Oct 19, 20, 2022

M9

Unit 4: Polynomials

4.1 Terminology, Definitions, and Concepts

- A polynomial is an algebraic expression that consists of a term or several terms added together.

- A term is an expression that consists of a real number coefficient multiplied by one or more variables, and the variable(s) is (are) raised to a non-negative integral exponent.

Real number — non negative integral exponents: 0, 1, 2, 3, ...

- $\mathbb{R} \cdot x$ ← variable is the pattern every term has to follow. (cannot be decimal or negative)

Term

- Some polynomials have specific names based on the number of terms they contain.

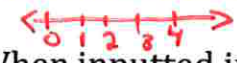
Examples: Determine the number of terms in each polynomial:

	x^2	$x^2 + 3x$	$x^5 + 3x - 2$	$x^5 + 3x^2 - 2x + 1$
Number of terms	1	2	3	4
Specific name	monomial	binomial	trinomial	N/A
	polynomial	polynomial	polynomial	polynomial

mono = 1 | bi = 2 | tri = 3 | Poly = More than 3

Vocabulary and Definitions

1. Real number = a number that can be plotted on a horizontal number line.



- When inputted in the calculator, a real number does not give an "error" message. It is any number "legal" in high school.

- The set of real numbers has a special symbol: $\mathbb{R}$

2. Variable = a value represented by any letter of the English alphabet, most often the lower case "x" that can be replaced (substituted) by any real number. (From English Alphabet)

- In a single polynomial, each variable is represented by a different letter.

Examples: Determine the number of variables in each polynomial:

Polynomial	$x^2 + x - y$	$-2a + 3b + a$	$-b + c$	$-2 + 3x^5 + x^3$
Number of variables (# of different letters)	2	2	2	1

3. Coefficient = a real number that is in front of a variable and multiplies the variable.

Example 1: Circle the coefficient(s) in each polynomial:

$2x$	$2x^4 + 45x$	$-x^4 + 7$	$-0.6x^3 + 5x + 1$	$8a^3 + 5x$

Example 2: Write each term separately and then write what the value of each coefficient.

Polynomial	$-1x^4 + 5x$	$-1x^4 - 6$ (has no variable → 7 (Not coefficient))	$0.3x^2 - 5x$	$8\pi - 1x$ (π Not Variable)
Individual terms separated by commas	$-1x^4, 5x$	$-1x^4, -6$	$0.3x^2, -5x$	$8\pi, -1x$
List of coefficients separated by commas	$-1, 5$	-1	$0.3, -5$	-1

4. Integral = an adjective form of the noun "integer"; = having the form of an integer.

5. Non-negative = positive OR zero.

6. Constant Term = a term that does not have a variable. → (has no letter)

- Note that the constant term meets the requirements of the definition (pattern) required for any term of a polynomial. How?

$5 = 5 \cdot x^0 = 5 \cdot (1) = 5$

- This term is called "the constant term" or the "constant" because it **remains unchanged** regardless of what value is substituted in for the variable.

Example: Circle the constant term in each polynomial; then write the constant term.

Polynomial	$1x^4 + 5x + 0$	$1x^4 - 6$	$0.3x + 1 - 5x$	$6 + 8\pi - 1x$
Constant term	None or $\emptyset$	-6	1	$6, 8\pi$

7. Degree of a term = the sum of all exponents of each variable in a single term.

- If a variable does not have an exponent, the exponent and the degree are equal to one.

$5x = 5x^1 \Rightarrow$ degree (1) Degree is the same as the exponent.

- A constant term has a degree of zero.

$-8 = -8x^0 \Rightarrow$ degree is (0)

Examples:

$5x^1y^2z^1$ (degree: $1+2+1 = 4$)

$-3a^4b^1c^2$ (degree: $4+1+2 = 7$)

$2^3x^2y^1$ ← degree: 2+1 = 3 $5^6x^1y^1$ ← degree: 1+1 = 2

! Exponent not connected To Variable !

