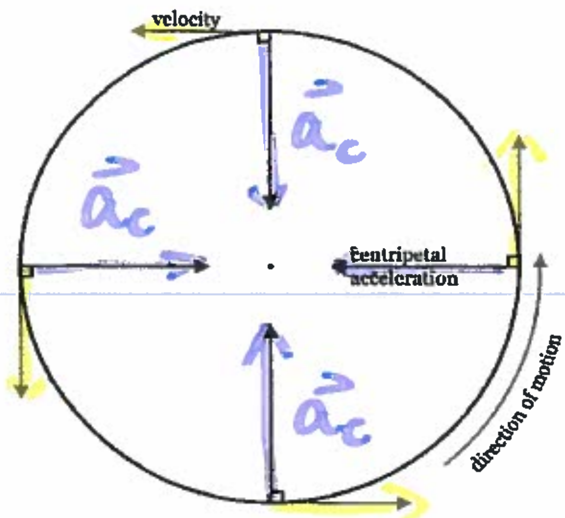
Notes + Practice

PHYSICS 12

DYNAMICS OF UNIFORM CIRCULAR MOTION



Uniform Circular Motion = motion of an object that is moving at a constant <u>speed</u> on a circular path.

Centripetal Acceleration of an object moving with speed v on a circular path with radius r has a magnitude a_c given by

$$\boxed{a_c = \frac{v^2}{r}}$$

Direction of a_c is always towards the center of the circle and it continually changes as the object moves along the circular path.

$$\boxed{v = \frac{2\pi r}{T}}$$

where V = speed [m/s]

T = period [s]

$2\pi r$ = circumference [m]

T = time needed to complete one revolution (cycle)

= period

= measured in seconds

Centripetal Force = is the net force needed to keep an object of mass m, moving at a speed v, on a circular path of radius r, and it has a magnitude of

$$\boxed{F_c = \frac{mv^2}{r}}$$

Direction – centripetal force is always directed towards the center of the circular path and in continually changes direction as the object moves.

FBD – centripetal force is never drawn in proper FBDs. Similarly to F_{net} and $F_{g\perp}$ and $F_{g//}$.

CENTRIPETAL FORCE

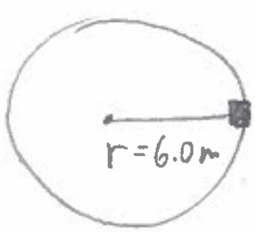
- When solving problems involving uniform circular motion, identify the type of the circular path the object is moving along. Decide whether the object moves along a **horizontal circle** (bird's view, example: a record player) or along a **vertical circle** (side view, example: a Ferris Wheel)
- Distinguishing the type of the circle is important for finding a solution:
  a) horizontal circle – DO NOT include gravity
  b) vertical circle – Must consider the force of gravity

Centripetal force can have various sources: gravitational force, normal force, force of tension or friction. Often, multiple forces contribute to the centripetal force at the same time.

$$\vec{F_c} = \frac{mv^2}{r}$$

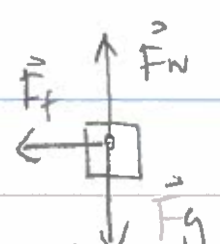
Example 1: A 2.0kg box is placed at the edge of the floor of a merry-go-round ride. The coefficient of friction between the box and the floor is 0.30. Determine the speed of the box if the diameter of the ride is 12.0 m.

Step1: diagram
Step 2: decide the type of the circle
Step 3: find all forces acting on the object
Step 4: write the formula for Newton's Second Law
Step 5: identify the force or forces that are keeping the object in the circle
Step 6: express the net force as a vector sum of all forces involved
Step7: choose an appropriate formula for a_c and solve for the unknown



$a_c = \frac{v^2}{r}$

• horizontal circle



• $F_{net} = m\vec{a}$

$\vec{F_c} = m\vec{a_c}$

$\vec{F_f} = m\vec{a_c}$

$F_N \mu = m a_c$

$\cancel{m} g \mu = \cancel{m} \frac{v^2}{r}$

$g\mu \cdot r = v^2$

$v = \sqrt{g \cdot \mu \cdot r}$

$v = \sqrt{(9.8)(0.30)(6.}$

$v = 4.2$ m/s

∴ The speed is 4.2 m/s.

