

# Answers

FMPC10

## SOLVING A SYSTEM OF LINEAR EQUATIONS

- There are four methods of solving a system of linear equations:
1. Trial-and-error: this is an easiest and least elegant way to solve a system of equations. It usually takes a long time.
  2. Substitution: this is a method most often used in sciences and applied math.
  3. Elimination: this is a method used in linear algebra.
  4. Graphing: this is often used in calculus and applied math. It usually requires graphing technology in order to obtain exact solutions.
- To find a solution to a system of linear equations is to find an ordered pair  $(x,y)$  that stands for the point of intersection of the two lines.

**Example 1:** Solve the system. Check your work by showing that left side=right side. (LS=RS). Do not move terms left-to-right and right-to-left when performing a check.

①  $y = 3x + 4$

②  $x - 2y - 2 = 0$

$$\frac{-2y}{-2} = \frac{-x+2}{-2}$$

$$y = \frac{1}{2}x - 1$$

$$x - 2(3x + 4) - 2 = 0$$

$$x - 6x - 8 - 2 = 0$$

$$-5x - 10 = 0$$

$$\frac{-5x}{-5} = \frac{10}{-5}$$

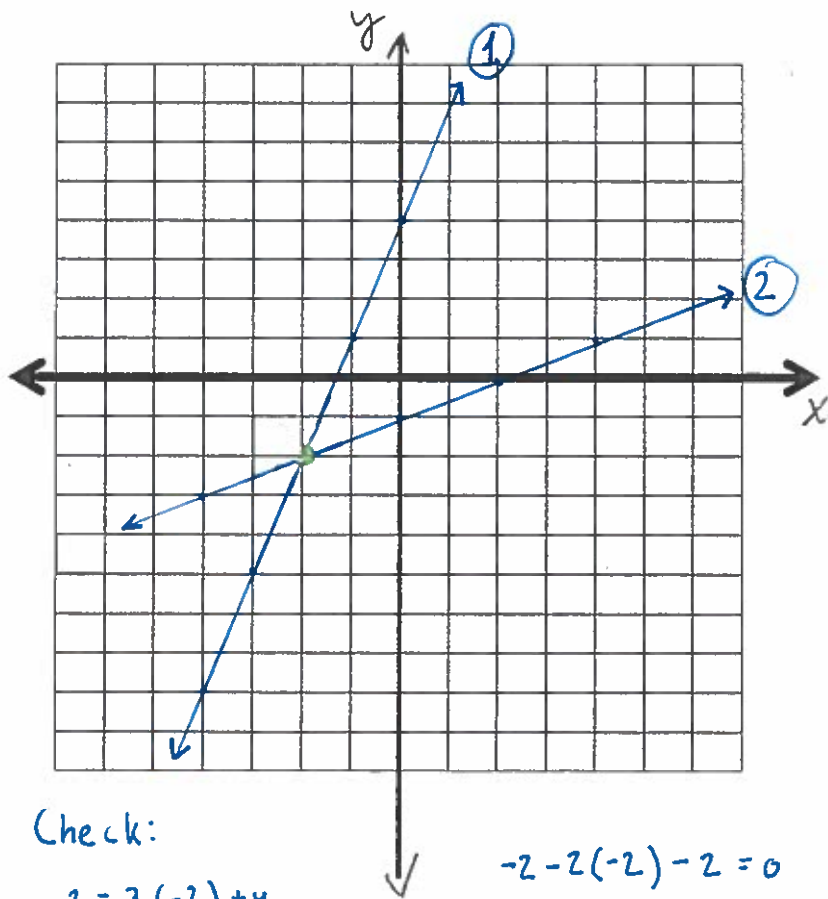
$$x = -2$$

$$y = 3(-2) + 4$$

$$y = -6 + 4$$

$$y = -2$$

∴ The solution is  $(-2, -2)$



Check:

$$-2 = 3(-2) + 4$$

$$-2 = -6 + 4$$

$$-2 = -2$$

$$LS = RS \checkmark$$

$$-2 - 2(-2) - 2 = 0$$

$$-2 + 4 - 2 = 0$$

$$0 = 0$$

$$LS = RS \checkmark$$

➤ If such point does not exist we say that the system does not have a real solution. This becomes apparent when the variables cancel out and the result is a false statement. If this system of equations was graphed, the lines would be parallel – that is they would have the same slope but a different y-intercept.

**Example 2:** Solve the system. Check your work by showing that left side=right side. (LS=RS). Do not move terms left-to-right and right-to-left when performing a check.

①  $y = \frac{2}{3}x - 4$

②  $2x - 3y + 3 = 0$

$$\begin{aligned} -3y &= -2x - 3 \\ \frac{-3y}{-3} &= \frac{-2x - 3}{-3} \\ y &= \frac{2}{3}x + 1 \end{aligned}$$

