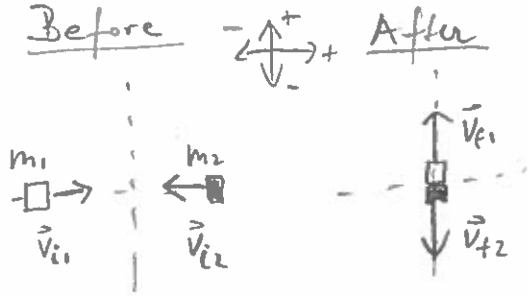
$\rightarrow$ assume a closed isolated system $\Leftrightarrow \sum \vec{p}_i = \sum \vec{p}_f$

PHYSICS 12

Collisions

1. Two balls of equal masses move with the same speed toward each other on the x-axis. When they collide, each ball deflects 90° from its original path, such that both balls are moving away from each other on the y-axis. What can be said about the final velocity of each ball?



$\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + \vec{p}_{f2}$

$[mv_{i1}, 0] + [-mv_{i2}, 0] = [0, mv_{f1}] + [0, -mv_{f2}]$

$[mv_i - mv_i, 0] = [0, mv_{f1} - mv_{f2}]$

- $\|\vec{v}_{i1}\| = \|\vec{v}_{i2}\| = v_i$
- $m_1 = m_2 = m$

$\sum \vec{p}_i = \sum \vec{p}_f$

$\frac{0}{m} = \frac{mv_{f1} - mv_{f2}}{m}$

$0 = v_{f1} - v_{f2}$

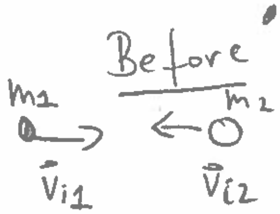
$\boxed{v_{f2} = v_{f1}}$

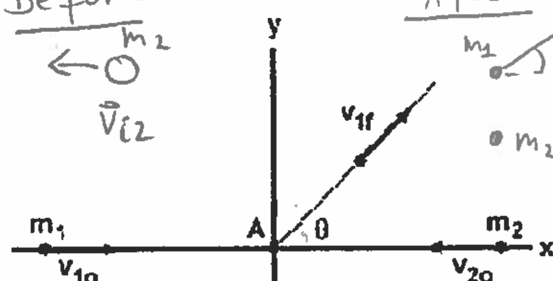
$\therefore$ Both objects move at the same speed but opposite direction.

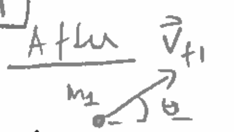
2. Two pool balls traveling in opposite directions collide. One ball travels off at an angle θ to its original velocity, as shown below. Is there any possible way for the second ball to be completely stopped by this collision? If so state the conditions under which this could occur.

$\Rightarrow \vec{p}_{f2} = [0, 0]$ kg·m/s

- $\boxed{m_1 = m_2 = m}$







$\sum \vec{p}_i = \sum \vec{p}_f$

$\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + [0, 0]$

$[mv_{i1}, 0] + [-mv_{i2}, 0] = \vec{p}_{f1} + [0, 0]$

$[mv_{i1} - mv_{i2}, 0] = [+mv_{f1}\cos\theta, +mv_{f1}\sin\theta]$

$\cancel{m}[v_{i1} - v_{i2}, 0] = \cancel{m}[v_{f1}\cos\theta, v_{f1}\sin\theta]$

$\boxed{0 = v_{f1}\sin\theta}$ (*)

(*) this equation holds only if $v_{f1} = 0$ m/s and/or if $\theta = n\pi, n \in \mathbb{Z}$. But $v_{f1} \neq 0$ m/s

Two particles collide at point A, and one moves off at an angle. Is there any way the other particle can remain at A after the collision?

and θ is given $0 < \theta < \frac{\pi}{2}$.

