CHAPTER

6 Motion in Two Dimensions

Practice Problems

6.1 Projectile Motion pages 147–152

page 150

1. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.

   a. How long does it take the stone to reach the bottom of the cliff?

   Since $v_y = 0$, $y - v_y t = -\frac{1}{2}gt^2$ becomes $y = -\frac{1}{2}gt^2$

   or $t = \sqrt{-\frac{2y}{g}}$

   $$= \sqrt{\frac{-(2)(-78.4\text{ m})}{9.80\text{ m/s}^2}}$$

   = 4.00 s

   b. How far from the base of the cliff does the stone hit the ground?

   $x = v_x t$

   $= (5.0\text{ m/s})(4.00\text{ s})$

   $= 2.0\times10^1\text{ m}$

   c. What are the horizontal and vertical components of the stone's velocity just before it hits the ground?

   $v_x = 5.0$ m/s. This is the same as the initial horizontal speed because the acceleration of gravity influences only the vertical motion. For the vertical component, use $v = v_i + gt$ with $v = v_y$ and v_i, the initial vertical component of velocity, zero.

   At $t = 4.00$ s

   $v_y = gt$

   $= (9.80\text{ m/s}^2)(4.0\text{ s})$

   $= 39.2\text{ m/s}$

2. Lucy and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of the conveyor belt and fall into a box below. If the box is 0.6 m below the level of the conveyor belt and 0.4 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?

   $y = v_{iy}t + \frac{1}{2}gt^2$; $t = \sqrt{\frac{-2y}{g}}$

   $x = v_x t = v_x\sqrt{\frac{-2y}{g}}$

   so $v_x = \frac{x}{\sqrt{\frac{-2y}{g}}} = \frac{0.4\text{ m}}{\sqrt{\frac{(-2)(-0.6\text{ m})}{9.80\text{ m/s}^2}}}$

   $= 1\text{ m/s}$

3. You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Stan, the short-order cook, slides his completed ice-cream sundaes down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundaes 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

   $x = v_x t$;

   $t = \frac{x}{v_x}$

   $y = -\frac{1}{2}gt^2$

   $= -\frac{1}{2}g\left(\frac{x}{v_x}\right)^2$

   $= -\frac{1}{2}(9.80\text{ m/s}^2)\left(\frac{0.070\text{ m}}{2.0\text{ m/s}}\right)^2$

   $= 0.0060$ m or 0.60 cm

Distance:

$x = v_i \cos\theta\, t$

$= (27.0\ \text{m/s})(\cos 60.0°)(4.77\ \text{s})$

$= 64.4\ \text{m}$

Maximum height:

at $t = \frac{1}{2}(4.77\ \text{s}) = 2.38\ \text{s}$

$y = v_i \sin\theta\, t - \frac{1}{2}gt^2$

$= (27.0\ \text{m/s})(\sin 60.0°)(2.38\ \text{s}) - \frac{1}{2}(+9.80\ \text{m/s}^2)(2.38\ \text{s})^2$

$= 27.9\ \text{m}$

6. A rock is thrown from a 50.0-m-high cliff with an initial velocity of 7.0 m/s at an angle of 53.0° above the horizontal. Find the velocity vector for when it hits the ground below.

$v_x = v_i \cos\theta$ $\quad t = \sqrt{\frac{2y}{g}}$ assumes $v_i = 0$

$v_y = v_i \sin\theta + gt$

$= v_i \sin\theta + g\sqrt{\frac{2y}{g}}$

$= v_i \sin\theta + \sqrt{2yg}$

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(v_i \cos\theta)^2 + (v_i \sin\theta + \sqrt{2yg})^2}$

$= \sqrt{((7.0\ \text{m/s})\cos 53.0°)^2 + \left((7.0\ \text{m/s})(\sin 53.0°) + \sqrt{(2)(50.0\ \text{m})(9.80\ \text{m/s}^2)}\right)^2}$

$= 37\ \text{m/s}$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$= \tan^{-1}\left(\frac{v_i \sin\theta_i + \sqrt{2yg}}{v_i \cos\theta_i}\right)$

$= \tan^{-1}\left(\frac{(7.0\ \text{m/s})(\sin 53.0°) + \sqrt{(2)(50.0\ \text{m})(9.80\ \text{m/s}^2)}}{(7.0\ \text{m/s})(\cos 53.0°)}\right)$

$= 83°$ from horizontal

Section Review

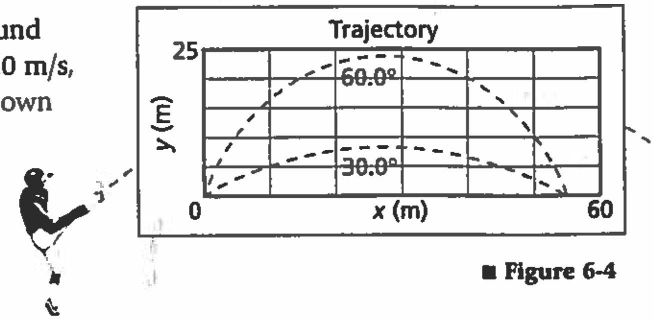
6.1 Projectile Motion pages 147–152

page 152

7. Projectile Motion Two baseballs are pitched horizontally from the same height, but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why does the faster ball not fall as far as the slower one?

page 152

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal, as shown in **Figure 6-4.** Find each of the following. Assume that air resistance is negligible.



■ Figure 6-4

a. the ball's hang time

$v_y = v_i \sin \theta$

When it lands, $y = v_y t - \frac{1}{2}gt^2 = 0$.

