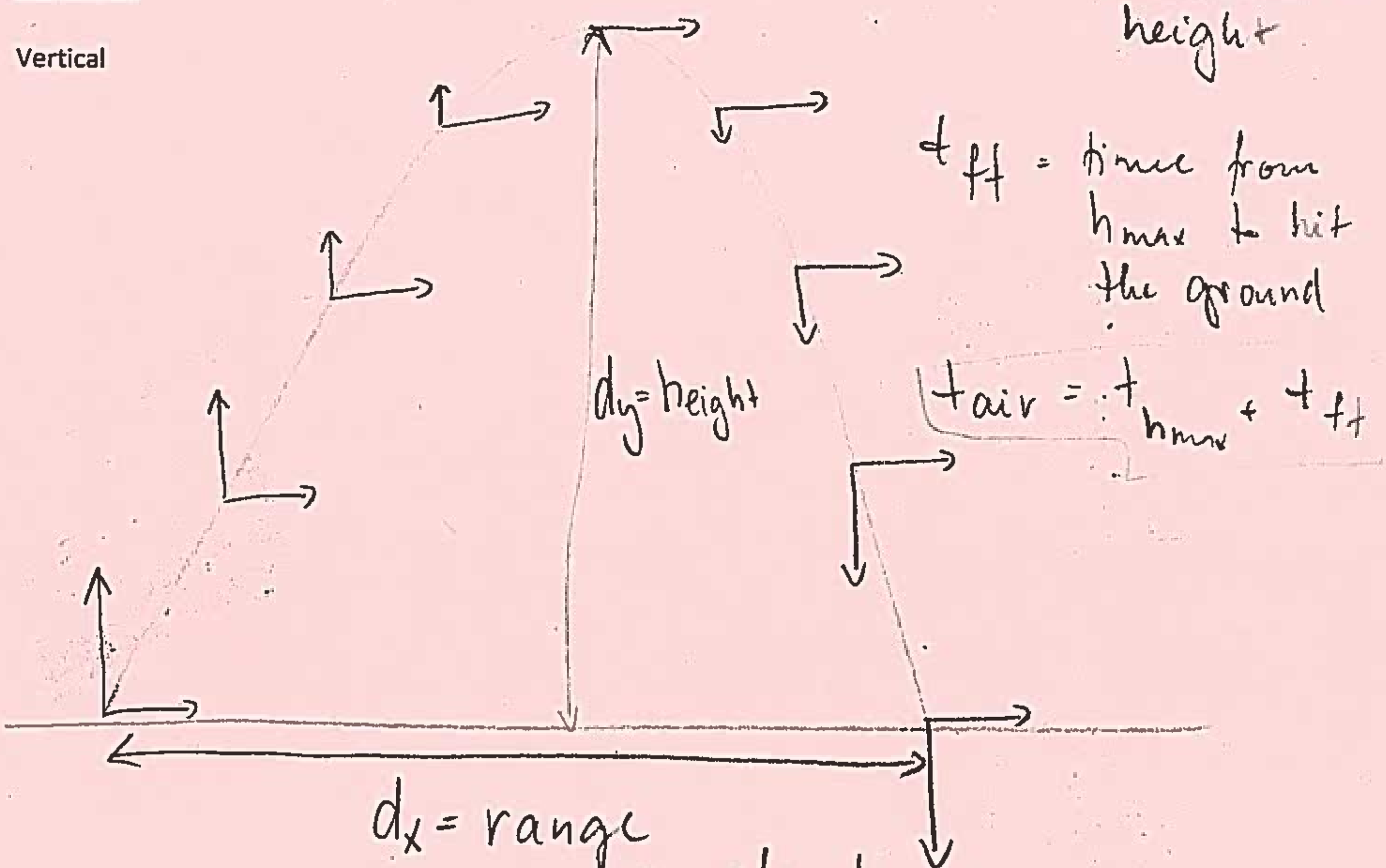
FREE FALL AND PROJECTILE MOTION

Projectile motion is motion in two dimension:

- Horizontal
- Vertical



The horizontal component of projectile motion is always constant

The vertical component of projectile motion always experiences acceleration due to gravity

