FMPC10

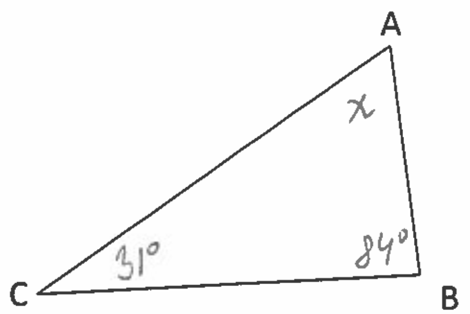
TRIGONOMETRY

Trigonometry is a branch of mathematics that studies properties of triangles and relationships between sides (lengths) and/or angles of those triangles.

REVIEW

- **In every triangle constructed on a 2-dimensional plane (=flat surface) the three interior angles will add up to __180°__ .**

Example: What is the degree measure of $\angle A$ if $\angle B = 84°$ and $\angle C = 31°$?

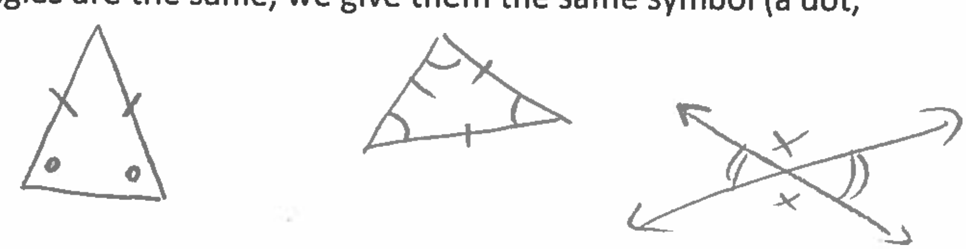


$x = 180° - 84° - 31°$

$x = 65°$

$\therefore \angle A = 65°$

- A triangle can have at most one angle greater than __90°__ . Such a triangle is called an obtuse triangle.
- A triangle that has all interior angles smaller than __90°__ is called an acute triangle.
- A triangle that has an angle of __90°__ is called a right-angled triangle, or a right triangle.
- A triangle that has at least two sides of the same size is called an __isosceles__ triangle. In such a triangle, at least two interior angles are also of the same size. A special case of such a triangle is a triangle that has all three sides of the same size.
- A triangle with all sides of the same size is called an __equilateral__ triangle. In this type of triangle, all three angles are the same and their degree measure is __60°__ .
- To show that sides are the same length, we cross the congruent (same) sides with a short stroke.
- To show that angles are the same, we give them the same symbol (a dot, cross, arc, …).



Rules for Labeling Angles and Sides

Every angle has a tip that is called a vertex.
To label an angle we can do one of these three things:

1. Use Greek letters:

α alpha

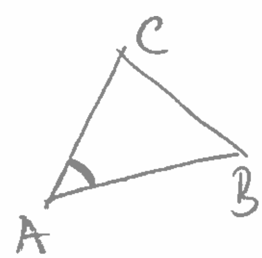
β beta

γ gamma

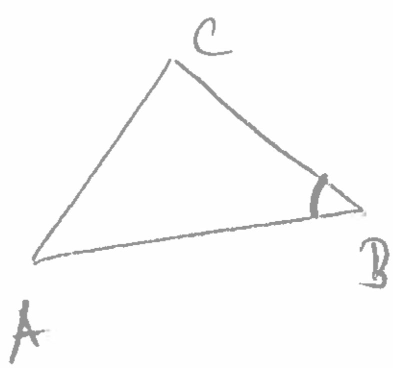
δ delta

θ theta

2. Use a symbol $\angle$ and a capital letter that is at the vertex: $\angle A$

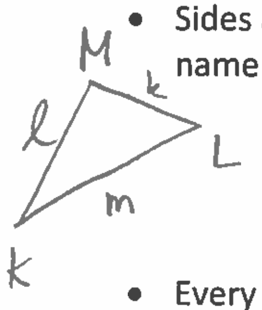


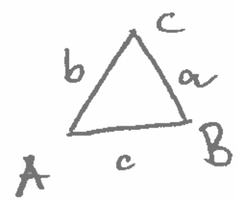
3. Use a symbol $\angle$ and three capital letters, where the middle letter is the vertex:



$\angle ABC$

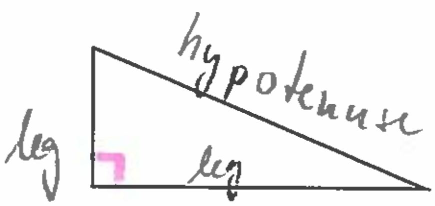
- Side lengths are line segments.
- Every side starts at a vertex of a triangle (corner) and goes to another vertex.
- Sides are labeled using **lower case letters.** The letter is the same as the name of the vertex opposite of the side.





Right-Angled Triangles

- Every right-angled triangle has its longest side across from the 90° angle.
- The longest side in the right-angled triangle is called the hypotenuse.
- The other two sides are called the legs.
- A special symbol in a diagram is used to show that the interior angle is the right angle.



