FF - vacuum
- only F_g
- no obstacles
- $g = 9.8\ m/s^2$ [D]

Notes:

PHYSICS 12 **FREEE FALL AND PROJECTILE MOTION**

Quantity	Free Fall	Projectile Motion
Initial velocity	zero	non zero
Acceleration	vertical due to $\vec{F_g}$	vertical due to $\vec{F_g}$
Horizontal motion	none	constant
Vertical motion	accelerated = changing	accelerated = changing
Velocity at maximum height	zero = $\vec{v_i}$	zero only in y-direction
Propulsion	none	none
Air resistance	none	none

Kinematics equations

- $t_{hmax} = \dfrac{v_{fy} - v_{iy}}{a_y}$
- $h_{max} = \dfrac{v_{fy}^2 - v_{iy}^2}{2a_y}$
- $d_x = v_{ix} \cdot t_{air}$
- $t_{air} = t_{hmax} + t_{ff}$ OR $t_{air} = 2t_{hmax}$
- $t_{ff} = \sqrt{\dfrac{2d_y}{a}}$
- $\vec{v_f} = -\sqrt{v_{iy}^2 + 2a_y d_y}$

$\vec{v}_{ix} = \|\vec{v_i}\| \cdot \cos\theta$

$\vec{v}_{iy} = \|\vec{v_i}\| \sin\theta$

- Slope on a position-time graph is velocity
- Slope on a velocity-time graph is acceleration
- Area below the curve of the velocity-time graph is displacement

1. a)Consider an object A thrown vertically with velocity 40m/s [up] and an object B that is thrown 1.03 seconds after object A. At what velocity [up] must be the object B thrown in order to reach its maximum height at the same instant as the object A?

Object A: $t_{hmax} = \frac{v_{fy} - v_{iy}}{a_y} = \frac{0 - 40}{-9.8} = 4.0816\,s$

Object B: $t_{hmax} = 4.0816 - 1.03 = 3.0516\,s$

this is zero as the object stops at h max

$v_f = v_i + at \Rightarrow \vec{v}_{iy} = \boxed{\vec{v}_{fy}} - at$

$= 0 - (-9.8)(3.0516)$

$\doteq 29.91\ m/s$ $\quad \therefore \vec{v}_{iyB} = 3.0 \times 10^1\ m/s\,[up]$

b) How does the maximum height of object A compare to the maximum height of object B?

object A: $h_{max} = \frac{v_{fy}^2 - v_{iy}^2}{2a_y} = \frac{0^2 - 40^2}{2(-9.8)} \doteq 81.6\,m$

object B: $h_{max} = \frac{0^2 - 29.91^2}{2(-9.8)} = 45.6\,m$

$\frac{h_{max}A}{h_{max}B} = \frac{81.6}{45.6} \doteq 1.8$ $\quad \therefore$ The maximum height of object A is 1.8 times greater than hmax of object B.

c) How would your answer from a) change if the object B was thrown at an angle above the horizontal?

→ $\vec{v}_i$ can be at any angle θ as long as $\vec{v}_{iy} = 29.91\ m/s$ the overall answer will not change.

→ $\vec{v}_{ixB}$ does not affect the answer.

2. Consider two objects that are thrown with the same speed of 30m/s. If one object is thrown vertically upwards at what velocity must be the other object thrown, so both objects reach their maximum height at the same instant even though they were thrown 8.5×10^{-2} seconds apart?

object A: $\vec{v}_i = 30 \text{ m/s } [U]$

object B: $\vec{v}_i = 30 \text{ m/s } \theta°$ above horizontal

(A)

$$t_{hmax} = \frac{v_{fy} - v_{iy}}{a_y} = \frac{0 - 30}{-9.8} = 3.06 \text{ s}$$

(B)

$$t^{(1)}_{hmax} = 3.06 - 0.085 = 2.975 \text{ s}$$

OR

$$t^{(2)}_{hmax} = 3.06 + 0.085 = 3.145 \text{ s}$$

(1) $v_{iy} = v_{fy} - a_y t$

$= 0 - (-9.8)(2.975)$

$= 29.155$ m/s

$v_{ix} = \sqrt{30^2 - 29.155^2}$

$\doteq 7.1$ m/s

$\theta = \tan^{-1}\left(\frac{29.155}{7.070}\right)$

$\theta \doteq 76°$

(2) $v_{iy} = v_{fy} - a_y t$

$= 0 - (-9.8)(3.145)$

$= 30.821$ m/s

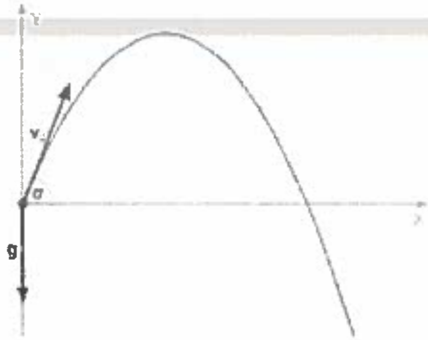
$\vec{v}_{ix} = \sqrt{30^2 - 30.82^2}$

$= \sqrt{-49...}$

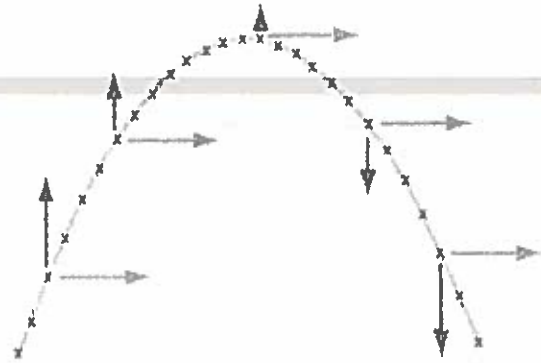
$\Rightarrow$ impossible ($v_{iy} > \|\vec{v}_i\| \Rightarrow$ impossible scenario)

$\therefore$ The second object is thrown at 30 m/s 76° above horizontal.