Example 2: A 0.335 kg ball is attached to a sting and swung in a vertical circle with radius 85.0 cm. The ball's constant speed is 3.25m/s. Calculate the tension in the string when the ball is at:

a) the top of its path

b)the bottom of its path

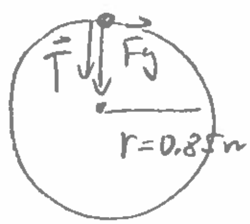
G: $m = 0.335 \text{ kg}$

$r = 85.0 \text{ cm} \rightarrow 0.85 \text{ m}$

$v = 3.25 \text{ m/s}$

R: $T = ? [N]$

A: a)



$$\vec{F_c} = \vec{T} + \vec{F_g}$$

$$-ma_c = -T - mg$$

$$T = ma_c - mg$$

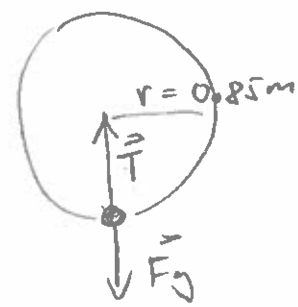
$$T = m\frac{v^2}{r} - mg$$

S: $T = (0.335)\left(\frac{3.25^2}{0.85}\right) - (0.335)(9.8)$

$T = 0.88 N$

S: The tension in the string is 0.88N [D]

A: b)



$$\vec{F_c} = \vec{T} + \vec{F_g}$$

$$F_c = T - mg$$

$$T = F_c + mg$$

$$T = m\frac{v^2}{r} + mg$$

S: $T = (0.335)\left(\frac{3.25^2}{0.85}\right) + (0.335)(9.8)$

$= 7.4 N$

S: The tension is 7.4 N[U].

Orbital radius: 1.50×10^{11} m

1. a) What would be the Earth's centripetal acceleration if the Earth's orbit around the Sun were a perfect circle with radius 1.5×10^{11} m ?

$a = \frac{v^2}{r}$ and $v = \frac{2\pi r}{T}$

$a = \frac{\frac{(2\pi r)^2}{T^2}}{r}$

$a = \frac{4\pi^2 r^{\cancel{2}}}{T^2} \times \frac{1}{\cancel{r}}$

$a = \frac{4\pi^2 r}{T^2}$

$a = \frac{4\pi^2 \times 1.5 \times 10^{11}}{(31557600)^2}$

$a = 0.005946 \ m/s^2$

$\boxed{\therefore a = 5.9 \times 10^{-3} \ m/s^2}$

$T: \frac{365.25 \text{ day}}{1} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}}$

31557600 s

b) What would be the magnitude of the centripetal force? Mass of Earth is approximately

$m_E = 5.98 \times 10^{24}$ kg

$F_c = ma_c$

$= (5.98 \times 10^{24})(0.005946)$

$= 3.6 \times 10^{22}$ N

$\therefore$ The magnitude of the centripetal force is 3.6×10^{22} N.

2. A 95-kg halfback makes a turn on the football field. The halfback sweeps out a path that is a portion of a circle with a radius of 12-meters. The halfback makes a quarter of a turn around the circle in 2.1 seconds. Determine the speed, acceleration and net force acting upon the halfback.

$m = 95 kg$

$r = 12 m$

$\frac{T}{4} = 2.1 s$

$T = 8.4 s$

$a_c = \frac{4\pi^2 r}{T^2}$

$a_c = \frac{4\pi^2 (12)}{8.4^2}$

$a_c \doteq 6.7 \, m/s^2$

$v = \frac{d}{t} = \frac{2\pi r}{T} = 8.9759\ldots$

$v \doteq 9.0 \, m/s$

$F_{net} = F_c = m a_c = (95)(6.7) \doteq 6.4 \times 10^2 N$

3. Anna Litical is practicing a centripetal force demonstration at home. She fills a bucket with water, ties it to a strong rope, and spins it in a circle. Anna spins the bucket when it is half-full of water and when it is quarter-full of water. In which case is more force required to spin the bucket in a circle?

By N2L: $\vec{F}_{net} = m\vec{a}$

$\vec{F}_c = m a_c$

$\vec{F}_c$ and m are directly proportional $\Rightarrow m \uparrow \; \vec{F}_c \uparrow$

$\therefore$ The fuller the bucket the more $\vec{F}$ is required to maintain same speed.

4. A Lincoln Continental and a Yugo are making a turn. The Lincoln is four times more massive than the Yugo. If they make the turn at the same speed, then how do the centripetal forces acting upon the two cars compare. Explain.

$F_c = m\frac{v^2}{r}$	$r_y = r_L$
$4 \cdot m_Y = m_L$	
$v_y = v_L$	

$F_y = m_y \cdot \frac{v_y^2}{r_y}$

$F_y = m_y \cdot \frac{v^2}{r}$

$F_L = m_L \frac{v^2}{r}$

$F_L = 4 m_y \frac{v^2}{r}$

$\boxed{F_L = 4 \cdot F_y}$

$\therefore$ Centripetal Force on Lincoln is 4x greater than the one on Yugo.