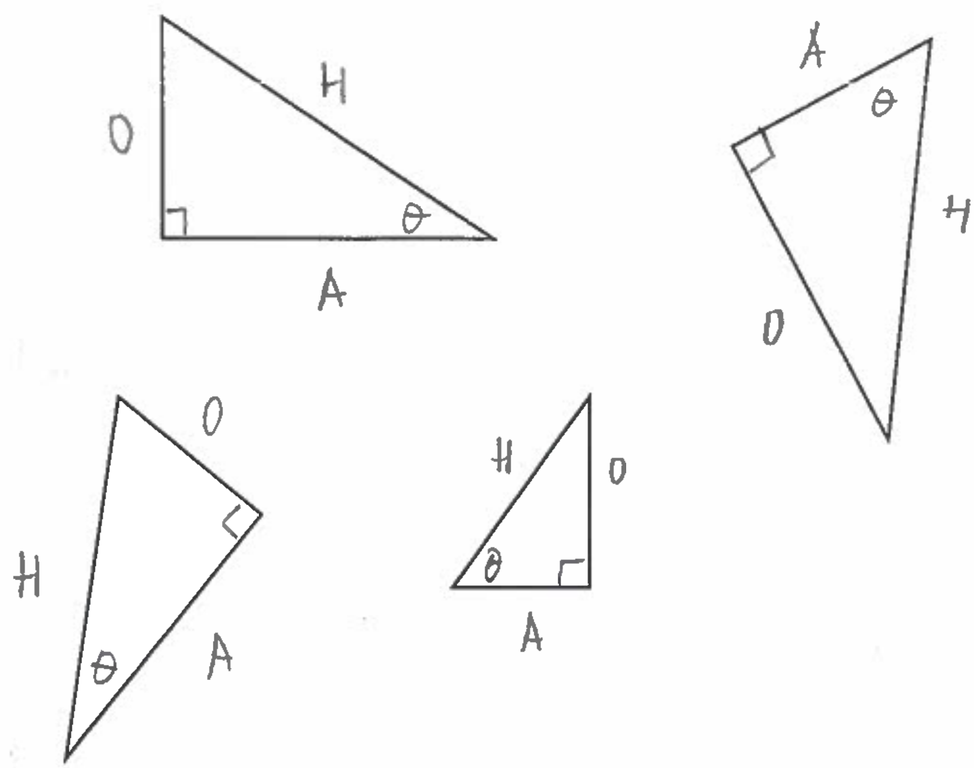
- In every right-angled triangle the following is true:

$$\boxed{\text{hypotenuse}^2 = \text{leg}^2 + \text{leg}^2}$$

$$\boxed{\text{leg}^2 = \text{hypotenuse}^2 - \text{leg}^2}$$

New labelling of right-angled triangles

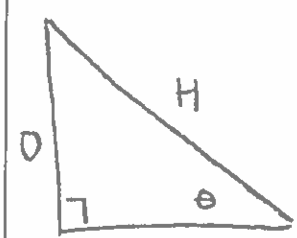
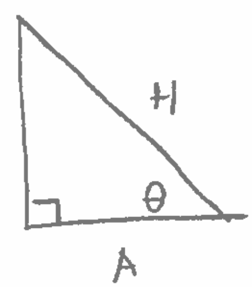
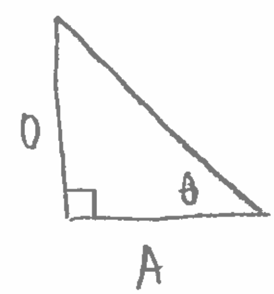
- When working with a right-angled triangle in which one of the acute angles is labeled. We label the sides in relation to this angle.
- For the longest side we use **H = hypotenuse** (this is the only label that has not changed).
- For the side that does not touch the labeled acute angle we use **O = opposite.**
- For the side that forms the angle but is shorter than the hypotenuse we use **A = adjacent.**



- **The relationship between the sides in a right-angled triangle goes beyond the Pythagorean Theorem.**

- **There are three specific ratios between two of the three sides. These ratios are called basic trigonometric ratios.**

BASIC TRIGONOMETRIC RATIOS

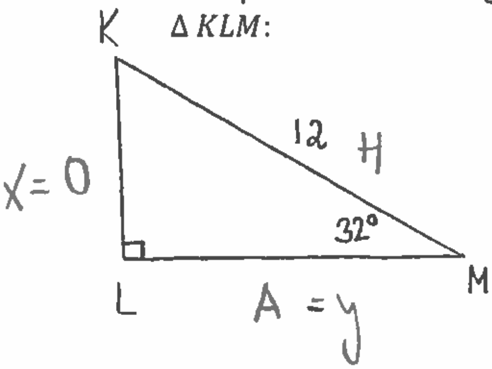
Name	Sine	cosine	tangent
Abbreviation	Sin	Cos	tan
Definition	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
Diagram			
Formula	$\sin\theta = \frac{O}{H}$	$\cos\theta = \frac{A}{H}$	$\tan\theta = \frac{O}{A}$

SOH – CAH – TOA

- The value of a trigonometric ratio does not have units. It is a number (a whole number, fraction or a decimal).
- Basic trigonometric ratios can be used to find the values for unknown side-lengths if one side and one acute angle in a right-angled triangle is known.
- **An inverse of a basic trigonometric ratio** can be used to find a degree measure of an unknown acute angle in a right-angled triangle when two sides are known.