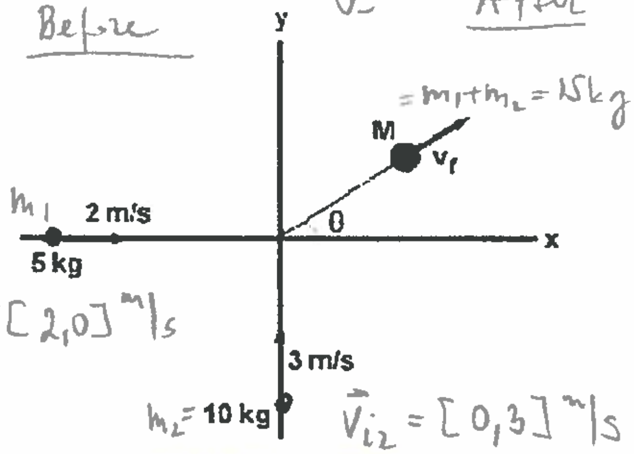
$\Rightarrow$ If m_1 moves at velocity $\vec{v}_{f1}$ [R θ° U] the other object has to move with some velocity $\vec{v}_{f2}$ that has a vertical component equal to $\|v_{f1} \cdot \sin\theta\|$. $\therefore$ m_2 can't stop at point A.

• Assume a closed isolated system.

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

3. Two objects are traveling perpendicular to each other, one moving at 2 m/s with a mass of 5 kg, and one moving at 3 m/s with a mass of 10 kg, as shown below. They collide and stick together. What is the magnitude and direction of the velocity of both objects?

Before — After



$\vec{V}_{i1} = [2, 0]$ m/s

$\vec{V}_{i2} = [0, 3]$ m/s

• $\sum \vec{p}_i = \sum \vec{p}_f$

$\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_f$

$[(5)(2), 0] + [0, (10)(3)] = [15v_f \cdot \cos\theta, 15v_f \sin\theta]$

$\frac{[10, 30]}{15} = \frac{15[v_f \cos\theta, v_f \sin\theta]}{15}$

$\left[\frac{10}{15}, \frac{30}{15}\right] = [v_f \cos\theta, v_f \sin\theta]$ ⊛

Two objects in an inelastic collision = an inelastic collision $\boxed{\sum KE_i \neq \sum KE_f}$

⊛ $\left[\frac{2}{3}, 2\right] = [v_f \cos\theta, v_f \sin\theta]$

$\frac{2}{3} = v_f \cos\theta$ — $2 = v_f \sin\theta$

$\cos\theta = \frac{2}{3v_f}$ — $\sin\theta = \frac{2}{v_f}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{v_f}}{\frac{2}{3v_f}} = \frac{\cancel{2}}{\cancel{v_f}} \times \frac{3\cancel{v_f}}{\cancel{2}} = 3$

$\rightarrow \theta = \tan^{-1}(3)$

$\doteq 71.5651°$

$\doteq \underline{\underline{72°}}$

$v_f = \frac{2}{\sin\theta}$

$v_f = \frac{2}{\sin 71.5651}$

$\underline{v_f = 2.1 \text{ m/s}}$

∴ The objects move at 2.1 m/s [R 72° U].

Assume a closed isolated system.

4. A common pool shot involves hitting a ball into a pocket from an angle. Shown below, the cue ball hits a stationary ball at an angle of 45°, such that it goes into the corner pocket with a speed of 2 m/s. Both balls have a mass of 0.5 kg, and the cue ball is traveling at 4 m/s before the collision.

a) Calculate the angle with which the cue is deflected by the collision.

b) Calculate the final velocity of the cue ball.

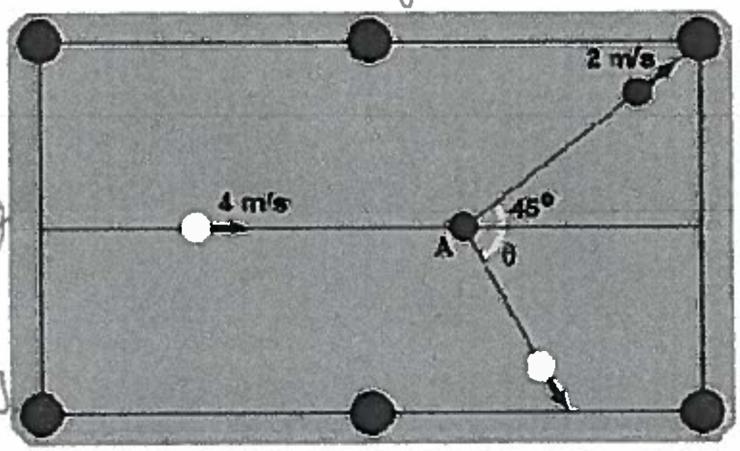
Before:

$m_1 = 0.5 kg$

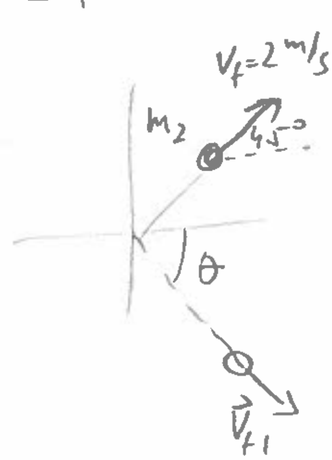
$\vec{v}_{i1} = [4,0]$ m/s

$m_2 = 0.5 kg$

$\vec{v}_{i2} = [0,0]$ m/s



After



The cue (white ball) collides with a stationary ball (black ball) at point A, sending the black ball into the corner pocket at an angle of 45°.

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + \vec{p}_{f2}$$

$$[(0.5)(4.0), 0] + [0,0] = [(0.5)(2)\cos 45°, (0.5)(2)\sin 45°] + [p_{f1x}, -p_{f1y}]$$

$$[2,0] = [0.7071, 0.7071] + [(0.5)v_{f1}\cdot\cos\theta, -(0.5)v_{f1}\sin\theta]$$

$$[2,0] - [0.7071, 0.7071] = 0.5[v_{f1}\cos\theta, -v_{f1}\sin\theta]$$

$$\frac{[1.2929, -0.7071]}{0.5} = [v_{f1}\cos\theta, -v_{f1}\sin\theta]$$

$$[2.5858, -1.4142] = [v_{f1}\cos\theta, -v_{f1}\sin\theta]$$

$$\frac{2.5858}{v_{f1}} = \cos\theta \quad \text{AND} \quad \frac{+1.4142}{+v_{f1}} = \sin\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1.4142}{2.5858} = 0.5469... \rightarrow \theta = \tan^{-1}(0.5469...)$$

$$\theta \doteq 28.6747° = \underline{29°}$$

$$v_{f1} = \frac{1.4142}{\sin 28.6747°}$$

$$v_{f1} \doteq 2.9 \text{ m/s}$$

$\therefore$ The cue ball moves at 2.9 m/s [R29°D].

$v_i = 0\,m/s \Rightarrow p = 0\,kg\,m/s$

Explosions (COLLISIONS IN REVERSE)

5. Two pop cans are at rest on a stand. A firecracker is placed between the cans and lit. The firecracker explodes and exerts equal and opposite forces on the two cans. Assuming the system of two cans to be isolated, the post-explosion momentum of the system _____.

a. is dependent upon the mass and velocities of the two cans

b. is dependent upon the velocities of the two cans (but not their mass)

