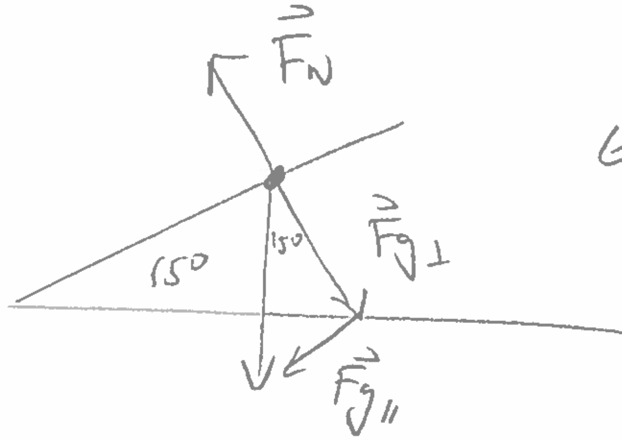
$\| \vec{F}_f \| < \| \vec{F}_{g_{\parallel}} \|$ → object in motion

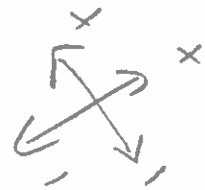
$$\vec{a} = \frac{-85,3868}{10}$$

$$\underline{\vec{a} = 8.5 \ m/s^2} \ [\text{down the plane}].$$

#2 $m = 50\ kg$

$\theta = 15°$





a) $F_g = (50)(9.8)$

$= 490 N$

$\vec{F}_{g\perp} = -490 \cos 15°$

$= \underline{-473.3037 N}$

$\vec{F}_{g\parallel} = -490 \sin 15°$

$= \underline{-126.8213 N}$

$\vec{F}_N = 473.3037 N$

$\vec{F}_{net} = \vec{F}_{g\perp} + \vec{F}_N + \vec{F}_{g\parallel}$

$= \vec{F}_{g\parallel}$

$= \underline{-126.8213 N}$ ⇒ object accelerates down the plane

$\vec{a} = \frac{F_{net}}{m} = \frac{-126.8213}{50.0} = 2.5\ m/s^2$ [down the plane].

#2 b) $\mu = 0.30$

$\vec{F}_f \leq F_N \cdot \mu$

$\leq (473.3037)(0.30)$

$\leq 141.9911\ N$

$\|\vec{F}_f\| > \|\vec{F}_{g_{\parallel}}\| \Rightarrow$ object is at rest

→ to accelerate down the plane:

$\vec{F}_{net} = \vec{F}_{g_{\parallel}} + \vec{F}_f + \underbrace{\vec{F}_N + \vec{F}_{g_{\perp}}}_{0} + \vec{F}_a$

$F_a \geq -126.8213 + 141.9911$

$\geq 15.1698\ N$

$\therefore \vec{F}_a \geq 15N$ [down the plane]

→ to accelerate up the plane:

$F_a \geq F_{g_{\parallel}} + F_f$

$\geq 126.8213 + 141.9911$

$\geq 268.8124\ N$

$\therefore \vec{F}_a \geq 2.7 \times 10^2\ N$ [up the plane]

#3

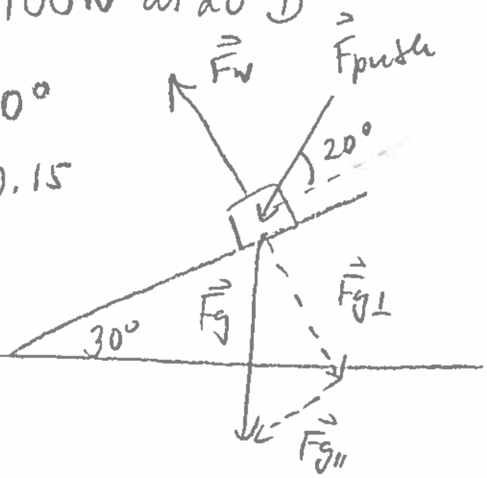
G: $m = 20\,kg$

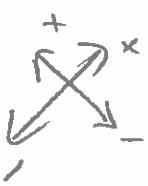
$\vec{F}_{push} = 100\,N$ at $20°$ D

$\theta = 30°$

$\mu = 0.15$

A:





$$\vec{F}_g = -mg$$
$$= -(20)(9.8)$$
$$= \underline{-196\,N}$$

$$\vec{F}_{g\parallel} = -196 \sin 30°$$
$$= \underline{-98\,N}$$

$$\vec{F}_{g\perp} = -196 \cos 30°$$
$$= \underline{-169.7410\,N}$$

$$\vec{F}_{push\perp} = -100 \cdot \sin 20°$$
$$= \underline{-34.2020\,N}$$

$$\vec{F}_{push\parallel} = -100 \cdot \cos 20°$$
$$= \underline{-93.9693\,N}$$

$$\vec{F}_N = \vec{F}_{g\perp} + \vec{F}_{push\perp}$$
$$= 169.7410 + 34.2020$$
$$= \underline{203.943\,N}$$

$$\vec{F}_f = +F_N \cdot \mu$$
$$= (203.943)(0.15)$$
$$= \underline{30.5915\,N}$$

$$\vec{F}_{net} = \underbrace{[-98, -169.7410]}_{\vec{F}_g} + \underbrace{[-93.9693, -34.2020]}_{\vec{F}_{push}} + \underbrace{[0, 203.943]}_{\vec{F}_N} + \underbrace{[30.5915, 0]}_{\vec{F}_f}$$

$[\parallel, \perp]$

$$= [-161.3778, 0]\,N$$

$$\vec{a} = \frac{[-161.3778, 0]}{20}$$

$$\vec{a} = -8.1\ m/s^2$$

$\therefore\ \vec{a} = 8.1\ m/s^2$ [down the plane].