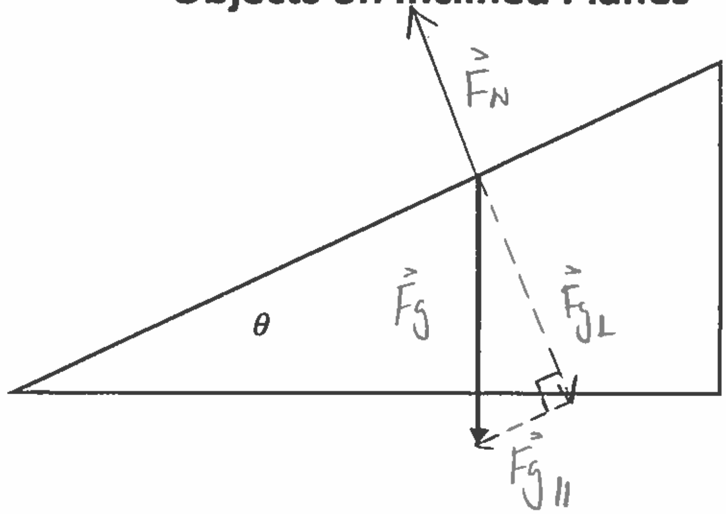
Notes

PHYSICS 11

Objects on Inclined Planes

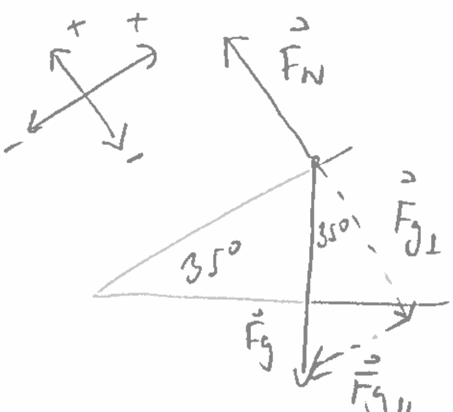


When analyzing scenarios that involve an object on an inclined plane you must pay attention to the following:

- The normal force vector is not colinear with the gravitational force vector.
- The normal force vector is colinear with the perpendicular component of the gravitational force.
- The parallel component of the gravitational force is the force that accelerates the object down the plane if no other forces parallel with the plane are involved.
- If force of friction is present, it may or may not keep the object from sliding down the inclined plane.
- The force of friction may act up the plane or it may act down the plane. This depends on the presence of other forces that are parallel with the plane. Or rather this depends on all the vector components that are parallel with the plane.
- If the vector sum of all vectors (vector components) that are parallel with the plane **not** including the for of friction is positive (up the plane) and the magnitude of this vector sum is greater than the magnitude of the fiction force, then the force of friction is acting down the plane.
- If the vector sum of all vectors (vector components) that are parallel with the plane **not** including the for of friction is negative (down the plane) and the magnitude of this vector sum is greater than the magnitude of the force of friction, then the force of friction is acting up the plane.
- If the magnitude of the net force of all parallel forces except the force of friction is equal to the magnitude of the force of friction, the object will not accelerate. (Either remains at rest or continues moving at constant velocity).
- If the magnitude of the net force of all parallel forces except the force of friction is less than the magnitude of the force of friction, the object will either remain at rest or if it is moving it will slow down to a stop.

Example 1:

A) Consider a 20.0 kg object on a frictionless inclined plane with an angle of inclination of 35°. What is the magnitude and direction of the object's acceleration? Include a situation diagram and an FBD.



$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

$$= \frac{112.42098}{20}$$

$$\doteq 5.6\ m/s^2$$

[down the plane]

$$\vec{F}_{net} = \underbrace{\vec{F}_{g\perp} + \vec{F}_N}_{0} + \vec{F}_{g\parallel}$$

$$F_{net} = F_{g\parallel}$$

$$= mg \sin 35°$$

$$= (20.0)(9.8)(\sin 35°)$$

$$= 112.42098\ N$$

$\therefore \vec{a} = 5.6\ m/s^2$ [down the plane].