c. is typically a very large value

d. can be a positive, negative or zero value

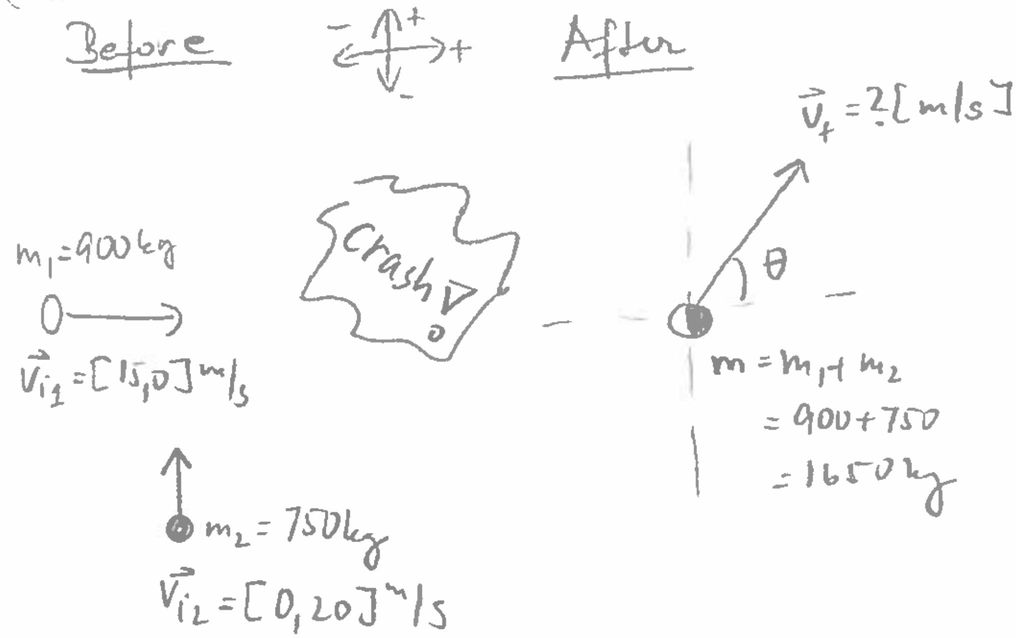
e. is definitely zero

$$\sum \vec{p_i} = \sum \vec{p_f}$$
$$[0,0] = [0,0]$$

Assume a closed isolated system.

Practice Problems:

1. A 900-kg car traveling east at 15 m/s collides with a 750-kg car traveling north at 20 m/s. The cars stick together. With what velocity does the wreckage move just after the collision?



$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$[(900)(15), 0] + [0, (750)(20)] = [1650\, v_f \cos\theta,\ 1650\, v_f \sin\theta]$$

$$\frac{[13500,\ 15000]}{1650} = \frac{1650[v_f \cos\theta,\ v_f \sin\theta]}{1650}$$

$$[8.1818,\ 9.0909] = [v_f \cos\theta,\ v_f \sin\theta]$$

$$8.1818 = v_f \cos\theta \qquad 9.0909 = v_f \sin\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{9.0909}{8.1818} \doteq 1.1111 \rightarrow \theta = \tan^{-1}(1.1111) \doteq \underline{48^\circ}$$

$$v_f = \frac{9.0909}{\sin 48^\circ}$$

$$v_f = 12.2330 \text{ m/s}$$

$\therefore$ The wrackage moves at 12 m/s [R 48° U].

Assume a closed isolated system. $\Sigma KE_i = \Sigma KE_f$.

2. A hockey puck moving at 0.45 m/s collides elastically with another puck that was at rest. The pucks have equal mass. The first puck is deflected 37 degrees to the right and moves off at 0.36 m/s. Find the speed and direction of the second puck after the collision.

Before — After

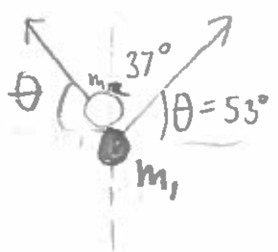
$m_1 = m_2 = m$

$v_{f1} = 0.36\ m/s$

$\Sigma \vec{p_i} = \Sigma \vec{p_f}$

$m_2 = m$

$v_{i2} = 0\ m/s$



$m_1 = m$

$v_{i1} = 0.45\ m/s$

$i: [0, m(0.45)] + [0, 0] = [0, 0.45m]$

$f: [m(0.36)\cos 53°, m(0.36)\sin 53°] + [-m v_{f2}' \cos\theta, m v_{f2} \sin\theta]$

$\Rightarrow [0, 0.45m] = [m(0.21665), m(0.28751)] + [-m v_{f2}\cos\theta, +m v_{f2}\sin\theta]$

$$[0, 0.45] = [0.21665, 0.28751] - [v_{f2}\cos\theta, v_{f2}\sin\theta]$$

$$[-0.21665, 0.1625] = [-v_{f2}\cos\theta, v_{f2}\sin\theta]$$

$-0.21665 = -v_{f2}\cos\theta$ AND $0.1625 = v_{f2}\sin\theta$

$\cos\theta = \dfrac{0.21665}{v_{f2}}$ $\qquad$ $\sin\theta = \dfrac{0.1625}{v_{f2}}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.1625}{0.21665} = 0.7501 \rightarrow \theta = \tan^{-1}(0.7501) \rightarrow \underline{\theta \doteq 37°}$$

$v_{f2} = \dfrac{0.1625}{\sin 36.8736°}$

$\underline{v_{f2} = 0.27\ m/s}$

$\therefore$ The second puck moves at 0.27 m/s [L 37° U].