5. The Cajun Cliffhanger at Great America is a ride in which occupants line the perimeter of a cylinder and spin in a circle at a high rate of turning. When the cylinder begins spinning very rapidly, the floor is removed from under the riders' feet. What affect does a doubling in speed have upon the centripetal force? Explain.

$F_c = m \cdot \frac{v^2}{r} \rightarrow v \rightarrow 2v \Rightarrow F_c = m \cdot \frac{(2v)^2}{r} = \frac{4mv^2}{r} = 4m\frac{v^2}{r}$

$\therefore$ Doubling the speed will quadruple the centripetal Force.

6. Determine the centripetal force acting upon a 40-kg child who makes 10 revolutions around the Cliffhanger in 29.3 seconds. The radius of the barrel is 2.90 meters. At what speed does the child move?

$m = 40 kg$

$T = \frac{29.3}{10} s$

$r = 2.90 m$

$F_c = ?$ [N]

$v = ?$ [m/s]

$v = \frac{d}{t} = \frac{2\pi r}{T}$

$v = \frac{2\pi(2.9)}{2.93}$

$v \doteq 6.22 m/s$

$F_c = m a_c$

$= m \frac{v^2}{r}$

$= (40)\left(\frac{6.22^2}{2.9}\right)$

$\doteq 5.3 \times 10^2 N$

7. The maximum speed with which a 945-kg car makes a 180-degree turn is 10.0 m/s. The radius of the circle through which the car is turning is 25.0 m. Determine the force of friction and the coefficient of friction acting upon the car.

$m = 945 kg$

$r = 25.0 m$

$v = 10.0 m/s$

$F_f = ?$ [N]

$\mu = ?$

$\boxed{F_f = F_c}$

$F_f = m a_c$

$= m \frac{v^2}{r}$

$= (945)\left(\frac{10.0^2}{25}\right)$

$\therefore F_f = 3780 N$

$F_f = F_N \cdot \mu$

$\mu = \frac{F_f}{F_N}$

$\mu = \frac{F_f}{mg}$

$= \frac{3780}{(945)(9.8)}$

$\therefore \mu \doteq 0.41$

8. The coefficient of friction acting upon a 945-kg car is 0.850. The car is making a 180-degree turn around a curve with a radius of 35.0 m. Determine the maximum speed with which the car can make the turn.

$F_f = F_N \cdot \mu$

$= (945)(9.8)(0.85)$

$= 7871.85 N$

$F_f = F_c$ (to make the turn)

$F_c = m \frac{v^2}{r}$

$\sqrt{\frac{F_c r}{m}} = v$

$v = \sqrt{\frac{(7871.85)(35.0)}{945}}$

$\therefore \boxed{v \doteq 17 m/s}$

9. A 1.50-kg bucket of water is tied by a rope and whirled in a circle with a radius of 1.00 m. At the top of the circular loop, the speed of the bucket is 4.00 m/s. Determine the acceleration, the net force and the individual force values when the bucket is at the top of the circular loop.

$m = 1.50\ kg$

$V = 4.00\ m/s$

$F_c = m a_c$ and $a_c = \frac{v^2}{r}$

$= \frac{4.00^2}{1.0}$

$= 16\ m/s^2$

$F_c = (1.50)(16)$

$\boxed{\therefore \vec{F_c} = 24N\ [D]}$

$F_g = (1.5)(9.8)$

$\boxed{\therefore F_g = 14.7\ N\ [D]}$

$T = F_c - F_g$

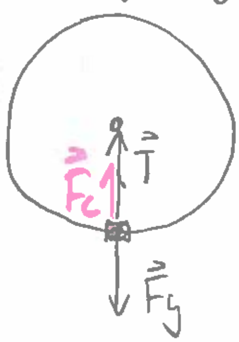
$T = 24 - 14.7$

$\boxed{\therefore T = 9.3\ N\ [D]}$

10. A 1.50-kg bucket of water is tied by a rope and whirled in a circle with a radius of 1.00 m. At the bottom of the circular loop, the speed of the bucket is 6.00 m/s. Determine the acceleration, the net force and the individual force values when the bucket is at the bottom of the circular loop.