B) What is the minimum coefficient of friction required to keep this object at rest on such an incline?

$$\|\vec{F}_{fs}\| = \|\vec{F}_{g\parallel}\|$$

$$F_{g\parallel} = F_N \mu_s$$

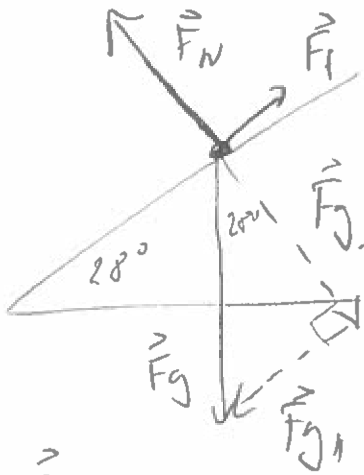
$$\mu_s = \frac{mg \sin 35°}{mg \cos 35°}$$

$$\mu_s = 0.70$$

$\therefore$ The minimum μ_s is 0.70.

Example 2:

Consider a 5.0 kg object on an inclined plane with an angle of inclination of 28°. What is the magnitude and direction of the object's acceleration if the coefficient of static friction is 0.023 and the coefficient of kinetic friction is 0.015? Include a situation diagram and an FBD.



$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F}_{net} = \underbrace{\vec{F}_{g\perp} + \vec{F}_N}_{} + \vec{F}_{g\parallel} + \vec{F}_f$$

$$= 0 - 23 + 0.64897$$

$$\doteq -22.3510\ N$$

$$\vec{a} = \frac{-22.3510}{5.0}$$

$$\vec{a} \doteq 4.5\ m/s^2\ [\text{down the plane}]$$

$$\vec{F}_{g\parallel} = mg\sin 28^\circ$$

$$\doteq 23\ N\ [\text{down the plane}]$$

$$\vec{F}_{f_s} = \vec{F}_N\ \mu_s$$

$$F_{fs} = (mg\cos 28^\circ)(0.023)$$

$$= 0.9951\ N\ [\text{up the plane}]$$

∴ The object accelerates 4.5 m/s^2 [down the plane].

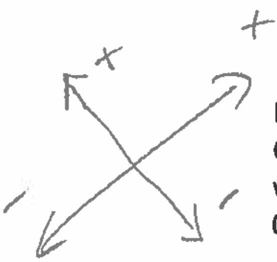
$$F_{fk} = \vec{F}_N \cdot \mu_k$$

$$= (mg\cos 28^\circ)(0.015)$$

$$= 0.64897\ N\ [\text{up the plane}]$$

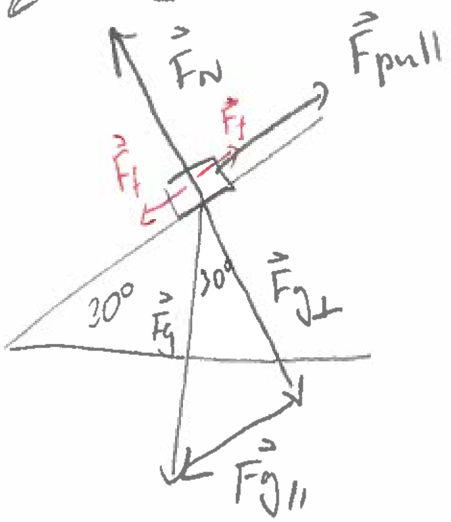
$F_{g\parallel} > F_{fs} \Rightarrow$ object will move

$\Rightarrow$ use $\vec{F}_{fk}$



Example 3:

Consider an 8.5 kg object pulled up an inclined plane of 30°with a force 120 N parallel with the plane. What is the acceleration of the object if the coefficient of static friction is 0.23 and the coefficient of kinetic friction is 0.12 ?