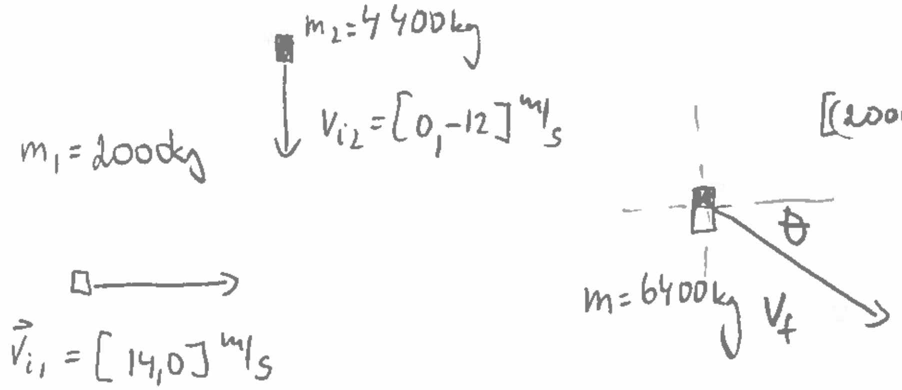
Assume a closed isolated system.

3. A 2000 kg truck moves east at 14m/s. It collides with a 4400 kg truck moving at 12m/s south. The trucks stick together and move as one unit after the collision. Determine the velocity of the wreckage.

Before After:

$$\sum \vec{p_i} = \sum \vec{p_f}$$

$$\vec{p_{i1}} + \vec{p_{i2}} = \vec{p_f}$$



$$[(2000)(14), 0] + [0, -(4400)(12)] = [p_{fx}, p_{fy}] \quad (x)$$

$$(x)\ [28000, 0] + [0, -52800] = 6400[v_f\cos\theta, -v_f\sin\theta]$$

$$\frac{[28000, -52800]}{6400} = [v_f\cos\theta, -v_f\sin\theta]$$

$$[4.375, -8.25] = [v_f\cos\theta, -v_f\sin\theta]$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-8.25}{4.375} = 1.8857 \rightarrow \theta \doteq \tan^{-1}(1.8857) \doteq \underline{62^\circ}$$

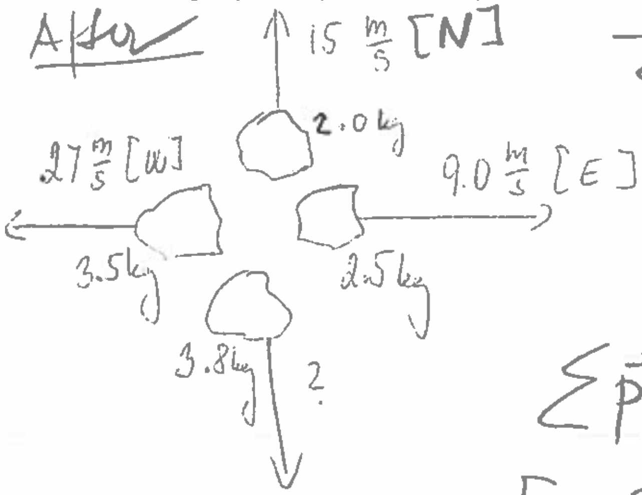
$$v_f = \frac{4.375}{\cos 62^\circ}$$

$$v_f \doteq 9.3\ m/s$$

$\therefore$ The wreckage moves at $9.3\ m/s\ [E62^\circ S]$.

Assum a closed isolated system.

