

**LCM – The least common multiple**

- To find the least common multiple of two numbers is to find the smallest number to which both numbers multiply to.
- There are two different methods to find the LCM
  - Listing all multiples before finding the common one.
  - Prime factorizing and selecting all unique factors and factors that repeat the most.

Example: Find the least common multiple of 6 and 14 by listing multiples.

$$6 \rightarrow 6, 12, 18, 24, 30, 36, \underline{42}, 48,$$

$$14 \rightarrow 14, 28, \underline{42}$$

$$\therefore \text{lcm}(6, 14) = 42.$$

Example: Using prime factorization, find the least common multiple of 126 and 420.

$$\begin{array}{c} 126 \\ \swarrow \quad \searrow \\ 2 \quad 63 \\ \swarrow \quad \searrow \\ 3 \quad 21 \\ \swarrow \quad \searrow \\ 3 \quad 7 \end{array}$$

$$\begin{array}{c} 420 \\ \swarrow \quad \searrow \\ 2 \quad 210 \\ \swarrow \quad \searrow \\ 2 \quad 105 \\ \swarrow \quad \searrow \\ 3 \quad 35 \\ \swarrow \quad \searrow \\ 5 \quad 7 \end{array}$$

$$126 = 2 \times \underline{3} \times \underline{3} \times 7$$

$$420 = \underline{2} \times \underline{2} \times 3 \times \underline{5} \times \underline{7}$$

$$2 \times 2 \times 3 \times 3 \times 5 \times 7 = \underline{\underline{1260}}$$

$$\therefore \text{lcm}(126, 420) = 1260.$$

Practice:

Using prime factorization, find the least common multiple for the given pairs of numbers.

20, 130	$\begin{array}{c} 20 \\ \swarrow \searrow \\ 2 \quad 10 \\ \quad \swarrow \searrow \\ \quad 2 \quad 5 \end{array}$ $\begin{array}{c} 130 \\ \swarrow \searrow \\ 2 \quad 65 \\ \quad \swarrow \searrow \\ \quad 5 \quad 13 \end{array}$	$20 = 2 \times 2 \times 5$ $130 = 2 \times 5 \times 13$ $2 \times 2 \times 5 \times 13 = 260$ <p><math>\therefore \text{lcm}(20, 130) = 260</math></p>
143, 1001	$\begin{array}{c} 143 \\ \swarrow \searrow \\ 11 \quad 13 \end{array}$ $\begin{array}{c} 1001 \\ \swarrow \searrow \\ 7 \quad 143 \\ \quad \swarrow \searrow \\ \quad 11 \quad 13 \end{array}$	$143 = 11 \times 13$ $1001 = 7 \times 11 \times 13$ $7 \times 11 \times 13 = 1001$ <p><math>\therefore \text{lcm}(143, 1001) = 1001</math></p>
42, 132	$\begin{array}{c} 42 \\ \swarrow \searrow \\ 2 \quad 21 \\ \quad \swarrow \searrow \\ \quad 3 \quad 7 \end{array}$ $\begin{array}{c} 132 \\ \swarrow \searrow \\ 2 \quad 66 \\ \quad \swarrow \searrow \\ \quad 2 \quad 33 \\ \quad \quad \swarrow \searrow \\ \quad \quad 3 \quad 11 \end{array}$	$42 = 2 \times 3 \times 7$ $132 = 2 \times 2 \times 3 \times 11$ $7 \times 2 \times 2 \times 3 \times 11 = 924$ <p><math>\therefore \text{lcm}(42, 132) = 924</math></p>
46, 575	$\begin{array}{c} 46 \\ \swarrow \searrow \\ 2 \quad 23 \end{array}$ $\begin{array}{c} 575 \\ \swarrow \searrow \\ 5 \quad 115 \\ \quad \swarrow \searrow \\ \quad 5 \quad 23 \end{array}$	$46 = 2 \times 23$ $575 = 5 \times 5 \times 23$ $2 \times 5 \times 5 \times 23 = 1150$ <p><math>\therefore \text{lcm}(46, 575) = 1150</math></p>