SOH - CAH - TOA

Example 1: Without using the Pythagorean Theorem, find the two unknown sides in the ΔKLM:



Find x:

$\sin\theta = \frac{O}{H}$

$\sin 32° = \frac{x}{12}$

$12 \cdot \sin 32° = x$

$6.36 \doteq x$

Find y:

$\cos\theta = \frac{A}{H}$

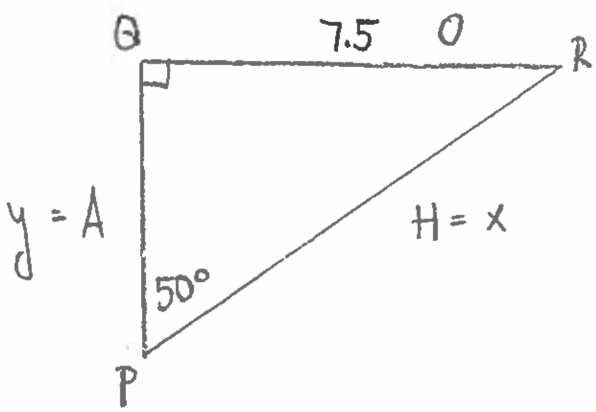
$\cos 32° = \frac{y}{12}$

$12 \cdot \cos 32° = y$

$10.18 \doteq y$

$\therefore$ the sides are : $x = 6.36$ and $y = 10.18$.

Example 2: Without using the Pythagorean Theorem, find the two unknown sides in the $\Delta\, PQR$:



Find x:

$\sin\theta = \frac{O}{H}$

$\sin 50° = \frac{7.5}{x}$

$x = \frac{7.5}{\sin 50°} \doteq 9.79$

Find y:

$\tan\theta = \frac{O}{A}$

$\tan 50° = \frac{7.5}{y}$

$y = \frac{7.5}{\tan 50°}$

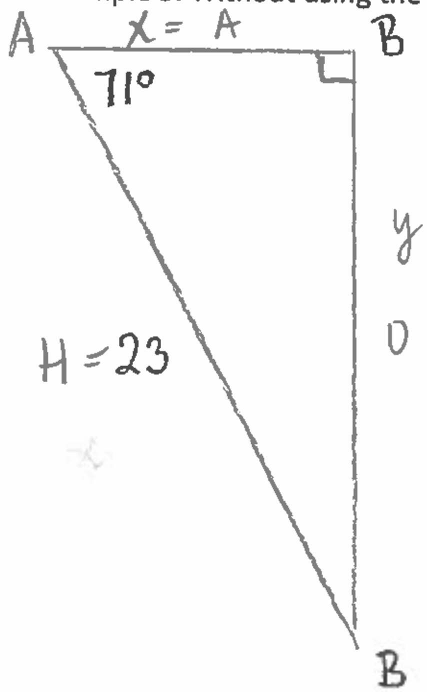
$y \doteq 6.29$

$\therefore$ the sides are : $x \doteq 9.79$

$y \doteq 6.29$

SOH - CAH - TOA

Example 3: Without using the Pythagorean Theorem, find the two unknown sides in the $\Delta\, ABC$:



Find x:

$$\cos\theta = \frac{A}{H}$$

$$\cos 71^\circ = \frac{x}{23}$$

$$23 \cdot \cos 71^\circ = x$$

$$x \doteq 7.49$$

Find y:

$$\sin\theta = \frac{O}{H}$$

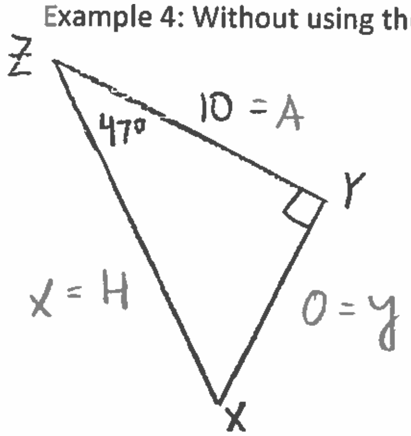
$$\sin 71^\circ = \frac{y}{23}$$

$$23 \cdot \sin 71^\circ = y$$

$$y \doteq 21.75$$

$\therefore$ The sides are: $x \doteq 7.49$ and $y \doteq 21.75$.

Example 4: Without using the Pythagorean Theorem, find the two unknown sides in the $\Delta\, XYZ$:



Find x:

$$\cos\theta = \frac{A}{H}$$

$$\cos 47^\circ = \frac{10}{x}$$

$$x = \frac{10}{\cos 47^\circ}$$

$$x \doteq 14.66$$

Find y:

$$\tan\theta = \frac{O}{A}$$

$$\tan 47^\circ = \frac{y}{10}$$

$$10 \cdot \tan 47^\circ = y$$

$$10.72 \doteq y$$

$\therefore$ The sides are $x \doteq 14.66$, $y \doteq 10.72$.