4. An 11.8-kg object explodes into 4 pieces as shown. Determine the velocity of the fourth piece.



Before

$m = 11.8\ kg$

$v_i = 0\ m/s$

$$\sum \vec{p_i} = \sum \vec{p_f}$$

$$[0,0] = [-(3.5)(27), 0] + [0, (2.0)(15)] + [(2.5)(9.0), 0] + [(3.8)\, v_4 \cos\theta, -(3.8)\, v_4 \sin\theta]$$

$$[0,0] = [-94.5, 0] + [0, 30] + [22.5, 0] + 3.8\,[v_4 \cos\theta, -v_4 \sin\theta]$$

$$[0,0] = [-72 + 3.8\, v_4 \cos\theta,\ 30 - 3.8\, v_4 \sin\theta]$$

$$0 = -72 + 3.8\, v_4 \cos\theta \qquad 0 = 30 - 3.8\, v_4 \sin\theta$$

$$\frac{72}{3.8} = v_4 \cos\theta \qquad \frac{-30}{-3.8} = \frac{-3.8\, v_4 \sin\theta}{-3.8}$$

$$\cos\theta = \frac{18.94737}{v_4} \qquad \sin\theta = \frac{7.89474}{v_4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{7.89474}{18.94737} \rightarrow \theta = \tan^{-1}(0.4166..) \doteq \underline{23^\circ}$$

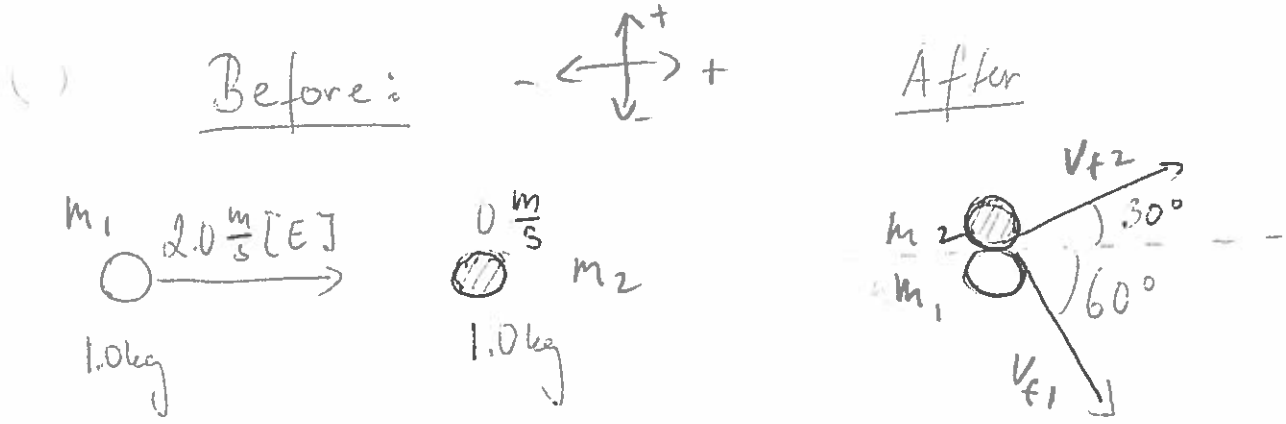
$$v_4 = \frac{18.94734}{\cos 22.6199}$$

$$v_4 = 20.526...\ m/s$$

$\therefore$ the forth piece moves at 21 m/s [E 23° S].

Assume a closed isolated system.

5. A collision occurs as shown. Determine the final velocity of each object.



$$\Sigma \vec{p_i} = \Sigma \vec{p_f}$$

$$[(1.0)(2.0),0] + [0,0] = [(1.0)v_{f1}\cos 60°, -(1.0)v_{f1}\sin 60°] +$$

$$+ [(1.0)(v_{f2})\cos 30°, (1.)(v_{f2})\sin 30°]$$

$$[2,0] = [0.5v_{f1}, -0.8660v_{f1}] + [0.8660v_{f2}, 0.5v_{f2}]$$

$2 - 0.5v_{f1} = 0.8660v_{f2}$ AND $0 + 0.8660v_{f1} = 0.5v_{f2}$

$$v_{f1} = \frac{0.5v_{f2}}{0.8660}$$

$$v_{f1} = 0.5774\,v_{f2}$$

$$= (0.5774)(1.7321)$$

$$\boxed{v_{f1} = 1.0\ m/s}$$

$$2 - 0.5(0.5774v_{f2}) = 0.8660v_{f2}$$

$$2 - 0.28868v_{f2} = 0.8660v_{f2}$$

$$2 = 1.1547v_{f2}$$

$$v_{f2} = \frac{2}{1.1547}$$

$$v_{f2} = 1.7321\ m/s$$

$$v_{f2} \doteq 1.7\ m/s$$

∴ Object 1 moves at 1.0 m/s [R60°D] and object 2 moves at 1.7 m/s [R30°U]