The horizontal displacement of a projectile is called range

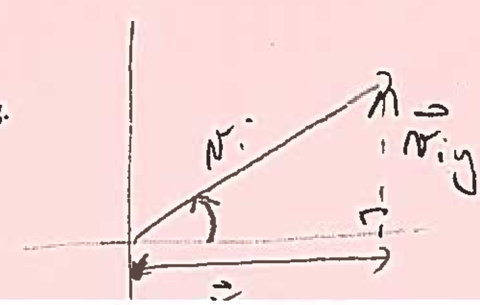
The vertical displacement of a projectile is its height

When a projectile reaches its maximum height, its velocity is zero only in the vertical direction, its horizontal velocity remains constant.

We assume that there are no forces acting on the object except the gravity, unless specifically mentioned otherwise, the gravitational pull is the one of the Earth.

\ projectile is an object that is launched and does not have any means of propelling.

NOTE: always include a sketch of the initial velocity vector and its decomposition.



Example 1: A ball is dropped from a height of 10 m. How long does it take to land?

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G: $\vec{a}_y = -9.8\ m/s^2$

$\vec{v}_i = 0\ m/s\ [D]$

$\vec{d}_y = -10\ m$

R: $t_{ff} = ?\ [s]$

A: $d = v_i t + \frac{1}{2} a t^2$

but $v_i = 0\ m/s$

$d = \frac{1}{2} a t^2 \rightarrow t = \sqrt{\frac{2d}{a}}$

S: $t_{ff} = \sqrt{\frac{2(-10)}{-9.8}}$

$t_{ff} \doteq 1.4\ s$

S: It will take 1.4 s to land.

Example 2: A ball is thrown horizontally from the height of 10 m with a velocity of 5.0 m/s. How long does it take to land? How far way does it land?

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G: $\vec{v}_i = 5.0\ m/s\ [R]$

$\vec{d}_y = -10\ m$

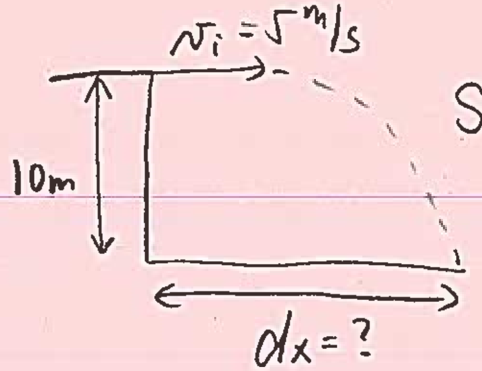
$a_y = -9.8\ m/s^2$

R: $t_{ff} = ?\ [s]$

$\vec{d}_x = ?\ [m][R]$

S: The ball will fall for 1.4 s and it will land 7.1 m horizontally from the launching point.

A: • $t_{ff} = \sqrt{\frac{2d_y}{a_y}}$ • $d_x = t_{ff} \cdot v_x$



S: $= \sqrt{\frac{2(-10)}{-9.8}}$

$= 1.428571429\ s$

S: $= \sqrt{\frac{2(-10)}{-9.8}} \times 5.00$

$\doteq 7.1\ m$

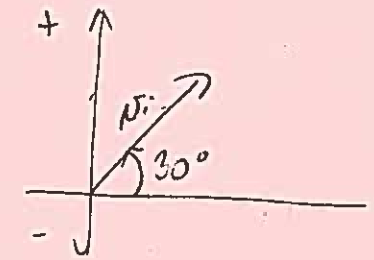
Example 4: An object is launched with an initial velocity of 20.0 m/s 30° above horizontal.

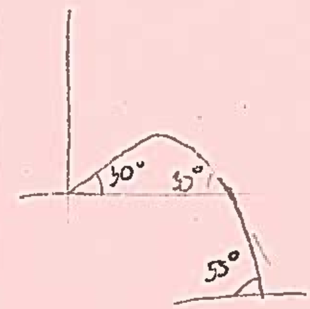
A) What is its maximum height?

B) How much time does the object spend in the air if it lands exactly at the same level it was launched from?

