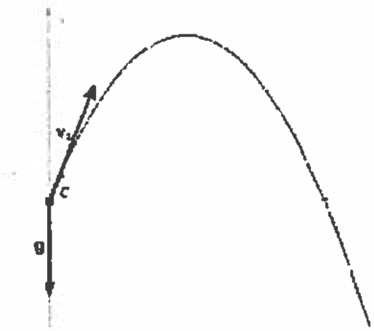
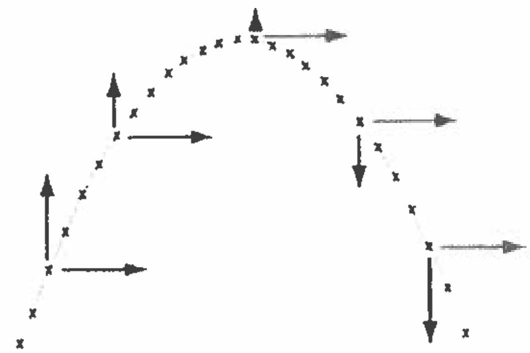


Projectile Motion:



1. List all given information. **Recall: acceleration is only in the vertical direction. Horizontal motion is at constant velocity.**

2. Sketch a labelled diagram that shows the path of the projectile.

3. Sketch a separate diagram of the initial velocity vector. Label the horizontal and vertical vector components of the initial velocity vector. Label the given angle.

4. When looking for maximum height and time needed the maximum height use the vertical component of the initial velocity.

5. When looking for the range use the horizontal component of the initial velocity and total time in the air.

6. Total time in the air is twice the time needed to reach the maximum height if and only if the projectile lands at the same level it was launched from.

7. Total time in the air for any other scenario is **time needed to reach the maximum height + time to free fall from the maximum height to the landing level.**

8. Practice and ask questions.

9. In the space below list the formulae you frequently use:

- $h_{max} = \frac{v_{fy}^2 - v_{iy}^2}{2g}$ where $v_{fy} = 0$
- $t_{FF} = \sqrt{\frac{d}{\frac{1}{2} g}} = \sqrt{\frac{2d}{g}}$
- $d_x = \text{range} = v_{ix} (t_{tot})$
- $v_{iy} = v_i (\sin\theta)$
- $v_{ix} = v_i (\cos\theta)$

$t_{hmax} = \frac{v_{fy} - v_{iy}}{g}$

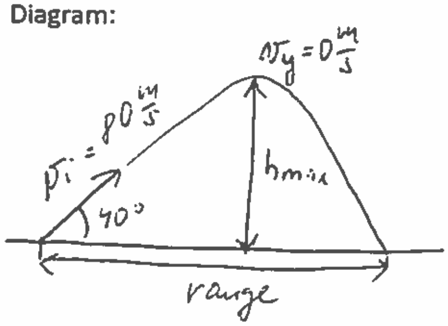
- $t_{tot} = 2 t_{hmax}$ or $t_{tot} = t_{hmax} + t_{FF}$

Example1: An object is thrown from the ground with initial velocity of 80m/s [40° above horizontal]. Find its maximum height and its range.

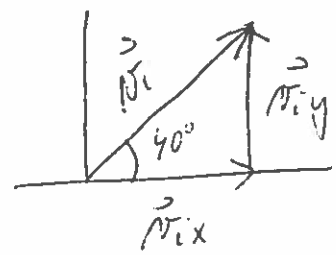
Given: $\vec{v}_i = 80\frac{m}{s}$ [40° above horizontal]

$\vec{a}_y = g = -9.8\ \frac{m}{s^2}$

Diagram:



Vector diagram:



Component

$\vec{v}_{ix} = 80(\cos 40°)$
$= 61.3\frac{m}{s}$ [R

$\vec{v}_{iy} = 80(\sin 40°)$
$= 51.4\frac{m}{s}$ [U

Time to reach maximum height:

$$\Delta t_{h_{max}} = \frac{v_{fy} - v_{iy}}{g} = \frac{0 - 51.4}{-9.8} = 5.2\ s$$

