

Exponent Laws

For any real number "a" and any integers "m" and "n" the following is true:

! It is very important that the base is the same for each power, the rules do not work otherwise! !

Law	Formula	Example
Product Law	$a^m \times a^n = a^{m+n}$ <p>if both bases are the same and multiplied, you can add the exponents together.</p>	$5^2 \times 5^4 = 5^{2+4} = 5^6$ <p>Proof: $5^2 \times 5^4$ $5 \times 5 \times 5 \times 5 \times 5 \times 5$ 6 times = 5^6</p>
Quotient Law	$a^m \div a^n = a^{m-n}$ <p>if both bases are the same and divided, you can subtract the exponents together.</p> $\frac{a^m}{a^n} = a^{m-n}$	$6^8 \div 6^3 = 6^{8-3} = 6^5$ <p>Proof: $\frac{6^8}{6^3} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6}$ 5 times 6^5</p>
Power Law	<p>if you have a power to a power you multiply the exponents</p> $(a^m)^n = a^{m \times n}$	$(7^2)^3 = 7^{2 \times 3} = 7^6$ <p>Proof: $(7^2)^3 = (7^2)(7^2)(7^2)$ $(7 \times 7)(7 \times 7)(7 \times 7)$ 6 times 7^6</p>

For any real numbers "a" and "b" and any integers "m" and "n" the following is true:

Law	Formula	Example
Power of a Product	$(a \times b)^m = a^m \times b^m$	$(2x)^3 = 2^3 \cdot x^3$ $= 8 \cdot x^3$
Power of a Quotient	$(a \div b)^m = a^m \div b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{3}{x}\right)^2 = 3^2 \div x^2$ $= 9 \div x^2$ or $\frac{3^2}{x^2} \rightarrow \frac{9}{x^2}$
Exponent of zero	$a \neq 0$ $a^0 = 1$	$15^0 = 1$
Base of positive one	$1^a = 1$	$1^{500} = 1$
Base of negative one	$(-1)^a = 1$ if and only if a is even $(-1)^a = -1$ if and only if a is odd	$(-1)^{20} = +1$ $(-1)^{31} = -1$

Rule	Formula	Example
<p>A base with a negative exponent</p>	<p>* $a^{-m} = \frac{1}{a^m}$ *</p> <p>→ Base goes to denominator</p> <p>→ Exponent changes to positive</p>	<p>Base $(5x)^{-1} = \frac{1}{(5x)^1} = \frac{1}{5x}$</p> <p>Base $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$</p> <p>Base $7x^{-3} = \frac{7}{x^3}$</p>
<p>A fraction with bases with one or more negative exponents</p>	<p>$b \neq 0$ and $a \neq 0$</p> <p>$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$</p> <p>$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$</p>	<p>$\frac{x^{-3}}{y^{-9}} = \frac{y^9}{x^3}$</p> <p>! $\frac{a^{-2}}{b^3} = \frac{1}{a^2 b^3}$ 'b' stayed put because it is positive exponent</p>
		<p>! $\frac{a^7}{b^{-9}} = \frac{a^7 b^9}{1}$ or $a^7 b^9$</p>

More examples

11. Write without negative exponents

$$a) 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$b) \left(\frac{1}{8}\right)^{-7} = \frac{8^7}{1} = 8^7$$

$$c) 2x^{-4} = \frac{2}{x^4}$$

$$d) (3x)^{-5} = \frac{1}{(3x)^5} = \frac{1}{3^5 x^5}$$

$$e) \frac{a^{-4}}{b^{-5}} = \frac{b^5}{a^4}$$

$$f) \frac{x^{-6}}{y^3} = \frac{1}{y^3 x^6}$$