Therefore,

$t^2 = \frac{2v_y t}{g}$

$t = \frac{2v_y}{g}$

$= \frac{2v_i \sin \theta}{g}$

$= \frac{(2)(27.0\ \text{m/s})(\sin 30.0°)}{9.80\ \text{m/s}^2}$

$= 2.76$ s

b. the ball's maximum height

Maximum height occurs at half the "hang time," or 1.38 s. Thus,

$y = v_y t - \frac{1}{2}gt^2$

$= v_i \sin \theta\ t - \frac{1}{2}gt^2$

$= (27.0\ \text{m/s})(\sin 30.0°)(1.38\ \text{s}) - \frac{1}{2}(+9.80\ \text{m/s}^2)(1.38\ \text{s})^2$

$= 9.30$ m

c. the ball's range

Distance:

$v_x = v_i \cos \theta$

$x = v_x t = (v_i \cos \theta)(t) = (27.0\ \text{m/s})(\cos 30.0°)(2.76\ \text{s}) = 64.5$ m

5. The player in problem 4 then kicks the ball with the same speed, but at 60.0° from the horizontal. What is the ball's hang time, range, and maximum height?

Following the method of Practice Problem 4,

Hangtime:

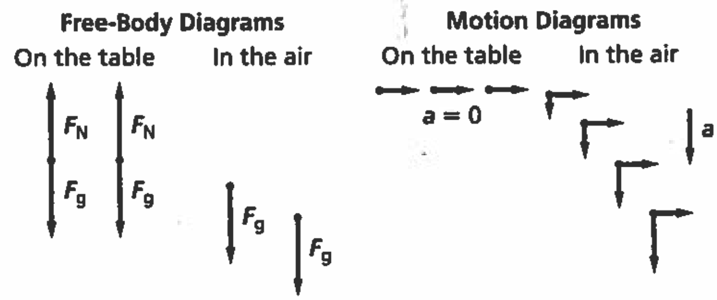
$t = \frac{2v_i \sin \theta}{g}$

$= \frac{(2)(27.0\ \text{m/s})(\sin 60.0°)}{9.80\ \text{m/s}^2}$

$= 4.77$ s

The faster ball is in the air a shorter time, and thus gains a smaller vertical velocity.

8. **Free-Body Diagram** An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points on the table and at two points in the air.



9. **Projectile Motion** A softball is tossed into the air at an angle of 50.0° with the vertical at an initial velocity of 11.0 m/s. What is its maximum height?

$v_f^2 = v_{iy}^2 + 2a(d_f - d_i)$; $a = -g$, $d_i = 0$

At maximum height $v_f = 0$, so

$$d_f = \frac{v_{iy}^2}{2g}$$

$$= \frac{(v_i \sin \theta)^2}{2g}$$

$$= \frac{((11.0\text{ m/s})(\sin 50.0°))^2}{(2)(9.80\text{ m/s}^2)}$$

$$= 3.62\text{ m}$$

10. **Projectile Motion** A tennis ball is thrown out a window 28 m above the ground at an initial velocity of 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?

$x = v_{0x}t$, **but need to find** t

First, determine v_{yf}:

$$v_{yf}^2 = v_{yi}^2 + 2gy$$

$$v_{yf} = \sqrt{v_{yi}^2 + 2gy}$$

$$= \sqrt{(v_i \sin \theta)^2 + 2gy}$$

$$= \sqrt{((15.0\text{ m/s})(\sin 20.0°))^2 + (2)(9.80\text{ m/s}^2)(28\text{ m})}$$

$$= 24.0\text{ m/s}$$

Now use $v_{yf} = v_{yi} + gt$ **to find** t.

$$t = \frac{v_{yf} - v_{yi}}{g}$$

$$= \frac{v_{yf} - v_i \sin \theta}{g}$$

$$= \frac{24.0\text{ m/s} - (15.0\text{ m/s})(\sin 20.0°)}{9.80\text{ m/s}^2}$$

$= 1.92\text{ s}$

$x = v_{xi}t$

$= (v_i \cos \theta)(t)$

$= (15.0\text{ m/s})(\cos 20.0°)(1.92\text{ s})$

$= 27.1\text{ m}$

11. Critical Thinking Suppose that an object is thrown with the same initial velocity and direction on Earth and on the Moon, where g is one-sixth that on Earth. How will the following quantities change?

a. v_x

will not change

b. the object's time of flight

will be larger on the moon; $t = \frac{-2v_y}{g}$

c. y_{max}

will be larger on the moon

d. R

will be larger on the moon

Practice Problems

6.2 Circular Motion pages 153–156

page 156

12. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal acceleration of the runner, and what agent exerts force on the runner?

$a_c = \frac{v^2}{r} = \frac{(8.8\text{ m/s})^2}{25\text{ m}} = 3.1\text{ m/s}^2$**, the frictional force of the track acting on the runner's shoes exerts the force on the runner.**

13. A car racing on a flat track travels at 22 m/s around a curve with a 56-m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and road is necessary for the car to round the curve without slipping?

$a_c = \frac{v^2}{r} = \frac{(22\text{ m/s})^2}{56\text{ m}} = 8.6\text{ m/s}^2$

Recall $F_f = \mu F_N$**. The friction force must supply the centripetal force so** $F_f = ma_c$**. The normal force is** $F_N = -mg$**. The coefficient of friction must be at least**

$$\mu = \frac{F_f}{F_N} = \frac{ma_c}{mg} = \frac{a_c}{g} = \frac{8.6\text{ m/s}^2}{9.80\text{ m/s}^2} = 0.88$$