

Notes

PC 11

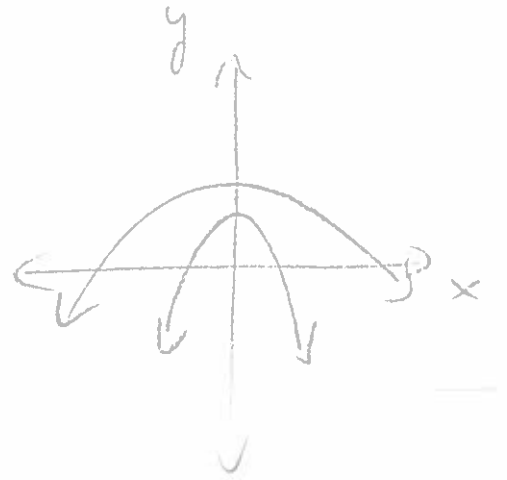
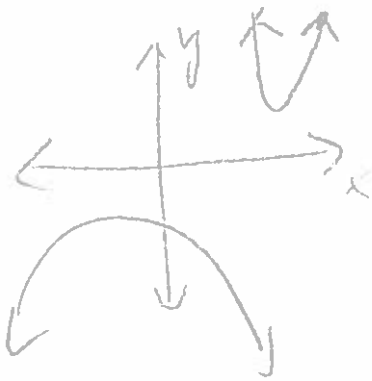
Solving a System of Quadratic Equations

- There are 4 possible scenarios when solving a quadratic-quadratic system

1. There are no real solutions to the system

- When graphed, the parabolas never intersect
- When solving algebraically, the variables cancel out and the remaining statement is false OR the resulting equation does not have real solutions.

Examples:



Solve:

$$\begin{aligned} 3x^2 &= y \\ -x^2 - 2x - 35 &= y \end{aligned}$$

$$3x^2 = -x^2 - 2x - 35$$

$$0 = -4x^2 - 2x - 35$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-4)(-35)}}{2(-4)}$$

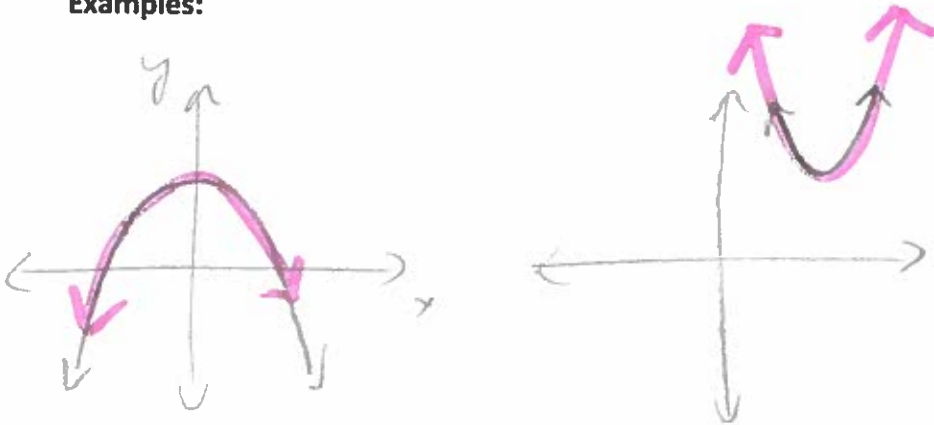
$$x = \frac{+2 \pm \sqrt{-556}}{-8} \rightarrow D < 0 \Rightarrow \text{no } \mathbb{R} \text{ solutions}$$

\therefore The system has no \mathbb{R} solutions.

2. There are infinitely many solutions to the system

- When graphed, the parabolas coincide (overlap)
- When solving algebraically, the variables cancel out and the remaining statement is true

Examples:



Solve:

$$(x-1)^2 - 9 = y$$

$$x^2 - 2x - 8 = y$$

$$(x-1)^2 - 9 = x^2 - 2x - 8$$

$$\cancel{x^2} - 2x + 1 - 9 = \cancel{x^2} - 2x - 8$$

$$-8 = -8$$

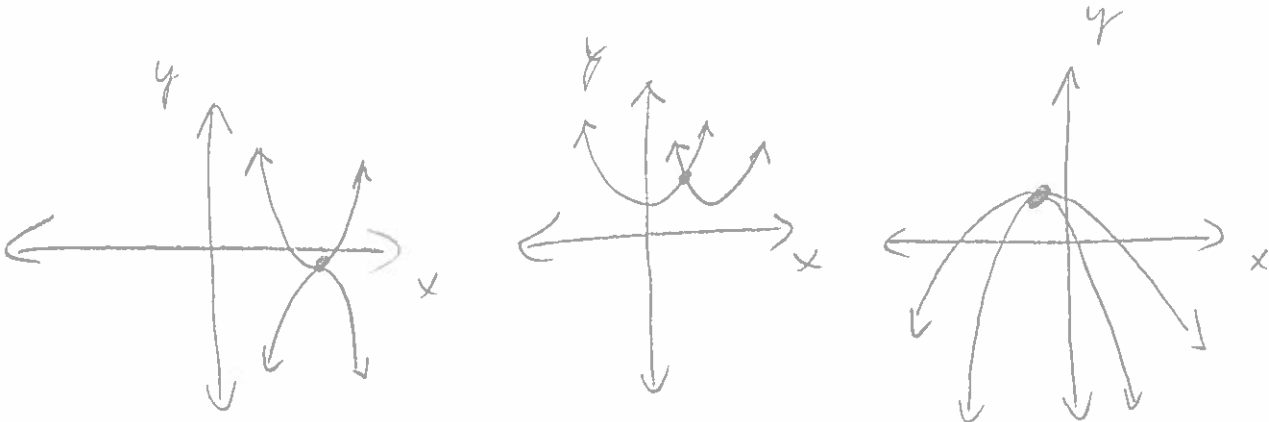
$$LS = RS \checkmark \Rightarrow \text{true statement}$$

\therefore The system has infinitely many solutions.

3. There is one solution to the system

- When graphed, the parabolas either touch or intersect only once
- When solving algebraically, the variables raised to the power of two cancel out and the remaining equation is linear OR the resulting equation is a perfect square trinomial.

Examples:



Solve:

$$x^2 - 8x + 19 = y$$

$$x^2 - 16x + 59 = y$$

$$\cancel{x^2} - 8x + 19 = \cancel{x^2} - 16x + 59$$

$$-8x + 16x = 59 - 19$$

$$\frac{8x}{8} = \frac{40}{8}$$

$$\boxed{x = 5}$$

Final y :

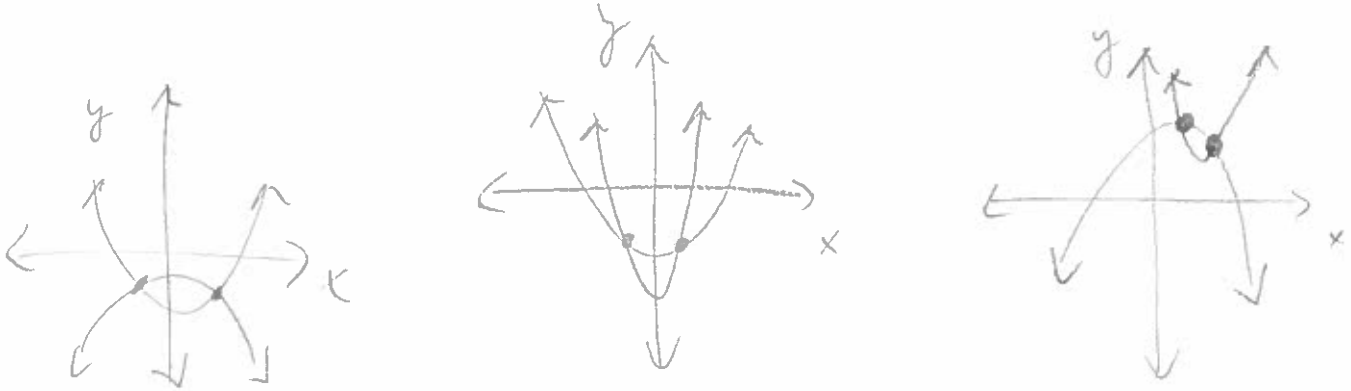
$$5^2 - 8(5) + 19 = y$$
$$25 - 40 + 19 = y$$
$$-15 + 19 = y$$
$$y = 4$$

\therefore The solution to the system is $x = 5$ (OR $(5, 4)$).

4. There are two solutions to the system

- When graphed, the parabolas intersect twice.
- When solving algebraically, the resulting equation has two solutions (discriminant is positive).

Examples:



Solve:

$$y = 3x^2 + 30x + 74$$

$$x^2 + 10x + 28 = y$$

$$3x^2 + 30x + 74 = x^2 + 10x + 28$$

$$2x^2 + 20x + 46 = 0$$

$$x^2 + 10x + 23 = 0$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(23)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{8}}{2} = \frac{-10 \pm 2\sqrt{2}}{2} = \frac{-5 \pm \sqrt{2}}{1} \begin{cases} x = -5 + \sqrt{2} \\ x = -5 - \sqrt{2} \end{cases}$$

\therefore the system has two solutions:
 $x = -5 + \sqrt{2}$ and
 $x = -5 - \sqrt{2}$