$$2x - 3\left(\frac{2}{3}x - 4\right) + 3 = 0$$

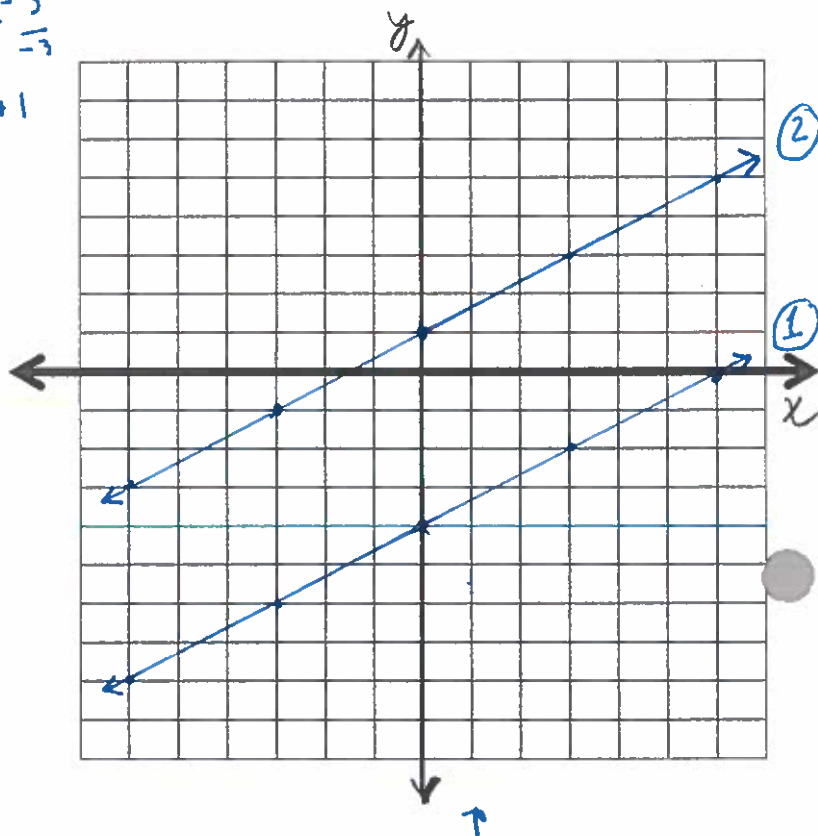
$$2x - 2x + 12 + 3 = 0$$

$$0 + 15 = 0$$

$$15 = 0$$

$$LS \neq RS$$

∴ There are No  $\mathbb{R}$  solutions



The lines have the same slope, they are parallel and do not intersect

- If the variables cancel out during the process of solving and the result is a true statement we say that the system has infinitely many real solutions. If this system of equations was graphed, the lines would be identical - that is they would overlap; they would have the same slope and same y-intercept.

➤ **Example 3:** Solve the system. Check your work by showing that left side=right side. (LS=RS). Do not move terms left-to-right and right-to-left when performing a check.

$$\begin{aligned} \textcircled{1} 1.5x + 3y &= 12 \rightarrow \frac{3y}{3} = \frac{-1.5x + 12}{3} \\ \textcircled{2} 2x + 4y - 16 &= 0 \end{aligned}$$

$$\frac{2x}{2} = \frac{16 - 4y}{2} \quad \frac{4y}{4} = \frac{-2x + 16}{4}$$

$$x = 8 - 2y \quad y = -\frac{1}{2}x + 4$$

$$y = -\frac{1}{2}x + 4$$

$$1.5(8 - 2y) + 3y = 12$$

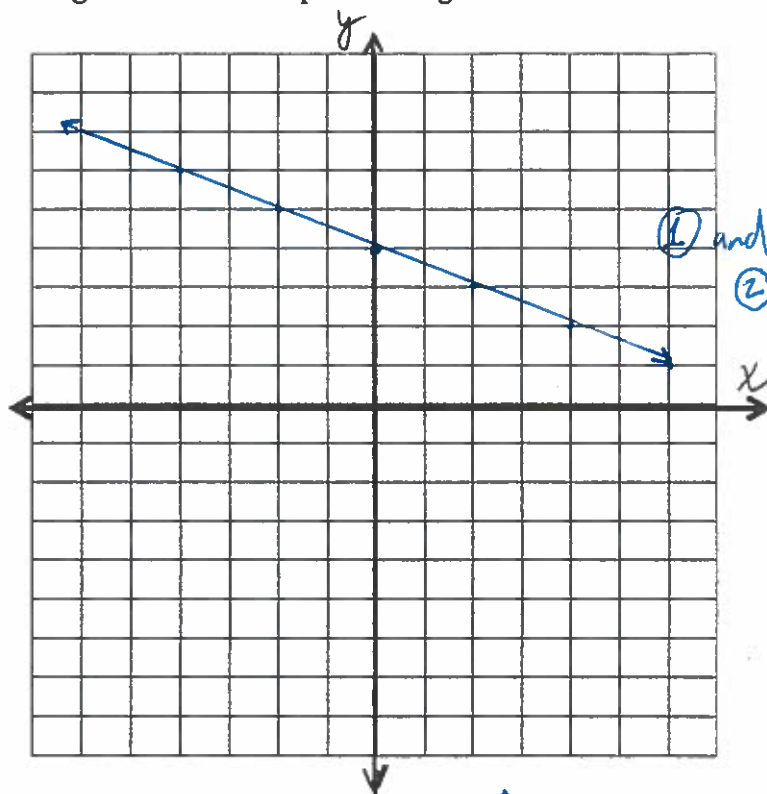
$$12 - 3y + 3y = 12$$

$$12 + 0 = 12$$

$$12 = 12$$

true statement  
and no variable

∴ There are infinitely many  
 $\mathbb{R}$  solutions



↑  
There are  $\infty$   
many points of  
intersection

