M9

# 1.2 Factoring Numbers, GCF, and LCM

**A factor** is a whole number that divides another whole number without leaving a remainder.

**A prime factor** is a prime number that divides another whole number without leaving a remainder.

- Every counting number greater than one has at least two factors: 1 and itself

!! only !!

★ **A prime number is a number that has exactly two factors – 1 and itself.** ★

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

**A composite number is a number greater than 1 that has more than two factors.**

Example of composite numbers:

4, 6, 8, 9, 10, 12, 14, 18, 20, 21, 22

**Recall: number 1 and number 0 are neither prime nor they are composite!**

Not prime or composite!

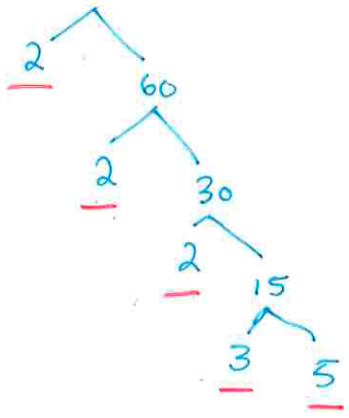
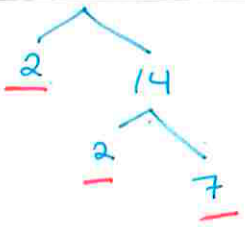
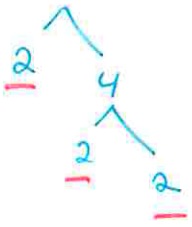
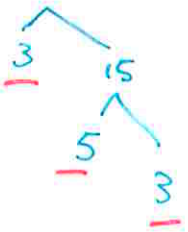
Task: List all the factors of each number:

| 36 | 56 | 28 |
|---|---|---|
| 1, 2, 3, 4, 6, 9, 12, 18, 36 | 1, 2, 4, 7, 8, 14, 28, 56 | 1, 2, 4, 7, 14, 28 |

**Prime Factorization – Writing Prime Factorization Tree**

- Remember to always start with the smallest prime number and if t does not work, use the prime number that follows. Do not skip primes when writing prime factorization tree. Always try if the prime works before moving onto the next prime.

Task: Write a prime factorization tree for each number. Then use the tree to write the prime factorization statement.

| 120 | 28 |
|---|---|
|   |   |
| $\therefore 120 = 2 \times 2 \times 2 \times 3 \times 5$ | $\therefore 28 = 2 \times 2 \times 7$ |
| **8** | **45** |
|   |   |
| $\therefore 8 = 2 \times 2 \times 2$ | $\therefore 45 = 3 \times 5 \times 3$ |

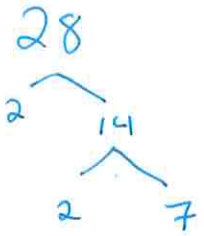
**GCF – greatest common factor**
"Noodle Method"

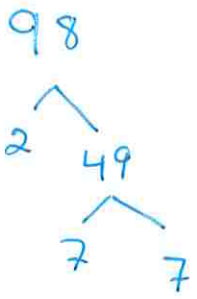
To find the greatest common factor of two or more numbers is to find the largest number that divides both or all given numbers.

Steps:

1. Write a prime factorization tree for each given number.
2. Write the prime factorization for each given number.
3. Circle factors that appear in both (or all) lists.
4. Find the product of these factors.
5. State the final answer.

Example: What is the greatest common factor of 28 and 98? Show all your work.



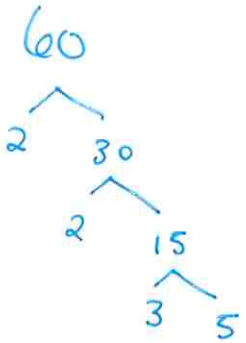



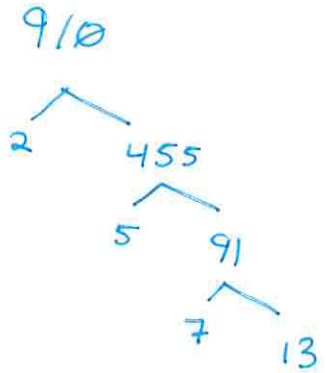

$28 = 2 \times 2 \times 7$

$98 = 2 \times 7 \times 7$

$2 \times 7 = \boxed{14}$

∴ therefor GCF (28, 98) = 14.




$60 = 2 \times 2 \times 3 \times 5$

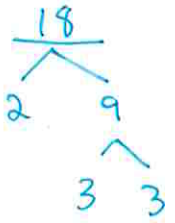
$910 = 2 \times 5 \times 7 \times 13$

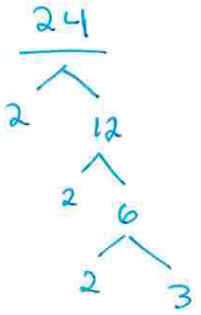
$2 \times 5 = \boxed{10}$

∴ GCF (60, 910) = 10.

Practice: Find the GCF for each pair of numbers. Show all your work.