Examples: Determine the degree of each term:

Term	$-x^4$	$45x^4yz^1$	$-75x^1$	6
Degree	4	4+1+1 6	1	∅
Term	$0.7x^9y^1z^3$	$\frac{1}{3}x^1y^1$	$-3d^1b^1c^9$	-5^4
Degree	9+1+3 13	1+1 2	1+1+9 11	∅

8. Degree of a polynomial = the highest degree of a term. the one with the biggest Exponent is the Boss

Examples: Determine the degree of each polynomial

Polynomial	$-3x^1+4x^0$	x^4+3z^1	$-7x^5+x^{13}$	$6x^1-0.5x^3$
Degree of the polynomial	1	4	13	3
Polynomial	$7x^9+x^1$	$\frac{1}{3}x^1+\frac{6}{17}x^2$	$-3x^0$	$3x^1+x^7-x^9$
Degree of the polynomial	9	2	∅	9

9. Leading term = term with the highest degree.

"boss term"

Examples: Determine the leading terms of each polynomial:

Polynomial	$-3x^1+4x^0$	x^2+3x^5	$-7x^5+x^3$	$6x^8-0.5x^5$
Leading term	$-3x$	$3x^5$	$-7x^5$	$6x^8$
Polynomial	$7x^9+9x^{12}$	$\frac{1}{3}x^1+\frac{6}{17}x^2$	$3x^1+x^2$	$3x^1+x^2-x^7$
Leading term	$9x^{12}$	$\frac{6}{16}x^2$	x^2	x^7

10. Leading coefficient = coefficient of the leading term.

Examples: Determine the leading coefficient for each polynomial:

Polynomial	$-3x+4$	$1x^2+3x^5$	$-7x^5+1x^3$	$6x^8-0.5x^5$
Leading coefficient	-3	3	-7	6
Polynomial	$7x^9+9x^{12}$	$\frac{1}{3}x+\frac{6}{17}x^2$	$3x+1x^2$	$3x+x^2-1x^7$
Leading coefficient	9	$\frac{6}{17}$	1	-1

$x^{10,9,8,7,6,5\ldots}$

11. Standard Form = a form of a polynomial in which the terms are written in the descending order of their degree = the leading term is written first followed by a term with the second highest degree; if the polynomial has a constant term different from zero, the constant term is always written last.

Examples: Write each polynomial in **standard form**, rearrange the terms of each polynomial if necessary.

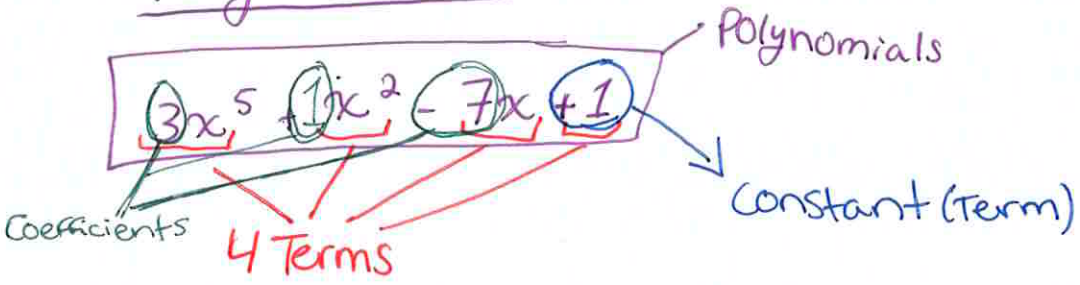
Polynomial	$-3x^1+4$	x^2+3x^5	$-7x^5+x^3-1$	$6x^8-0.5x^5$
Standard form	$-3x+4$	$3x^5+x^2$	$-7x^5+x^3-1$	$6x^8-0.5x^5$
Polynomial	$7x^9+9x^{12}$	$\frac{1}{3}x^1+x^2$	$3x^1+x^2+4$	$3x^1+x^2-x^7$
Standard form	$9x^{12}+7x^9$	$x^2+\frac{1}{3}x$	x^2+3x+4	$-x^7+x^2+3x$

Example: $x^8+x^{10}+x^9-3(x^0)$

Highest to Lowest

$x^{10}+x^9+x^8-3x^0$

Polynomials



Real number = $\mathbb{R}$

Pattern of every term has to follow $\mathbb{R} \cdot x^{0,1,2,3}$

↳ the coefficient must be real

↳ the variable is a letter from the English alphabet

↳ Exponents are not allowed to be negative, or a decimal, or a fraction. ⇒ in other words, exponents are positive integers or a Zero.