b) Energy released = ΔKE

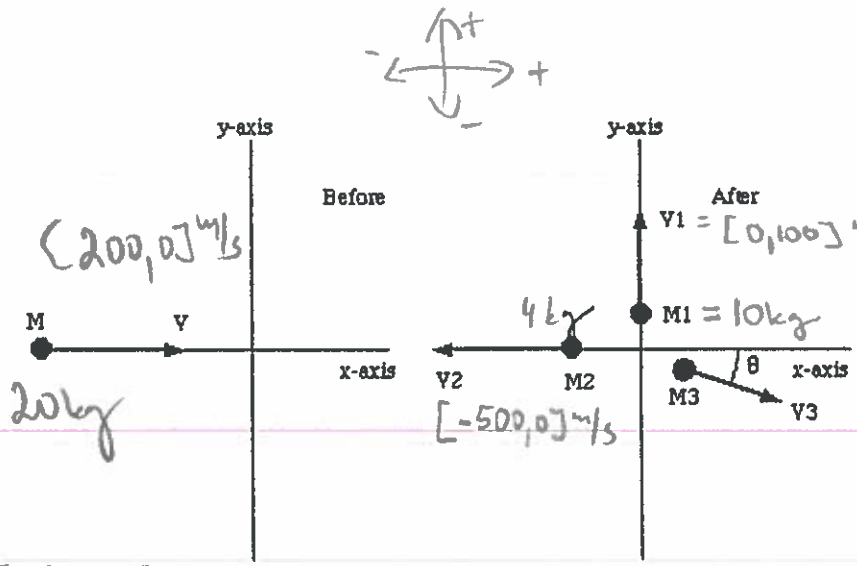
6. A 20 kg body is moving in the direction of the positive x-axis with a speed of 200 m/s when, owing to an internal explosion, it breaks into three parts. One part, whose mass is 10 kg, moves away from the point of explosion with a speed of 100 m/s along the positive y-axis. A second fragment, with a mass of 4 kg, moves along the negative x-axis with a speed of 500 m/s.

a)What is the speed of the third (6 kg) fragment ?

b) How much energy was released in the explosion (ignore gravity) ?

• Assume a closed isolated system.

$$\sum \vec{p_i} = \sum \vec{p_f}$$



• $m_3 = 20-10-4 = 6.0\,kg$

$$\sum \vec{p_f} = [0, (10)(100)] + [-(4)(500), 0] + [(6)v_{f3}\cos\theta, -(6)(v_{f3})\sin\theta]$$

$$= [0, 1000] + [-2000, 0] + [6v_{f3}\cos\theta, -6v_{f3}\sin\theta]$$

$$\sum \vec{p_i} = [20(200), 0]\ kg\ m/s$$

$$= [4000, 0]$$

$$[4000, 0] = [-2000, 1000] + 6[v_{f3}\cos\theta, -v_{f3}\sin\theta]$$

$$\frac{6000}{6} = \frac{6v_{f3}\cos\theta}{6} \qquad -1000 = -6v_{f3}\sin\theta$$

$$1000 = v_{f3}\cos\theta \qquad \sin\theta = \frac{166.\bar{6}}{v_{f3}}$$

$$\cos\theta = \frac{1000}{v_{f3}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{166.\bar{6}}{1000} = 0.1\bar{6} \rightarrow \theta = \tan^{-1}(0.1\bar{6}) \doteq 9.4623° \doteq 9.5°$$

$$v_3 = \frac{1000}{\cos 9.4623} \doteq 1014\ m/s$$

∴ ~~The 6kg fragment moves~~ at 1.0×10^3 m/s [R 9° D].

#6 b)

$\Delta KE = KE_f - KE_i$

$= \left[\frac{1}{2}(10)(100)^2 + \frac{1}{2}(4)(500)^2 + \frac{1}{2}(6)(1014)^2\right] - \frac{1}{2}(20)(200)^2$

$= 3\,634\,588 - 400\,000$

$= 3\,234\,588$

$\therefore$ 3.2×10^6 J of energy was released during the explosion.