Maximum height:

$$h_{max} = \frac{v_{fy}^2 - v_{iy}^2}{2g} = \frac{0^2 - 51.4^2}{2(9.8)} = 134.8\ m = 1.3\times10^2\ m$$

Time in the air: $t_{tot} = 2t_{h_{max}} = 2(5.2) = 10.4\ s$

Range:

$$d_x = (v_{ix})(t_{tot})$$
$$= (61.3)(10.4)$$
$$= 637.5\ m = 6.4\times10^2\ m$$

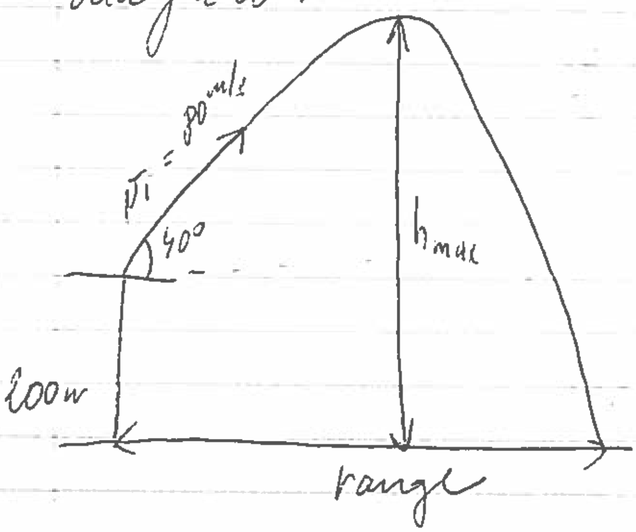
Example 2: In your notebook solve the above problem with a projectile launched 200 m above the ground level. Assume that the projectile lands on the ground. Compare your results with ex. 1.

Ex 2 : G : $\vec{v}_i = 80 \frac{m}{s}$ [40° above h.]

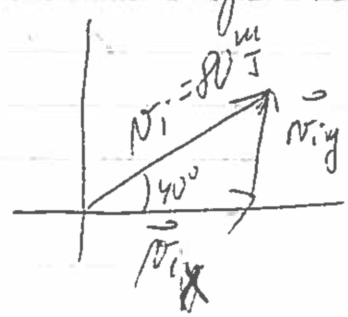
$\vec{d}_{iy} = 200$ m [up]

$a_y = g = -9.8 \frac{m}{s^2}$

diagram :



Vector diagram



Components:

$v_{ix} = 80(\cos 40)$
$= 61.3 \frac{m}{s}$

$\vec{v}_{iy} = 80(\sin 40)$
$= 51.4$ m/s

$$t_{hmax} = \frac{v_{fy} - v_{iy}}{g} = \frac{0 - 51.4}{-9.8} = \underline{5.2 \text{ s}}$$

$$h_{max} = d_{iy} + v_{iy}\Delta t + \frac{1}{2} g (\Delta t)^2$$

$$= 200 + (51.4)(5.2) + (\tfrac{1}{2})(-9.8)(5.2)^2 = \underline{3.3 \times 10^2 \text{ m}}$$

or $\frac{0^2 - 51.4^2}{2(9.8)} + 200 = \ldots$

time in the air : $t_{hmax} + t_{FF} = t_{tot}$

$$t_{FF} = \sqrt{\frac{d}{\frac{1}{2}g}} = \sqrt{\frac{335}{4.9}} = 8.3 \text{ s} \Rightarrow t_{tot} = 5.2 + 8.3 = \underline{13.5 \text{ s}}$$

Range : $(t_{tot})(v_{ix}) = d_x$

$$d_x = (13.5)(61.3) = 827.6 = \underline{8.3 \times 10^2 \text{ m}}$$

P11

Derive formula to find time needed to fall freely when height is known and an object is falling from its maximum height in the air.

G: $v_i = 0\ m/s$
v_f = unknown
$a = g = -9.8\ \frac{m}{s^2}$
$d = d\ m$

R: Δt_{FF}

$$d = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$0$$

$$d = \frac{1}{2} g \Delta t^2$$

$$\frac{d}{\frac{1}{2}g} = t^2$$

$$t = +\sqrt{\frac{d}{\frac{1}{2}g}} = +\sqrt{\frac{2d}{g}}$$