$m = 1.5\ kg$

$V = 6.0\ m/s$



$F_c = m a_c$

$= m \frac{v^2}{r}$

$= (1.5)\left(\frac{6^2}{1.0}\right)$

$\boxed{\therefore \vec{F_c} = 54N\ [U]}$

$\vec{F_g} = mg$

$= (1.5)(9.8)$

$\boxed{\therefore \vec{F_g} = 14.7N\ [D]}$

$\vec{F_c} = \vec{T} + \vec{F_g}$

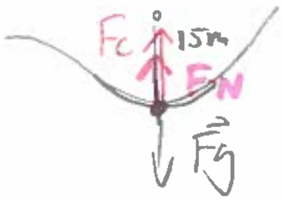
$F_c = T - F_g$

$F_c + F_g = T$

$54 + 14.7 = T$

$\boxed{\therefore T = 69N\ [U]}$

11. Anna Litical is riding a "woody" roller coaster. Anna encounters the bottom of a small dip having a radius of curvature of 15.0 m. At the bottom of this dip, Anna is traveling with a speed of 16.0 m/s and experiencing a much larger than usual normal force. Use Newton's second law to determine the normal force acting upon Anna's 50-kg body.



$r = 15\,m$

$v = 16.0\,m/s$

$m = 50\,kg$

$F_c = ma_c$

$\vec{F_c} = \vec{F_g} + \vec{F_N}$

- $F_c = (50)\left(\frac{16.0^2}{15}\right)$
  $= 853.\overline{3}\,N$
- $F_c = F_N - F_g$
  $F_N = F_c + F_g$
  $F_N = 853.\overline{3} + (50)(9.8)$
  $\boxed{\therefore \vec{F_N} = 1.3\times10^3\,N\ [up]}$

12. What is the frequency and period of the uniform circular motion if an object moves along a circle with radius 5.0 m with centripetal acceleration of 400 m/s²?

$r = 5.0\,m$

$a_c = 400\,m/s^2$

$T = ?\,[s]$

$f = ?\,[Hz]$

$f = \frac{1}{T}$

$a_c = \frac{v^2}{r} = \frac{\frac{(2\pi r)^2}{T^2}}{r} \Rightarrow a_c \cdot r = \frac{4\pi^2 r^2}{T^2}$

$\rightarrow T = \sqrt{\frac{4\pi^2 r^{\cancel{2}}}{a_c \cancel{r}}}$

$T = \sqrt{\frac{4\pi^2(5.0)}{400}}$

$\boxed{\therefore T = 0.702\,s}$

$f = \frac{1}{T}$

$f = \frac{1}{0.702}$

$\boxed{\therefore f = 1.4\,Hz}$

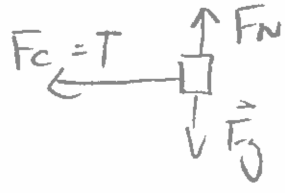
13. If a 1.5-kg object attached to a rope and moving along a horizontal circle with frequency of 3 Hz requires a force of tension of 450N in order to maintain the uniform circular motion, what is the radius of the circle?

$m = 1.5\,kg$

$f = 3\,Hz$

$F_T = 450\,N$

$r = ?\,[m]$



$F_c = ma_c = m\frac{v^2}{r} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r}$

$F_c = \frac{m4\pi^2 r^{\cancel{2}}}{T^2 \cancel{r}} \rightarrow r = \frac{F_c \cdot T^2}{m4\pi^2}$

$f = \frac{1}{T} \rightarrow T = \frac{1}{f} = \frac{1}{3}\,s$

$r = \frac{(450)\left(\frac{1}{3}\right)^2}{(1.5)(4)(\pi^2)}$

$\boxed{\therefore r = 0.84\,m}$

$v = \frac{2\pi r}{T}$

14. Find the mass of an object attached to a cable and rotating along a horizontal circle with a radius of 45.0cm are requiring tension of 560N. It takes 1 minute for the object to complete 100 full rotations.

$r = 45.0\ cm = 0.450\ m$

$F_T = 560N$

$\Delta t = 1 min = 60s$

$T = \frac{\Delta t}{100} = 0.60\ s$

$F_T = F_c = ma_c = m\frac{v^2}{r}$

$\Rightarrow m = \frac{F_T \cdot r}{v^2}$

$m = \frac{F_T \cdot \cancel{r} \cdot T^2}{2^2 \pi^2 r^{\cancel{2}}}$

$m = \frac{F_T \cdot T^2}{4\pi^2 r}$

$m = \frac{(560)(0.60)^2}{4\pi^2(0.45)}$

$\boxed{\therefore m \doteq 11\ kg}$

15. Explain how acceleration of an object may be a result either of change in speed, change in direction or change in both. Give an example for each scenario.

- Changein speed only ⇒ a car that has its breakes engaged while moving on a leveled straight road.

- change in direction only ⇒ a car that does not speed up when going out of a curve.

- change in direction and speed ⇒ a car slowing down when entering a roundabout.