18, 24




$18 = 2 \times 3 \times 3$

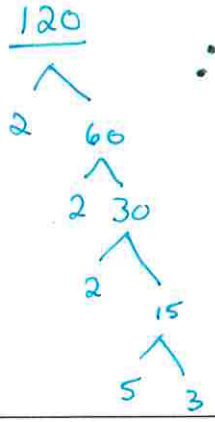
$24 = 2 \times 2 \times 2 \times 3$

GCF = 2x3

GCF = 6

∴ the GCF(18,24) = 6

35, 120

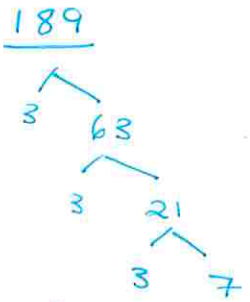

∴ the GCF(35,120) = 5

$35 = 5 \times 7$

$120 = 2 \times 2 \times 2 \times 5 \times 3$

GCF = 5

45, 189


$45 = 5 \times 3 \times 3$

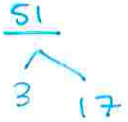
$189 = 3 \times 3 \times 3 \times 7$

GCF = 3x3

GCF = 9

∴ the GCF (45, 189) = 9

26, 51


$26 = 2 \times 13$

$51 = 3 \times 17$

*There is no noodles Therfore 1 is GCF*

∴ the GCF(26,51) = 1

**LCM – The least common multiple**

- To find the least common multiple of two numbers is to find the smallest number to which both numbers multiply to.
- There are two different methods to find the LCM
    - Listing all multiples before finding the common one.
    - Prime factorizing and selecting all unique factors and factors that repeat the most.

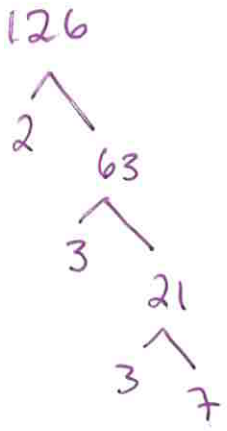
Example: Find the least common multiple of 6 and 14 by listing multiples.

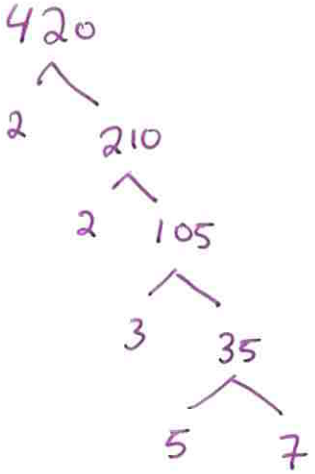
6: 6, 12, 18, 24, 30, 36, (42), 48, 54...

14: 14, 28, (42),

∴ lcm (6, 14) = 42

Example: Using prime factorization, find the least common multiple of 126 and 420.




126 = 2 × 3 × 3 × 7

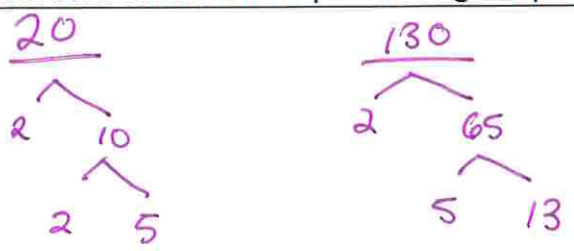
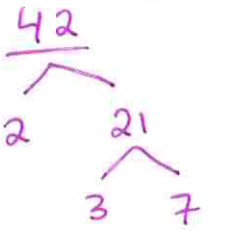
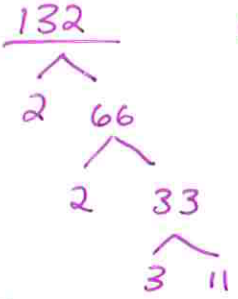
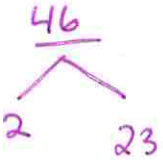
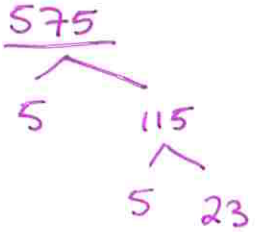
420 = 2 × 2 × 3 × 5 × 7

lcm = 3 × 3 × 2 × 2 × 5 × 7 = 1260

∴ Lcm (126, 420) = 1260

Practice:

Using prime factorization, find the least common multiple for the given pairs of numbers.

| | |
|---|---|
| 20, 130 |  <br>$20 = 2 \times 2 \times 5$<br>$130 = 2 \times 5 \times 13$<br>lcm $= 2 \times 2 \times 5 \times 13$<br>$= 260$<br>$\therefore$ lcm $(20, 130) = 260$ |
| 143, 1001 |  <br>lcm: $11 \times 13 \times 7 = 1001$<br>$\therefore$ lcm $(143, 1001) = 1001$ |
| 42, 132 |  <br>  <br>lcm $= 2 \times 2 \times 3 \times 7 \times 11$<br>lcm $= 924$<br>$42 = 2 \times 3 \times 7$<br>$132 = 2 \times 2 \times 3 \times 11$<br>$\therefore$ lcm $(42, 132) = 924$ |
| 46, 575 |  <br>  <br>$46 = 2 \times 23$<br>$575 = 5 \times 5 \times 23$<br>lcm $= 2 \times 5 \times 5 \times 23$<br>lcm $= 1150$<br>$\therefore$ lcm $(46, 575) = 1150$ |