## Unit 4: Polynomials

### 4.1 Terminology, Definitions, and Concepts

$>$ A polynomial is an algebraic expression that consists of a term or several terms added together.
$>$ A term is an expression that consists of a real number coefficient multiplied by one or more variables, and the variable(s) is (are) raised to a non-negative integral exponent.
$\qquad$ is the pattern every term has to follow.
> Some polynomials have specific names based on the number of terms they contain.

Examples: Determine the number of terms in each polynomial:

|  | $x^{2}$ | $x^{2}+3 x$ | $x^{5}+3 x-2$ | $x^{5}+3 x^{2}-2 x+1$ |
| :--- | :---: | :---: | :---: | :---: |
| Number of <br> terms |  |  |  |  |
| Specific <br> name |  |  |  | N/A |
|  | polynomial | polynomial | polynomial | polynomial |

## Vocabulary and Definitions

1. Real number $=$ a number that can be plotted on a horizontal number line.
> When inputted in the calculator, a real number does not give an "error" message. It is any number "legal" in high school.

The set of real numbers has a special symbol: $\qquad$
2. Variable = a value represented by any letter of the English alphabet, most often the lower case " $x$ " that can be replaced (substituted) by any real number.
$>$ In a single polynomial, each variable is represented by a different letter.

Examples: Determine the number of variables in each polynomial:

| Polynomial | $x^{2}+x-y$ | $-2 a+3 b+a$ | $-b+c$ | $-2+3 x^{5}+x^{3}$ |
| :--- | :--- | :--- | :--- | :--- |
| Number of variables |  |  |  |  |

3. Coefficient $=$ a real number that is in front of a variable and multiplies the variable.

Example 1: Circle the coefficient(s) in each polynomial:

| $2 x$ | $2 x^{4}+45 x$ | $-x^{4}+7$ | $-0.6 x^{3}+5 x+1$ | $8 a^{3}+5 x$ |
| :--- | :--- | :--- | :--- | :--- |

Example 2: Write each term separately and then write what the value of each coefficient.

| Polynomial | $-x^{4}+5 x$ | $-x^{4}-6$ | $0.3 \mathrm{x}^{2}-5 x$ | $8 \pi-x$ |
| :---: | :---: | :---: | :---: | :---: |
| Individual terms <br> separated by commas |  |  |  |  |
| List of coefficients <br> separated by commas |  |  |  |  |

4. Integral = an adjective form of the noun "integer"; = having the form of an integer.
5. Non-negative = positive OR zero.
6. Constant Term = a term that does not have a variable.
$>$ Note that the constant term meets the requirements of the definition (pattern) required for any term of a polynomial. How?
$\qquad$ $=$ $\qquad$ $=$ $\qquad$
$>$ This term is called "the constant term" or the "constant" because it remains unchanged regardless of what value is substituted in for the variable.

Example: Circle the constant term in each polynomial; then write the constant term.

| Polynomial | $-x^{4}+5 x$ | $-x^{4}-6$ | $0.3 x+1-5 x$ | $6+8 \pi-x$ |
| :---: | :---: | :---: | :---: | :---: |
| Constant term |  |  |  |  |

7. Degree of a term = the sum of all exponents of each variable in a single term.
$>$ If a variable does not have an exponent, the exponent and the degree are equal to one.
> A constant term has a degree of zero.

Examples: Determine the degree of each term:

| Term | $-x^{4}$ | $45 x^{4} y z$ | $-75 x$ | 6 |
| :---: | :---: | :---: | :---: | :---: |
| Degree |  |  |  |  |
| Term | $0.7 x^{9} y z^{3}$ | $\frac{1}{3} x y$ | $-3 a b c^{9}$ | $-5^{4}$ |
| Degree |  |  |  |  |

8. Degree of a polynomial $=$ the highest degree of a term.

Examples: Determine the degree of each polynomial

| Polynomial | $-3 x+4$ | $x^{4}+3 z$ | $-7 x^{5}+x^{13}$ | $6 x-0.5 x^{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| Degree of the <br> polynomial |  |  |  |  |
| Polynomial | $7 x^{9}+x$ | $\frac{1}{3} x+\frac{6}{17} x^{2}$ | -3 | $3 x+x^{7}-x^{9}$ |
| Degree of the <br> polynomial |  |  |  |  |

9. Leading term = term with the highest degree.

Examples: Determine the leading terms of each polynomial:

| Polynomial | $-3 x+4$ | $x^{2}+3 x^{5}$ | $-7 x^{5}+x^{3}$ | $6 x^{8}-0.5 x^{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| Leading term |  |  |  |  |
| Polynomial | $7 x^{9}+9 x^{12}$ | $\frac{1}{3} x+\frac{6}{17} x^{2}$ | $3 x+x^{2}$ | $3 x+x^{2}-x^{7}$ |
| Leading term |  |  |  |  |

10. Leading coefficient = coefficient of the leading term.

Examples: Determine the leading coefficient for each polynomial:

| Polynomial | $-3 x+4$ | $x^{2}+3 x^{5}$ | $-7 x^{5}+x^{3}$ | $6 x^{8}-0.5 x^{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| Leading coefficient |  |  |  |  |
| Polynomial | $7 x^{9}+9 x^{12}$ | $\frac{1}{3} x+\frac{6}{17} x^{2}$ | $3 x+x^{2}$ | $3 x+x^{2}-x^{7}$ |
| Leading coefficient |  |  |  |  |

11. Standard Form $=$ a form of a polynomial in which the terms are written in the descending order of their degree $=$ the leading term is written first followed by a term with the second highest degree; if the polynomial has a constant term different from zero, the constant term is always written last.

Examples: Write each polynomial in standard form, rearrange the terms of each polynomial if necessary.

| Polynomial | $-3 x+4$ | $x^{2}+3 x^{5}$ | $-7 x^{5}+x^{3}-1$ | $6 x^{8}-0.5 x^{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| Standard form |  |  |  |  |
| Polynomial | $7 x^{9}+9 x^{12}$ | $\frac{1}{3} x+x^{2}$ | $3 x+x^{2}+4$ | $3 x+x^{2}-x^{7}$ |
| Standard form |  |  |  |  |