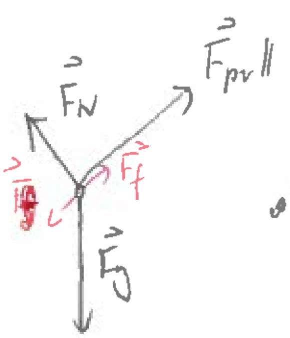


- $F_{g\perp} = F_N = mg\cos 30° = (8.5)(9.8)\cos 30°$
  $\doteq 72.1399\ N$
- $F_{g\parallel} = mg\sin 30° = (8.5)(9.8)(\sin 30°)$
  $\doteq 41.65\ N$
- $F_{fs} = F_N \mu_s$
  $= (72.1399)(0.23)$
  $\doteq 16.5922\ N$
- $F_{fk} = F_N \mu_k$
  $= (72.1399)(0.12)$
  $\doteq 8.6568\ N$

- $\vec{F}_f$ and its direction can be determined only when $F_{g\parallel}$ is known and compared with F_{pull}

! Notice $F_{pull} >>> F_{g\parallel} \Rightarrow$ Object moves up the plane $\Rightarrow$ use F_{fk} (as the difference between F_{pull} and $F_{g\parallel} > F_{fs}$) and $\vec{F}_{fk}$ is down the plane.

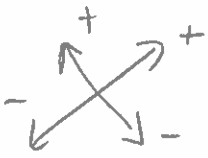
FBD



- $\vec{F}_{net} = \underbrace{\vec{F}_N + \vec{F}_{g\perp}} + \vec{F}_f + \vec{F}_{pull} + \vec{F}_{g\parallel}$
  $= 0 - 8.6568 + 120 - 41.65$
  $\doteq 69.6932\ N$

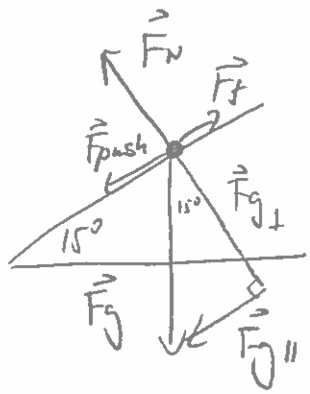
- $\vec{a} = \frac{\vec{F}_{net}}{m}$
  $= \frac{69.6932}{8.5}$
  $\doteq 8.2\ m/s^2$ [up the plane].

$\therefore \vec{a} \doteq 8.2\ m/s^2$ [up the plane].



Example 4:

Consider a 15.5 kg object pushed down an inclined plane of 15° with a force of 87 N parallel with the plane. What is the acceleration of the object if the coefficient of static friction is 0.39 and the coefficient of kinetic friction is 0.20 ?



$$\vec{F}_{net} = \underbrace{\vec{F}_{g\perp} + \vec{F}_N}_{0} + \vec{F}_{g\parallel} + \vec{F}_{push} + \vec{F}_{fk}$$

$$= 0 - 39.3146 - 87 + 29.3448$$

$$= -96.9698 \text{ N}$$

$$\vec{a} = -\frac{96.9698}{15.5}$$

$$\doteq -6.3 \text{ m/s}^2$$

- $\vec{F}_{fk} = \vec{F}_N \mu_k$
  $= (mg\cos 15°)(0.20)$
  $= 29.3448 \text{ N [up the plane]}$

- $\vec{F}_{fs} = \vec{F}_N \mu_s$
  $= (mg\cos 15°)(0.39)$
  $= 57.2224 \text{ N [up the plane]}$

- $\vec{F}_{g\parallel} = -mg\sin 15°$
  $= -39.3146 \text{ N}$
  $= 39.3146 \text{ N [down the plane]}$

$\therefore \vec{a} = 6.3 \text{ m/s}^2$ [down the inclined plane].

- As $(F_{g\parallel} + F_{push}) > F_{fs}$ object moves $\Rightarrow$ use $\vec{F}_{fk}$