Answer the following questions: p 47 #13-16 and p 52 #1,3, 6, 10, 13,19,22,24 and p53 #13 **Support your answers.**



Projectile Motion:



1. List all given information. **Recall: acceleration is only in the vertical direction. Horizontal motion is at constant velocity.**

2. Sketch a labelled diagram that shows the path of the projectile.

3. Sketch a separate diagram of the initial velocity vector. Label the horizontal and vertical vector components of the initial velocity vector. Label the given angle.

4. Find the horizontal and vertical component of the initial velocity.

5. When looking for maximum height and time needed the maximum height use the vertical component of the initial velocity.

6. When looking for the range use the horizontal component of the initial velocity and total time in the air.

7. Total time in the air is twice the time needed to reach the maximum height if and only if the projectile lands at the same level it was launched from.

8. Total time in the air for any other scenario is **time needed to reach the maximum height + time to free fall from the maximum height to the landing level.**

9. Practice and ask questions.

10. In the space below list the formulae you frequently use:

- $h_{max} = \frac{v_{fy}^2 - v_{iy}^2}{2g}$
- $t_{air} = \begin{cases} 2 \cdot t_{hmax} \\ t_{hmax} + t_{ff} \end{cases}$
- $t_{hmax} = \frac{v_{fy} - v_{iy}}{g}$
- $t_{ff} = \sqrt{\frac{2d_y}{g}}$

$$\boxed{\begin{aligned} \vec{v}_{ix} &= \|\vec{v}_i\| \cdot \cos\theta \\ \vec{v}_{iy} &= \|\vec{v}_i\| \cdot \sin\theta \end{aligned}}$$

- $v_{fy} = -\sqrt{v_{iy}^2 - 2gd_y}$
- $d_x = v_{ix} \cdot t_{air}$

Example1: An object is thrown from the ground with initial velocity of 80m/s [40° above horizontal]. Find its maximum height and its range.

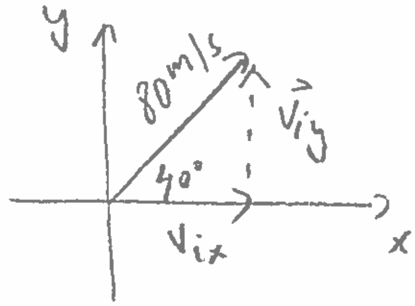
Given: $\vec{v}_i = 80\ m/s\ [40°\ \text{above horizontal}]$

$a_y = g = -9.8\ m/s^2$

$\uparrow +$ $\downarrow -$

Diagram:

Vector diagram of v_i:



Vector components of v_i:

$\vec{v}_{ix} = 80 \cdot \cos 40°$
$= 61.2836\ m/s\ [\rightarrow]$

$\vec{v}_{iy} = 80 \cdot \sin 40°$
$= 51.4230\ m/s\ [\uparrow]$

Time to reach maximum height:

$$t_{h_{max}} = \frac{v_{fy} - v_{iy}}{g} = \frac{0 - 51.4230}{-9.8} = 5.2472s \doteq \underline{\underline{5.2\ s}}$$

Maximum height:

$$h_{max} = \frac{v_{fy}^2 - v_{iy}^2}{2a_y} = \frac{0^2 - 51.4230^2}{2(-9.8)} = 134.914\ldots m \doteq \underline{1.3 \times 10^2\ m}$$

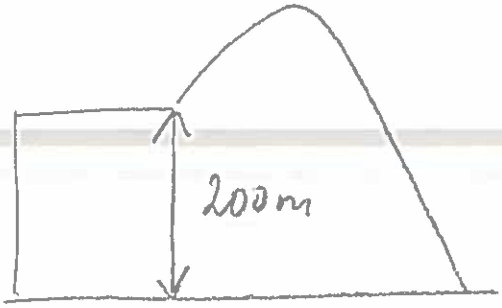
Time in the air:

$$t_{air} = 2t_{h_{max}} = 2(5.2472\ldots) = \underline{1.0 \times 10^1\ s} \quad \text{OR } 10.5\ s$$

Range:

$$d_x = v_{ix} \cdot t_{air}$$
$$= (61.2836)(10.4945) \doteq \underline{6.4 \times 10^2\ m}$$

Example 2: In your notebook solve the above problem with a projectile launched 200 m above the ground level. Assume that the projectile lands on the ground. Compare your results with ex. 1.



⇒ lands below it launching level ⇒ $t_{ff} \uparrow$ ⇒ $t_{air} \Uparrow$

$\vec{d_y} = (134.9145 + 200)\,m$
$= \underline{-334.9145\,m}$

$t_{ff} = \sqrt{\dfrac{2(-334.9145)}{-9.8}}$

$\underline{t_{ff} \doteq 8.2674\,s}$

$t_{air} = t_{h\,max} + t_{ff}$
$= 5.2472 + 8.2674$
$= 13.5146\,s$
$\doteq \underline{14\,s}$

range: $d_x = v_{ix} \cdot t_{air}$
$= (61.2836)(13.5146)$
$\doteq 828.2233\,m$
$\doteq \underline{8.3 \times 10^2\,m}$

∴ Comparison: the range increases from $6.4 \times 10^2\,m$ to $8.3 \times 10^2\,m$ because the time total increases due to longer freefall from 10.5 s to 13.5 s.