C) What is the object's final velocity if it lends 25.0 m below its launching point?

A)





$h_{max} \Rightarrow \vec{v}_{fy} = 0$ m/s [down]

$\vec{a} = -9.8$ m/s²

$v_{ix} = (\cos 30°)(20.0)$

$= 17.32$ m/s

$\dfrac{v_{fy}^2 - v_{iy}^2}{2a} = d$

$v_{iy} = (\sin 30°)(20.0)$

$= 10.0$ m/s

$h_{max} = \dfrac{0^2 - 10.0^2}{2(-9.8)}$

$= \underline{\underline{5.1 \text{ m}}}$

B) $t_{air} = 2t_{hmax}$

$t_{hmax} = ?$

$t = \dfrac{v_{fy} - v_{iy}}{a_y}$

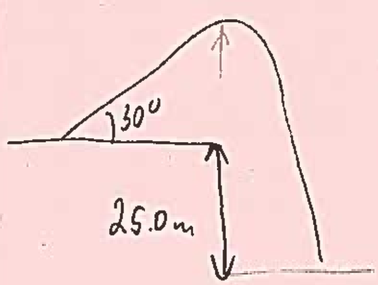
$t = \dfrac{0 - 10.0}{-9.8}$

$t = 1.0204$ s

$t_{air} = 2(1.0204)$

$= \underline{\underline{2.0 \text{ s}}}$

C)



$\vec{d}_y = 5.1 + 25.0 = 30.1$ m [down]

$a_y = -9.8$ m/s²

$v_{fx}^2 = v_i^2 + 2ad$

$v_{fy} = \sqrt{0 + 2(-9.8)(-30.1)}$

$v_{fy} = \sqrt{589.96}$

$v_{fx} = v_{ix} = 17.32$

$\vec{v}_{fy} = 24.3$ m/s [down]

$\vec{v}_f = [17.32, -24.3]$ m/s

Example 3: An object is thrown from a height ~~to~~ of 10 m with initial upward velocity of 3.0 m/s. How long does it take to land on the ground? What is its maximum height?

G: $\vec{d}_{iy} = 10\,m$

$\vec{a}_{iy} = -9.8\,m/s^2$

$\vec{v}_{iy} = 3.0\,m/s\,[u]$

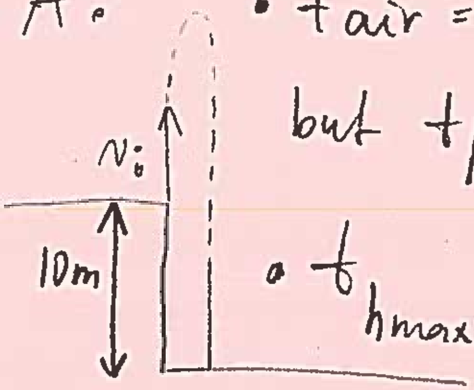
$\vec{v}_f$ @ $h_{max} = 0\,m/s\,[u]$

(+ ↑ / − ↓)

R: • $t_{air} = ?\,[s]$

• $h_{max} = ?\,[m]$

A:



• $t_{air} = t_{hmax} + t_{ff}$

but $t_{ff} > t_{hmax}$

• $t_{hmax} = \dfrac{v_f - v_i}{a}$

• $h_{max} = d_{iy} + \vec{v}_{iy} t_{hmax} + \frac{1}{2} a_y t$ (t_{hmax})

S: • $t_{hmax} = \dfrac{0 - 3.0}{-9.8}$

$= 0.306122449\,s$

• $h_{max} = 10 + (3.0)(t_{hmax}) + \frac{1}{2}(-9.8)(t_{hmax})$

$= 10.45918...$

$\doteq 10.46\,m$

• $t_{ff} = \sqrt{\dfrac{2d}{a}}$

$= \sqrt{\dfrac{2(-10.45918367)}{-9.8}}$

$= 1.46\,s$

• $t_{air} = t_{ff} + t_{hmax} = \underline{1.8\,s}$

S: It takes 1.8s to hit the ground and its maximum height is 10.5 m above ground or 4.6×10^{-1} m above the launching point.